

**Sauder School of Business  
COMM 295 (107)**

**Solution to Midterm**

**PART I. MULTIPLE CHOICE ANSWERS (Use letters A, B, C, or D.) Put your answers here. Multiple choice answers placed elsewhere will not be marked.**

- |       |       |
|-------|-------|
| 1. C  | 11. A |
| 2. C  | 12. B |
| 3. D  | 13. A |
| 4. C  | 14. B |
| 5. B  | 15. B |
| 6. D  | 16. D |
| 7. A  | 17. A |
| 8. B  | 18. B |
| 9. C  | 19. A |
| 10. D | 20. D |

**Total Multiple Choice Marks: \_\_\_\_\_ / 40**

**PART II: Marks for Longer Questions: Put your answers in the indicated place on each page. Choose 4 questions out of 5.**

Question 1. \_\_\_\_\_

Question 2. \_\_\_\_\_

Question 3. \_\_\_\_\_

Question 4. \_\_\_\_\_

Question 5. \_\_\_\_\_

**Total Longer Question Marks: \_\_\_\_\_ / 60**

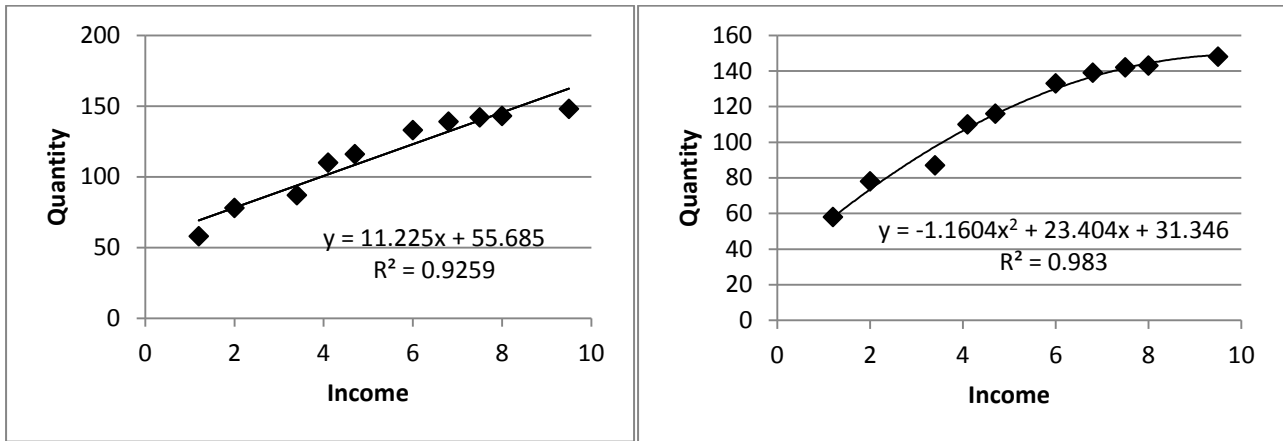
**Overall Mark: \_\_\_\_\_ / 100**

## Part II

### LONGER QUESTIONS

**Choose 4 questions out of 5.** If you do all 5 questions the last question will not be marked. Show your working and provide (brief) explanations where appropriate. Confine your answers to the space provided in the question. You may cross out an answer and do the entire answer somewhere else if necessary but the total space used cannot exceed the original space provided.

**1. Regression and Elasticity.** A supermarket chain used data from focus groups to estimate the effect of income on demand for gourmet ice cream in Vancouver. The following two diagrams show two regressions that were used by the market research team to assess the relationship between income and demand.



a. What are the main points illustrated by these two regression diagrams? List three points and explain them briefly. (Hint: What do these diagrams tell us about the relationship between income and quantity and what do you learn by comparing the two diagrams?) (7 pts)

- 1) Both graphs show that quantity demanded is increasing in income. Gourmet ice cream is therefore a normal good.
- 2) A quadratic regression line fits the data better than a linear regression line as reflected by the fact that the  $R^2$  is higher for the quadratic regression (on the right) than for the linear regression (on the left).
- 3) On the right graph--the effect of income on quantity demanded is diminishing in income but on the right graph it is constant.
- 4) A linear regression line causes us to overestimate demand at high income levels.

b. You are asked to provide your **best** estimate of the point income elasticity of demand at an income level of 10. What is your answer? (Hint: Recall that any point elasticity can be calculated using the form  $(dy/dx)(x/y)$ . In this case  $y$  is quantity and  $x$  is income.) Also explain briefly in words how you would calculate the arc income elasticity of demand for an increase in income from  $x_1$  to  $x_2$ . Provide a formula if you can. (8 pts).

We should use the quadratic version because it gives better estimate.

The elasticity  $E = dy/dx (x/y)$ .

$$dy/dx = 2(-1.1604)x + 23.404$$

$$\text{At } x = 10, dy/dx = 0.196 \text{ and } y = -116.04 + 234.04 + 31.346 = 149.346$$

$$\text{Thus } E = (0.196)10/149.346 = 0.013$$

To calculate an arc elasticity for a change from  $x_1$  to  $x_2$  we need to find the corresponding values of  $y_1$  and  $y_2$  and determine percentage change in quantity ( $y$ ) and the percentage change in income ( $x$ ). We divide the percentage change in quantity by the percentage change in income.

$$\text{Arc elasticity} = ((y_2 - y_1)/y^*)/(x_2 - x_1)/x^* \text{ where } y^* = (y_1 + y_2)/2 \text{ and } x^* = (x_1 + x_2)/2.$$

## 2. Cost Minimization

A bicycle parts company has production function  $Q = L K^2$  where  $K$  = machine units and  $L$  = labor hours. The wage rate,  $w$ , is \$15 per hour, and the rental rate on machines,  $r$ , is \$60 per unit.

a) At the current level of output, the marginal product of machines is 300 and the marginal product of labor is 200. Draw an appropriate diagram and explain how the firm should change the ratio of the two inputs to lower its cost of producing the current level of output. (7 pts)

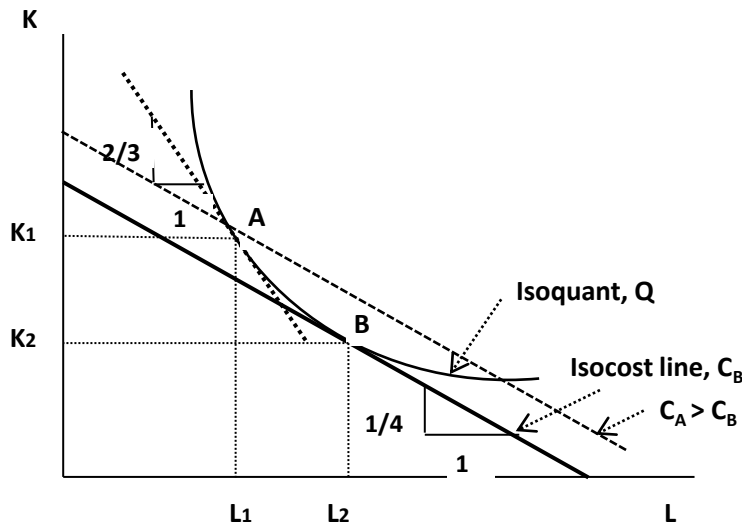
$$\text{Slope of isocost line } w/r = 15/60 = 1/4$$

At the current production level (point A in the graph),

$$\text{slope of isoquant, } MPL / MPK = 200/300 = 2/3$$

Since at A,  $MP_L / MP_K > w/r$  (i.e., one dollar spent on  $L$  produces more than a dollar spent on  $K$ ), the company should hire more  $L$  and less  $K$  (i.e., move towards B) to lower the cost of producing the given output  $Q$ .

(Alternative: At present  $MP_L/w = 200/15 = 13.33$ ;  $MP_K/r = 300/60 = 5$ . As the marginal product per dollar is higher for labour than for capital the firm can lower costs by using more labour and less capital – reducing the capital to labor ratio.)



b) Determine how much L and K the firm would need to use to minimize the cost of producing 2000 units. What is this cost? (8 pts)

$$\begin{aligned} \text{For cost minimization: } MP_L / w &= MP_K / r \\ K^2 / 15 &= 2LK / 60 \\ L &= 2K \end{aligned}$$

$$\begin{aligned} \text{Substitute this value into production function, } Q &= L K^2 = 2000 \\ 2K^3 &= 2000 \\ K &= 10 \text{ and } L = 20 \end{aligned}$$

$$\text{Min Cost of producing 2000 units} = wL + rK = 15 \cdot 20 + 60 \cdot 10 = \$900$$

### 3. Perfect Competition.

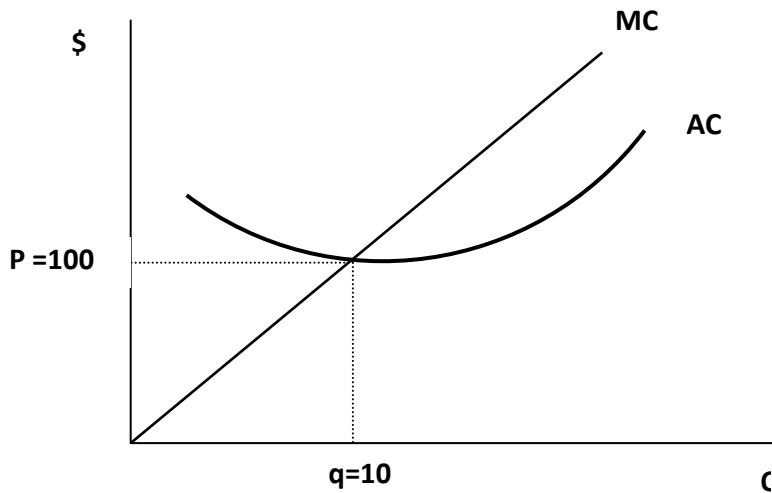
a) The long run equilibrium price in a perfectly competitive market is \$100 and the total quantity traded is  $Q=5000$ . Each firm in the market has a cost of production given by  $C(q) = 500 + 5q^2$ . Derive a firm's average cost curve and marginal cost curve and illustrate these curves in an appropriate diagram. Assuming the industry is in long run equilibrium, calculate and show the quantity produced by each firm. How many firms are in the market? (8 pts).

$$\begin{aligned} AC &= C/q = 500/q + 5q \\ MC &= dC/dq = 10q \end{aligned}$$

$$\begin{aligned} \text{At long run equilibrium, } p &= MC = AC \\ 100 &= 10q \\ q &= 10 \end{aligned}$$

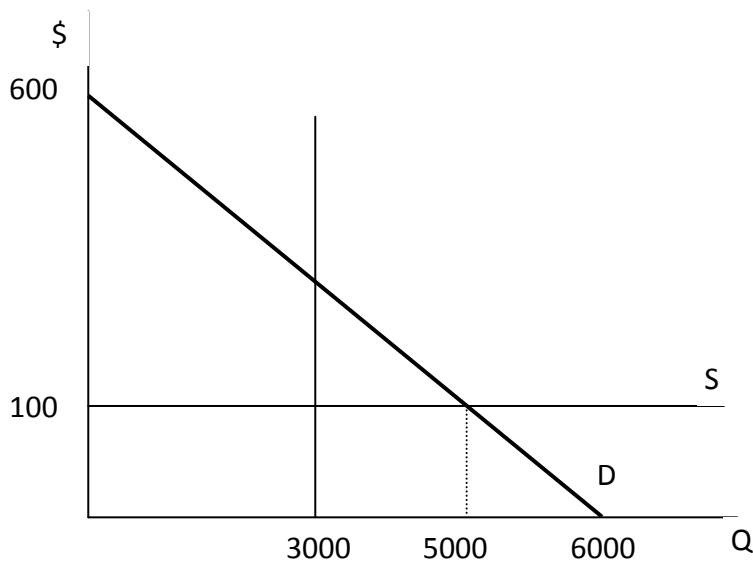
$$\text{Number of firms in the market } n = Q/q = 5000/10 = 500.$$

(Or set  $MC = AC$  to get  $10q = 500/q + 5q$  so  $q = 10$ ; you could also minimize  $AC$  to get  $dAC/dq = -500/q^2 + 5 = 0$  so  $q = 10$ )



b) In a perfectly competitive market, the market demand curve is  $Q = 6000 - 10p$  and the supply curve is perfectly elastic at a price of 100. Illustrate the solution in a supply-demand diagram. Determine total consumer surplus and producer surplus in this market and explain why total surplus falls if output is restricted to  $Q=3000$ . (7 pts)

At  $P = 100$ ,  $Q = 6000 - 10(100) = 5000$ .



$CS = \frac{1}{2}(600 - 100)5000 = 1,250,000$ .

$PS = 0$  (note area under P and above MC. Since P and MC coincide  $PS = 0$ )

If output is restricted 3000, total surplus falls as shown in diagram because for  $Q = 3000$  to  $5000$  marginal willingness to pay as shown by the demand curve exceeds marginal cost as shown by the supply curve but these quantities are not produced, leading to deadweight loss.

#### 4. Pricing

a. A local theatre has found that students have a different demand curve from everybody else. Student inverse demand has been estimated to be  $p_s = 20 - 0.1Q_s$ . The inverse demand curve for all other consumers is  $p_e = 50 - 0.25Q_e$ . The movie theatre's cost function is  $C(Q) = 200 + 5Q$  where  $Q = Q_s + Q_e$ . Find the profit-maximizing price to charge each group. (No diagram is needed.) Briefly explain what the resale problem is and how it can be avoided in this case. (8 pts).

$$MC=5$$

For profit maximization,  $MR_s = MC$  and  $MR_e = MC$

Students:  $MR_s = MC$   
 $20 - 0.2Q_s = 5$   
 $Q_s = 75$   
 $P_s = \$12.50$

Others:  $MR_e = MC$   
 $50 - 0.5Q_e = 5$   
 $Q_e = 90$   
 $P_e = \$27.50$

Resale problem: If students can buy tickets for \$12.50 they would be able to resell them to everyone else for some amount greater than \$12.50 but not more than \$27.50. This can be avoided by asking for ID when entering the theatre to ensure the ticket holder is a student.

b. Joey's snack shack is trying to come up with a strategy to price its customer's favorite snacks (crackers and cheese). The table below shows the willingness to pay of the four different types of consumers that buy snacks at Joey's. Assume costs are zero.

Consumer type	Crackers	Cheese
A	3	1
B	3	2.50
C	2.50	3
D	1	3

Determine Joey's maximum profit if he prices crackers and cheese individually, if he uses pure bundling, and if he uses mixed bundling. Which approach yields the highest profit. Show your working. (7 pts)

Individual pricing: profit max at  $P_{\text{crackers}} = \$2.50$  and  $P_{\text{cheese}} = 2.50$   
 $\Pi = 3 * 2.50 + 3 * 2.50 = \$15$

Pure bundling: profit max at  $P_{\text{bundle}} = \$4$   
 $\Pi = 4 * 4 = \$16$

Mixed bundling: profit max at  $P_{\text{bundle}} = \$5.50$  and  $P_{\text{crackers}} = \$3$  and  $P_{\text{cheese}} = \$3$   
 $\Pi = 2 * 5.50 + 1 * 3.00 + 1 * 3.00 = \$17$

Therefore mixed bundling would yield the highest profit.

**5. Cournot Oligopoly.** Consider a market with linear demand and two identical Cournot duopoly firms (A and B). The firms produce a homogenous commodity with a constant marginal cost of 10. Market demand is given by inverse demand function  $P = 110 - Q$  where  $P$  is the market price and  $Q$  is total production by the two firms (i.e.,  $Q = q_A + q_B$ ).

a. Derive the Cournot best response curves for the two firms. Calculate the Nash-Cournot equilibrium price and quantity. (No diagram is needed.) (8 pts)

$$P = 110 - q_A - q_B.$$

$$\text{Set } MR_A = MC$$

$$110 - 2q_A - q_B = 10.$$

The best response curve is therefore  $q_A = 50 - 0.5q_B$ .

By symmetry,  $q_B = 50 - 0.5q_A$ .

Substitute  $q_B = 50 - 0.5q_A$  into  $q_A = 50 - 0.5q_B$  to obtain:

$$q_A = 50 - 0.5(50 - 0.5q_A)$$

$$q_A^* = 100/3 = 33.33 \text{ and } q_B = 50 - 0.5q_A = 33.33$$

$$Q = 33.333 + 33.333 = 66.666$$

Substitute the value of  $Q$  into  $P = 110 - Q = \$43.333$ .

b. Find the profit-maximizing price and quantity if the two firms operate as a cartel. How would formation of a cartel affect consumer surplus? Illustrate this effect using a diagram containing a demand curve and the relevant prices and quantities under the two situations: Cournot duopoly and a profit-maximizing cartel. Show the difference in consumer surplus on the diagram. (7 pts).

$$P = 110 - Q, \text{ so } MR = 110 - 2Q.$$

Profit maximization requires  $MR = MC$

$$110 - 2Q = 10.$$

$$Q^* = 50 \text{ and } P^* = 110 - Q = 110 - 50 = \$60.$$

