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* ALWAYS REMEMBER TO ADD C *

MAT1330 DGD

Nov 22

Newton's Method

Recall: Intermediate Value Theorem (IVT),

Let f be cts on [a, b] and let $y \in [f(a), f(b)]$
Then there is an $x \in [a, b]$ s/t $f(x) = y$

Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

① Lets show $f(x) = x^3 + x^2 + 3x + 2$, $x \in (-\infty, \infty)$, has exactly one zero.

a) Use the IVT to show there is exists (at least) one zero
b) Use the Mean Value Theorem (MVT) to show if there are two (or more) zeros, then f has (at least) one critical point
c) Show that f does not have a critical pt.
d) Put a, b, c together to conclude f has exactly one zero
e) Estimate the root, accurate to 3 decimals using Newton's Method.

Recall: Mean Value Theorem;

If f is cts on [a, b], differentiable on (a, b), there exists some $c \in (a, b)$ s/t

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Rolle's Theorem: If $f(a) = f(b)$, there is a c s/t $f'(c) = 0$

Solution:

a) $f(-1) = -1 + 1 - 3 + 2$ → any point
$f(0) = 2 > 0$ → any point

∴ @ [−1, 0], f is cts, so by IVT, since $0 \in [f(-1), f(0)]$
$= [-1, 2]$, there is som $x \in [-1, 0]$ s/t $f(x) = 0$

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b) Suppose there's at least two zeros say $a \neq b$ s/t $f(a)=f(b)=0$ (assume $a<b$)
So by Rolle's Thm, there is a c, $a<c<b$ s/t
$f'(c)=0$

So f has a critical pt: (at least one, at c).

* has to be an interval that goes from ⊕ to ⊖, or ⊖ to ⊕

* critical points are not the zeros *

c) Let's show there is no critical point
$f'(x) = 3x^2+2x+3$
③ $b^2-4ac = 4-4\cdot3\cdot3 = -20<0$
So $f'(x) \neq 0$ for all x

(b)+(c) ∴ If there are two zeros, then there must be critical pt. But we know there are no critical pts

* e) Let $x_0 = -0.5$ (We know there is a zero between -1 and 0) *
$f(x) = x^3+x^2+3x+2$
$f'(x) = 3x^2+2x+3$
② So $x_{n+1} = x_n - \frac{x_n^3+x_n^2+3x_n+2}{3x_n^2+2x_n+3}$
Plug into calculator
$x_1 \doteq -0.727272\ldots$
$x_2 \approx -0.715277944$
a) $x_3 \approx -0.71522524$
$x_4 \approx -0.7152225238$
(do it till the 1st 3 decimals are the same)

So Newton's Method gives the roots approx $x \approx -0.715$

* A and B statement *
$A = \neg B$
* If there is two zeros, then there is a critical point.
If there is no critical points, then there is no more than one zeros
at most one. (if $f'(x) \neq$ ...
A = "it is night time"
B = "the sun is down"
If the "sun is up" = "sun is not down"
∴ then it must be "daytime" "not night"

* Since there are no critical points, there are no more than 1 zeros (i.e. at most 1)
So there are at least one and at most one zeros, so f has exactly one zero