University of Ottawa
Department of Mathematics and Statistics

MAT1322

Calculus II

Midterm test 2

October 30, 2019

Instructor: Vadim Kaimanovich

Duration: 75 minutes

Read the following information before starting the test:

- Verify that your copy of the test contains 5 pages, including this one.
- Write your name and student number on this page.
- Work the problems in the space provided. Use the back-pages and the blank sheet attached at the end for rough work. Do not use any other paper. Before submitting the test remove the rough work page 5.
- Show all work, clearly and in order, if you want to get full credit. Points may be taken off if it is not clear how you arrived at your answer (even if your final answer is correct).
- Please keep your written answers brief; be clear and to the point. Points may be taken off for rambling and for incorrect or irrelevant statements.
- Circle your final answers.

The Faculty of Science requires that you read and sign the following statement:

Cellular phones, calculators or other electronic devices and course notes are not allowed during this test. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the test.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

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Last Name: ______________________________

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Problem	1(2)	2(2)	3(2)	4(2)	5(2)	6(2)	Total(12)
Points							

ID checked:

1. Determine whether the sequence $a_n = \ln(2n^2 - 1) - \ln(n^2 + 3)$ converges or diverges; if it converges, find its limit L.

A. converges, $L = -\ln 2$ B. diverges, $L = \ln 2$ C. converges, $L = \infty$ D. converges, $L = \ln(-2)$
E. converges, $L = 2$ F. diverges G. converges, $L = \ln 2$ H. none of the above

$$a_n = \ln \frac{2n^2-1}{n^2+3} = \ln \frac{2 - 1/n^2}{1 + 3/n^2} \to \ln 2$$

2. Write a formula for the n-th entry of the sequence $a_1 = -1/3, a_2 = 2/9, a_3 = -4/27, a_4 = 8/81, \ldots$ and find $S = \sum_{n=1}^{\infty} a_n$.

A. $a_n = (-1)^n 2^n / 3^n, S = -2/5$ B. $a_n = (-1)^{n-1} 2^{n-1} / 3^n, S = -1/5$ C. $a_n = 2^n / 3^n, S = -1/5$
D. $a_n = (-1)^n 2^n / 3^n, S = -1/5$ E. $a_n = (-1)^n 2^{n-1} / 3^n, S = -1/5$ F. none of the above

The sequence a_n is a geometric series with the initial term $a_1 = -\frac{1}{3}$ and ratio $r = -\frac{2}{3}$, so that

$$a_n = a_1 \cdot r^{n-1} = -\frac{1}{3} \cdot \left(-\frac{2}{3}\right)^{n-1} = (-1)^n \frac{2^{n-1}}{3^n};$$

$$S = \frac{a_1}{1-r} = \frac{-1/3}{5/3} = -\frac{1}{5}$$

3. Which of the following series converge? (I) $\sum_{k=1}^{\infty} \frac{k \cos k}{1+k^3}$, (II) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{n}{n+2}$ (III) $\sum_{n=1}^{\infty} \frac{3n^2+n}{2n^3-1}$

A. all these series B. none of these series C. (I) only D. (II) only E. (III) only
F. (I) and (II) G. (I) and (III) H. (II) and (III) I. none of the above

(I): $\left|\frac{k \cos k}{1+k^3}\right| \le \frac{k}{1+k^3} \le \frac{1}{k^2} \Rightarrow$ the series converges by comparison with $\sum \frac{1}{k^2}$

(II) $\left|(-1)^n \frac{n}{n+2}\right| = \left|\frac{n}{n+2}\right| \to 1 \neq 0 \Rightarrow$ the series diverges

(III) $\frac{3n^2+n}{2n^3-1} : \frac{1}{n} = \frac{3n^3+n^2}{2n^3-1} \to \frac{3}{2} \Rightarrow$ the series diverges by comparison with the harmonic series

4. Choose one of the following descriptions of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n \sin n}{3^n+2n}$

A. alternating absolutely convergent B. absolutely conditionally convergent C. alternating divergent
D. absolutely divergent E. geometric divergent F. alternating conditionally convergent
G. absolutely convergent H. geometric convergent I. none of the above

The series is not alternating as $\sin n$ changes the sign irregularly; since $\left|(-1)^n \frac{2^n \sin n}{3^n+2n}\right| \le \frac{2^n}{3^n+2n}$, and by applying the ratio test to $\sum \frac{2^n}{3^n+2^n}$, it is absolutely convergent;
The series is not geometric as the ratio of the consequtive terms is non-constant

5. What is the minimal number of terms n of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ one should take so that the error R_n be less than 0.01?

A. 10 B. 11 C. ∞ D. 15 E. 14 F. 8 G. 7 H. none of the above

Here $a_n = f(n)$ with $f(x) = \frac{1}{x^3}$ which is non-negative, monotone, so that

$R_n \le \int_n^\infty f(x)\,dx = \frac{1}{2n^2}$, so that one has to find the minimal n with $\frac{1}{2n^2} < 0.01$, i.e. $n^2 > 50$, whence $n = 8$

6. Estimate the error when the series $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \dots$ is approximated by its first 50 terms.

A. $\frac{1}{50}$ B. $-\frac{2}{101}$ C. $\frac{2}{103}$ D. $\frac{1}{103}$ E. 0 F. $\frac{2}{101^2}$ G. none of the above

This is an alternating series with $a_1 = \frac{2}{3}$, $a_2 = -\frac{2}{5}$, —, $a_{50} = -\frac{2}{101}$, $a_{51} = \frac{2}{103}$, whence $R_{50} \le |a_{51}| = \frac{2}{103}$

extra page for calculations (please remove it when submitting the test!)