Random Variables

Independence:

Two random variables, X and Y, are said to be **independent** if

$$P(X \text{ and } Y) = P(X) \cdot P(Y)$$

for **all** possible values of X and Y.

Example:

Consider the experiment of tossing a coin two times. A convenient sample space is
{HH, HT, TH, TT}.

Let X denote the number of heads on the first toss and Y the number of heads on the second toss. Then the **joint distribution of X and Y** is given by:

		y	
		0	1
x	0	.25	.25
	1	.25	.25

Are X and Y independent? To determine this, compute the marginal probabilities P(x) and P(y). Each is given by P(0) = 1/2, P(1) = 1/2.

Entering this in the table we have:

		y		
		0	1	P(x)
x	0	.25	.25	.5
	1	.25	.25	.5
	P(y)	.5	.5	1.0

Hence for all cells in the joint probability distribution, the probability is equal to the product of the marginals. This means that X and Y are independent as expected.

What does independence tell us? Essentially, that knowledge of the value of one random variable tells us nothing about the value of the second random variable, or that the two variables vary separately.

Problems:

7.3.a Given the bivariate probability distribution:

		y		
		1	2	3
x	1	.2	.1	.2
	2	.3	.2	0

Compute

(i) the marginal distribution of X, the marginal distribution of Y
(ii) E(X) and E(Y)
(iii) the probability X=1 and Y=2
(iv) the probability X=2 and Y$\leq$2
(v) E(X+Y)
(vi) σ^2(X+Y)

7.3.b A farm equipment dealer has obtained the probabilities of selling X tractors on day 1 and Y tractors on day 2. His findings are summarized below.

y_j / x_i	0	1	2
0	0.1	0.1	0
1	0.1	0.2	0.2
2	0	0.2	0.1

(i) Find the marginal distributions.
(ii) Are the two random variables X and Y independent?
(iii) Find the average number of tractors sold on day 1. Find the variance of the number of tractors sold on day 1.
(iv) Find the distribution of the sales on two successive days. Calculate the expectation and the variance of this distribution.
(v) Suppose that each tractor sold has a selling price of $35,000 and a cost of $20,000. Also assume that daily overhead costs are $3,000 per day. Determine the average profit over the two day period. Determine the standard deviation of profit over the two day period.

7.3.c A construction project consists of four jobs - A, B, C, and D. Jobs B and C cannot be begun until A is completed, and D cannot be begun until C is completed. The project is completed when all four jobs are completed. The completion times of the four jobs are uncertain. The completion time of job A is independent of the completion times of B, C, and D, and the completion time of B is independent of the completion times of A, C, and D. The distributions of the completion times of job A and B are as follows:

Job A

Completion Time	1 week	2 weeks
Probability	0.4	0.6

Job B

Completion Time	3 weeks	4 weeks
Probability	0.5	0.5

The completion times of jobs C and D are dependent on one another but independent of the completion times for A and B. The joint probabilities of these two are given in the following probability table:

		Job C Completion Time	
		1 week	2 weeks
Job D Completion Time	2 weeks	0.2	0.4
	3 weeks	0	0.4

(i) What is the probability distribution for the total project completion time?

The firm undertaking this project will receive $10,000 for completing the job, unless it takes more than 5 weeks, in which case they get only $9,500. Total costs will be $4,000, plus $500 for every week that it takes to complete the project (labor costs), plus $500 for every week that it takes to complete job C (because C requires the rental of machinery).

(ii) What is the probability distribution of the firm's net contribution for this project?

7.4.c Two firms are bidding for the contract to repair the heating and air conditioning system in the H. Angus building. The maintenance director in charge of the H. Angus building has assessed the probability distribution for the bid price of two firms, Point Grey Plumbing and Heating (PGPH) and University Contractors Ltd. (UCL) as:

PGPH		UCL	
Bid Price	Probability	Bid Price	Probability
10,000	0.5	10,000	0.2
13,000	0.5	12,000	0.4
		15,000	0.4

Assume independence between the bid prices.

a) Why is the independence assumption between the two bid prices a reasonable assumption?

b) Find the probability distribution for the winning (lowest) bid price.

c) What is the expected value of the winning bid?

d) What is the variance of the winning bid?

e) Is the winning bid independent of the PGPH bid? Provide satisfactory evidence.

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