14 QUESTION ONE/QUESTION THREE

7 a) Test for model significance. What are your conclusions? Use $\alpha = 0.05$.

H_0: The fitted model is not significant
H_1: The fitted model is significant 1

Test Statistic: $F = \dfrac{MSreg}{MSres}$ 1

Rejection Region: Reject the null if $F > F_{p,n-p-1,\alpha}$ 1

$$F = \frac{828.66}{17.53} = 47.27$$ 2

$F_{p,n-p-1,\alpha} = F_{3,8,0.05} = 4.07$ 1

Since 47.27 > 4.07, reject the null and conclude that the fitted model is significant at a 5% level of significance. 1

7 b) Test for model adequacy. What are your conclusions? Use $\alpha = 0.05$.

H_0: The fitted model has no bias due to model inadequacy
H_1: The fitted model has bias due to model inadequacy 1

Test Statistic: $F = \dfrac{MSLF}{MSPE}$ 1

Rejection Region: Reject the null if $F > F_{\nu LF,\nu PE,\alpha}$ 1

$$F = \frac{23.87}{6.97} = 3.42$$ 2

$F_{p,n-p-1,\alpha} = F_{5,3,0.05} = 9.01$ 1

Since 3.42 < 9.01, do not reject the null and conclude that the fitted model has no bias due to model inadequacy at a 5% level of significance. 1

22 QUESTION TWO

9 a) Using the provided information and the matrix solution to linear least squares, estimate the coefficients in the proposed model and write out the fitted model.

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{Y}$$

$$\mathbf{X} = \begin{bmatrix} 1 & 1.12 \\ 1 & 1.235 \\ 1 & 1.27 \\ 1 & 1.38 \\ 1 & 1.465 \\ 1 & 1.49 \end{bmatrix} \quad 2$$

$$\mathbf{X}^T\mathbf{Y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1.12 & 1.235 & 1.27 & 1.38 & 1.465 & 1.49 \end{bmatrix} \begin{bmatrix} 1.00 \\ 1.20 \\ 1.35 \\ 1.65 \\ 1.90 \\ 2.05 \end{bmatrix}$$

$$= \begin{bmatrix} 9.15 \\ 12.4315 \end{bmatrix} \quad 3$$

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} 17.2572 & -12.8823 \\ -12.8823 & 9.7103 \end{bmatrix} \begin{bmatrix} 9.15 \\ 12.4315 \end{bmatrix}$$

$$= \begin{bmatrix} -2.243 \\ 2.841 \end{bmatrix} \quad 3$$

Therefore, $\hat{Y} = -2.243 + 2.841X$. 1

9 b) From a previous experiment, you have an estimate of the pure error variance of $(s_e)^2 = 0.0061$ with 10 degrees of freedom. Using this additional information, calculate the 95% confidence interval for the predicted mean clotting time for a heparin concentration of 1.2 mmol/L.

The covariance matrix for the parameter estimates is $\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\hat{\sigma}^2$ or $\left(\mathbf{X}^T\mathbf{X}\right)^{-1}s_e^2$.

$$(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}s_e^2 = \begin{bmatrix} 17.2572 & -12.8823 \\ -12.8823 & 9.7103 \end{bmatrix}(0.0061)$$

$$= \begin{bmatrix} 0.1052 & -0.0786 \\ -0.0786 & 0.0592 \end{bmatrix} \quad 2$$

$$\begin{aligned} \hat{y}_k &= -2.243 + 2.841x_k \\ &= -2.243 + 2.841(1.2) \\ &= 1.17 \quad 1 \end{aligned}$$

$$\begin{aligned} \mathrm{var}(\hat{y}_k) &= (1)^2\,\mathrm{var}(\hat{\beta}_0) + (1.2)^2\,\mathrm{var}(\hat{\beta}_1) + 2(1)(1.2)\mathrm{cov}(\hat{\beta}_0, \hat{\beta}_1) \\ &= 0.1052 + 1.44(0.0592) + 2.4(-0.0786) \\ &= 0.0018 \quad 3 \end{aligned}$$

95% confidence interval is:

$$\begin{aligned} \hat{y}_k \pm t_{v,\alpha/2}\sqrt{\mathrm{var}(\hat{y}_k)} &= 1.17 \pm t_{10,0.025}\sqrt{0.0018} \\ &= 1.17 \pm 2.228(0.0424) \\ &= 1.17 \pm 0.09 \\ &\Rightarrow [1.08, 1.26] \quad 2 \end{aligned}$$

$$t_{10,0,025} = 2.228 \quad 1$$

4 c) Using $(s_e)^2$ from part (b), calculate a 95% confidence interval for the slope. What are your conclusions?

95% confidence interval for the slope is:

$$\begin{aligned} \hat{\beta}_1 \pm t_{v,\alpha/2}s_{\hat{\beta}_1} &= 2.841 \pm t_{10,0.025}\sqrt{0.0592} \\ &= 2.841 \pm 2.228(0.2433) \\ &= 2.841 \pm 0.542 \\ &\Rightarrow [2.299, 3.383] \quad 3 \end{aligned}$$

Conclusion: The slope is significant since the 95% CI does not contain zero. 1

14 QUESTION THREE/QUESTION ONE

5 a) Estimate the pure error variance for this experiment. How many degrees of freedom does it have?

$$\overline{X}_e = \frac{85+89+88}{3} = \frac{262}{3} = 87.333 \quad 1$$

$$s_e^2 = \frac{(85-87.333)^2 + (89-87.333)^2 + (88-87.333)^2}{3-1} = \frac{8.667}{2} = 4.333 \quad 3$$

The pure error variance has 2 degrees of freedom. 1

7 b) Complete the ANOVA table corrected for the mean.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Regression	302.750		50.458
Residual	11.795		2.949
Lack of Fit			1.564
Pure Error			4.333
Total		10	

$$df_{reg} = \frac{SS_{reg}}{MS_{reg}} = \frac{302.75}{50.458} = 6 \quad 1$$

$$df_{resud} = \frac{SS_{resid}}{MS_{resid}} = \frac{11.795}{2.949} = 4 \quad \text{or} \quad df_{resid} = df_{tot} - df_{reg} = 10 - 6 = 4 \quad 1$$

$df_{PE} = 2$ from part (a) 1

$df_{LF} = df_{resid} - df_{PE} = 4 - 2 = 2$ 1

$SS_{LF} = df_{LF}(MS_{LF}) = 2(1.565) = 3.13$ 1

$SS_{PE} = df_{PE}(MS_{PE}) = 2(4.333) = 8.666$ or $SS_{PE} = SS_{resid} - SS_{LF} = 11.795 - 3.13 = 8.665$ 1

$SS_{tot} = SS_{reg} + SS_{resid} = 302.75 + 11.795 = 314.545$ 1

2 c) What is the R^2 for the model fit? What does it mean?

$$R^2 = \frac{SS_{reg}}{SS_{tot}} = \frac{302.75}{314.545} = 0.963 \quad 1$$

This means that the fitted model is explaining 96.3% of the variation in the response. 1