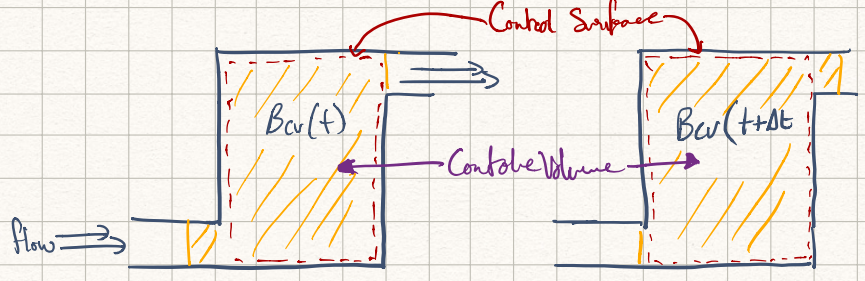
Controle volume / Continuity

$$\frac{dB_{sys}}{dt} = \frac{dB_{cv}}{dt} + (B_{out} - B_{in})$$

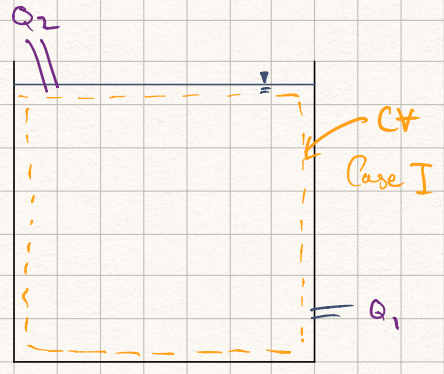
[Rate of change of Property B of system] = [Rate of change of Prop B in Control volume] + [net out flow of property B through Control surface]



$$0 = \frac{dm_{cv}}{dt} + (\dot{m}_{out} - \dot{m}_{in})$$

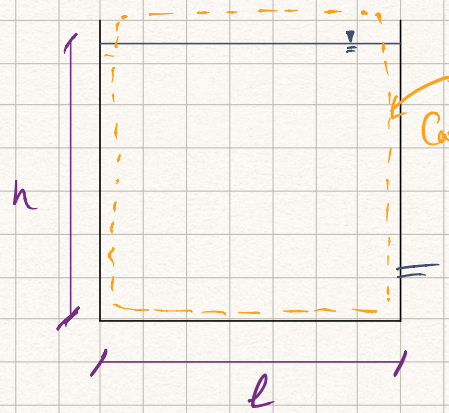
$$\dot{m} = \rho Q$$

$$\left[\frac{kg}{s} = \frac{kg}{m^3} \cdot \frac{m^3}{s}\right]$$



① C.V ← Chose C.V

② Term By Term



② (Case I)

$$\frac{dm_{cv}}{dt} = 0$$

$$(\dot{m}_{out})_{cs} = (\rho Q_{out})_{cs} = \rho Q_1$$

$$(\dot{m}_{in})_{cs} = (\rho Q_{in})_{cs} = \rho Q_2$$

$$0 = 0 + (\rho Q_1 - \rho Q_2) \Rightarrow Q_1 = Q_2$$

② (Case II)

$$dM_{cv} \neq 0 \quad \Rightarrow \quad M_{cv} = \rho \forall = \rho(h l 1)$$

$$\frac{d}{dt}\left(M_{cv} = \rho(hl)\right)$$

$$\frac{dM_{cv}}{dt} = \frac{d}{dt}(\rho h l) = \boxed{\rho l \frac{dh}{dt}}$$

$$(\dot{m}_{out})_{cs} = \rho Q_1$$

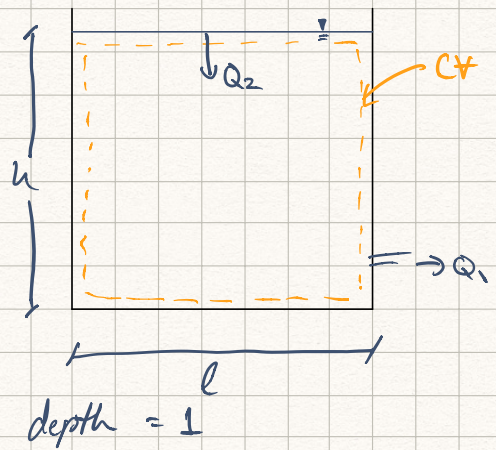
$$(\dot{m}_{in})_{cs} = 0$$

$$0 = \frac{dM_{cv}}{dt} + (\dot{m}_{out} - \dot{m}_{in})$$

$$0 = \rho l\left(\frac{dh}{dt}\right) + (\rho Q_1 - 0)$$

$$\frac{dh}{dt} = \frac{-Q_1}{l}$$

(Case 3)



$$\frac{d}{dt}(M_{cv}) = 0$$

$$(\dot{m}_{out})_{cs} = \rho Q_1$$

$$(\dot{m}_{in})_{cs} = \rho Q_2$$

$$0 = 0 + (Q_1\rho - Q_2\rho)$$

$$Q_1 = Q_2$$

$$Q_2 = \frac{\Delta \forall}{\Delta t} = vA$$

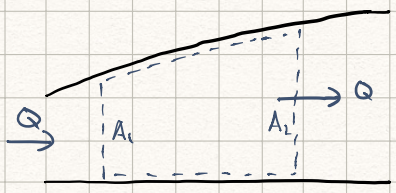
$$\begin{cases} Q_2 = vA_2 = v(ld) = vl \\ Q_2 = lv \\ v = \dfrac{dh}{dt} \end{cases} \Rightarrow \frac{dh}{dt}(l) = Q_2$$

$$\frac{Q_1}{l} = \frac{dh}{dt}$$

Common Mistakes

$\frac{dm_{cv}}{dt}$

$(\dot{m}_{in})_{cs}$



$$\frac{d}{dt}(m_{cv}) = 0$$

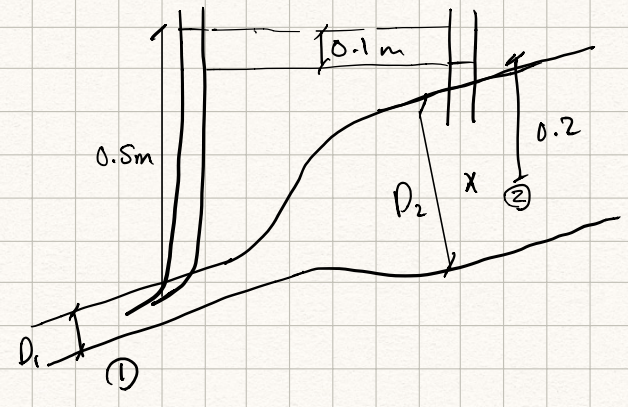
$$(\dot{m}_{out})_{cs} = \rho Q_1$$

$$(\dot{m}_{in})_{cs} = \rho Q_2$$

$$Q_1 = Q_2$$

$$v_1 A_1 = v_2 A_2$$

Example



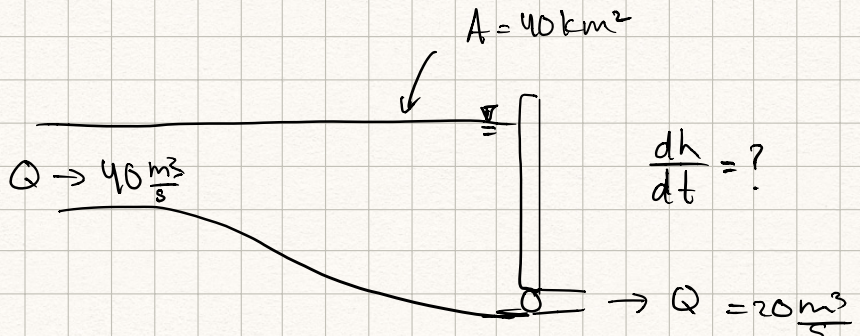
$$\text{loss} = 0$$

$$D_1 = \frac{1}{2} D_2$$

$$Q_1 = Q_2$$

$$4A_1 = v_2 A_2$$

$$v_1 = 4 v_2$$



$\frac{dh}{dt} = ?$

$$0 = \frac{d(m_{cv})}{dt} + (\dot{m}_{out} - \dot{m}_{in})$$

$\frac{dm_{cv}}{dt} \neq 0$

$\dot{m}_{out} = 20\rho$

$\dot{m}_{in} = 40\rho$

$$\frac{d}{dt}(m_{cv} = \rho h A)$$

$$\frac{dm_{cv}}{dt} = \frac{d}{dt}(\rho h A)$$

$$= \left(\frac{dh}{dt}\right) \rho A$$