MAT 1341

Fall 2017

Midterm #1, Version #1

1. (1 point)

To find the vectors that are perpendicular to *both* (-1,1,5) and (2,1,2), we will take the cross product of these vectors. ***Any multiple*** of the resulting vector is then perpendicular to the original two.

$$u = (-1,1,5)$$
$$v = (2,1,2)$$

$$u \times v = (-1,1,5) \times (2,1,2) = \begin{vmatrix} i & j & k \\ -1 & 1 & 5 \\ 2 & 1 & 2 \end{vmatrix} = (-3,12,-3)$$

Of the answers, only *D* is a multiple of this vector.

(Can't see it? Divide the resulting vector by 3, and then multiply by *t*).

Answer: D

2. (1 point) First, we will calculate the projection of *u* along *v*:

$$
\begin{aligned}
proj_v u &= \frac{u \cdot v}{v \cdot v} v \\
&= \frac{(3,3,6)\cdot(2,-1,1)}{(2,-1,1)\cdot(2,-1,1)}(2,-1,1) \\
&= \frac{6-3+6}{4+1+1}(2,-1,1) \\
&= \frac{9}{6}(2,-1,1) \\
&= \frac{3}{2}(2,-1,1) \\
&= \left(3,-\frac{3}{2},\frac{3}{2}\right)
\end{aligned}
$$

The result is a vector. Now, we want to determine the length of the vector. In other words, its magnitude. For this, we use the Pythagorean Theorem:

$$
\begin{aligned}
\left\| proj_v u \right\| &= \sqrt{3^2 + \left(-\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2} \\
&= \sqrt{9+\frac{9}{4}+\frac{9}{4}} \\
&= \sqrt{9+\frac{18}{4}} \\
&= \sqrt{\frac{18}{2}+\frac{9}{2}} \\
&= \sqrt{\frac{27}{2}} \\
&= \sqrt{9\left(\frac{3}{2}\right)} \\
&= 3\frac{\sqrt{3}}{\sqrt{2}}
\end{aligned}
$$

Hmmm… doesn't match any of the possible answers… let's try a stupid trick…

Multiply the top and bottom by $\sqrt{2}$ (this technique is often used to "get rid of" rationals in the denominator):

$$\begin{aligned}\|proj_v u\| &= 3\frac{\sqrt{3}}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{3}{2}\sqrt{6}\end{aligned}$$

Aha!

<u>Answer: B</u>

3. (1 point)

A homogeneous system of linear equations can have infinitely many solutions.

It is not specified the number of equations nor the number of unknowns… only that the system is homogeneous. Therefore, there are two possibilities for the number of solutions: unique or infinite. (Homogeneous systems NEVER have no solutions).

Therefore, TRUE.

It is possible that a system of linear equations with coefficients in R has exactly 2 solutions.

There are only 3 possibilities for the number of solutions that a linear system can have: a unique solution (in other words, exactly one solution), infinite solutions, or no solutions. Exactly 2 solutions is not in that list.

Therefore, FALSE.

If a linear system is inconsistent, then it cannot be homogeneous.

In this class "inconsistent" is a synonym for "non-homogeneous". Therefore, if a system is inconsistent, it cannot be homogeneous.

TRUE.

There exists a linear system of three equations such that its coefficient matrix has rank 5.

Let's attempt to draw this out… The system has to have three equations and a rank (number of leading 1's once row-reduced) of 5. We are not told how many unknowns it has. (It doesn't actually matter to answer the question. Choose a number, any number. I'll pick 6.)

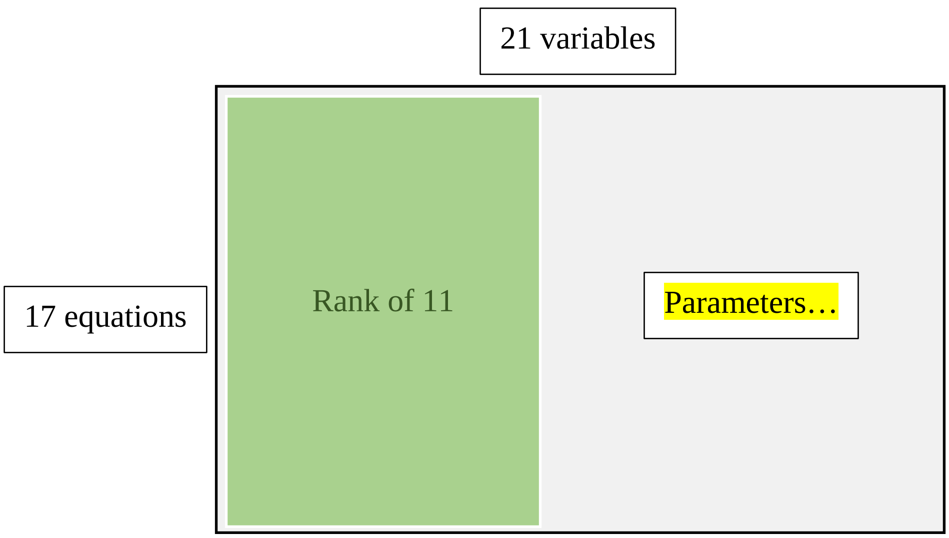
$$\begin{bmatrix} \underline{|1} & 0 & 0 & 1 & 0 & 6 \\ 0 & \underline{|1} & 0 & 6 & 4 & 0 \\ 0 & 0 & \underline{|1} & 2 & 4 & 5 \end{bmatrix}$$

No matter how you draw it, the maximum number of leading 1's a matrix with 3 rows (i.e. 3 equations) can have is *3*. So, its max rank is 3. Ain't gonna be 5.

Therefore, FALSE.

4. (1 point)

Let's draw this out.



Strictly based on the "geometry" of this diagram, we can figure out the # of parameters, which is exactly how we'll do it!

$$\begin{aligned} Rank + Parameters &= 21 \\ Parameters &= 21 - Rank \\ &= 21 - 11 \\ &= 10 \end{aligned}$$

<u>Answer: B</u>

5. (5 points)

$$\left[\begin{array}{ccc|c} 2 & -2 & e & f \\ 1 & 0 & 1 & -1 \\ 3 & 1 & 2 & -1 \end{array}\right]$$

a) Let's row reduce the matrix as much as possible.

$$\left[\begin{array}{ccc|c} 2 & -2 & e & f \\ 1 & 0 & 1 & -1 \\ 3 & 1 & 2 & -1 \end{array}\right]$$

$$\xrightarrow{\text{swap } R_1, R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 2 & -2 & e & f \\ 3 & 1 & 2 & -1 \end{array}\right]$$

$$\xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & -2 & e-2 & f+2 \\ 0 & 1 & -1 & 2 \end{array}\right]$$

$$\xrightarrow{\text{swap } R_2, R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & -2 & e-2 & f+2 \end{array}\right]$$

$$\xrightarrow{2R_2+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & e-4 & f+6 \end{array}\right]$$

This is as far as we should take the row reduction. If we try to row reduce further… 😫

Now we have this system:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & e-4 & f+6 \end{array}\right]$$

Focusing on just the coefficient *A* matrix:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & e-4 \end{bmatrix}$$

If $e = 4$, then we end up with $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ and a rank of 2.

If $e \neq 4$, then *A* has a rank of 3.

Focusing on the entire A|b system:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & e-4 & f+6 \end{array}\right]$$

If $e = 4$ and $f = -6$, we end up with $\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ and a rank of 2.

With any other set of numbers, we end up with a rank of 3.

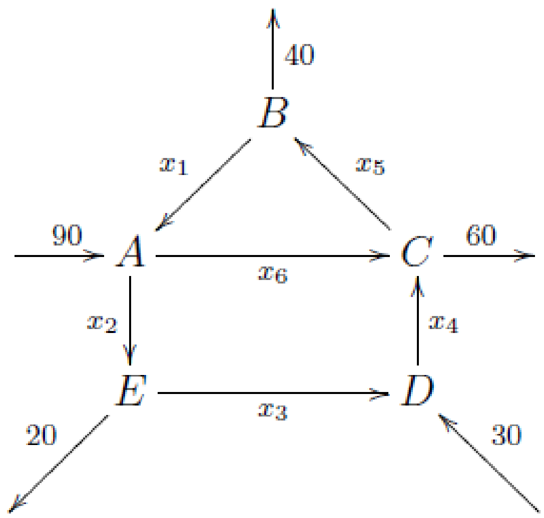
b) For these answers, I am referring back to the table in the course booklet in the section called "***How to determine the number of solutions in a linear system using the rank***". What a convenient section…

i) To get a unique solution, we need rank[A] = rank[A|b] = n. We have a 3x3 matrix, so $n = 3$. For this to happen, we need $e \neq 4$ and f can be any real number ($f \in R$).

ii) To get infinite solutions, we need rank[A] = rank[A|b] < n. So, for this to happen, we need $e = 4,\ f = -6$.

iii) To get no solutions, we need rank[A] < rank[A|b]. For this to happen, we need $e = 4,\ f \neq -6$.

6.



a) We will base the system of linear equations around each of the intersections (A, B, C, D, and E) by writing ***in*** = ***out*** equations for each intersection.

Intersection	***in = out***
A	$90 + x_1 = x_2 + x_6$
B	$x_5 = x_1 + 40$
C	$x_4 + x_6 = x_5 + 60$
D	$30 + x_3 = x_4$
E	$x_2 = 20 + x_3$

Note: If you "clean up" the equations by moving all of the variables to one side and all of the numbers to the other, this is fine. But, it is not necessary to be marked "correct" on the test.

The constraints are ***always the same***: 1. These are one-way streets; 2. You can only have a "whole number" of cars.

Write the constraints in "math language" as follows:

1. One-way streets, therefore $x_i \geq 0$ for $i = 1, 2, \ldots, 6$ (***<u>this number may change</u>***, depending on the question).

2. Only "whole number" of cars, therefore, x_i must be an integer; equivalently $x_i \in Z$.

b) The RREF matrix is

$$\left[\begin{array}{cccccc|c} \underline{1} & 0 & 0 & 0 & \boxed{-1} & \boxed{0} & -40 \\ 0 & \underline{1} & 0 & 0 & \boxed{-1} & \boxed{1} & 50 \\ 0 & 0 & \underline{1} & 0 & \boxed{-1} & \boxed{1} & 30 \\ 0 & 0 & 0 & \underline{1} & \boxed{-1} & \boxed{1} & 60 \\ 0 & 0 & 0 & 0 & \boxed{0} & \boxed{0} & 0 \end{array}\right]$$

The general solution is

$$\begin{aligned} x_1 - x_5 &= -40 \\ x_2 - x_5 + x_6 &= 50 \\ x_3 - x_5 + x_6 &= 30 \\ x_4 - x_5 + x_6 &= 60 \\ x_5 &= s \\ x_6 &= t \end{aligned}$$

Rewriting in terms of the parameters s and t…

$$\begin{aligned} x_1 &= -40 + s \\ x_2 &= 50 + s - t \\ x_3 &= 30 + s - t \\ x_4 &= 60 + s - t \\ x_5 &= s \\ x_6 &= t \end{aligned}$$

c) If ED is closed, this is equivalent to saying $x_3 = 0$. Then, we want to determine the minimum flow along AC (which is x_6).

Let's rewrite the general solution using $x_3 = 0$.

$$x_3 = 0 = 30 + s - t \rightarrow s - t = -30$$

Therefore, $t = s + 30$.

Note that $x_6 = t$

If we can determine the restriction on *s*, we can then solve for the restriction on *t*, which in turns solves the question.

OK…

Let's analyze all of the streets and determine the MINIMUM allowable values on *s*. Recall that the one-way streets must always have a minimum flow of 0 cars.

Also, since we already have $s - t = -30$, streets x_2, x_3, and x_4 are already solved.

$$\begin{aligned} x_1 &= -40 + s \rightarrow s \geq 40 \\ x_2 &= 50 + s - t = 20 \\ x_3 &= 30 + s - t = 0 \\ x_4 &= 60 + s - t = 30 \\ x_5 &= s \rightarrow s \geq 0 \\ x_6 &= t = s + 30 \rightarrow s \geq 0 \quad (\text{s can't be negative}) \end{aligned}$$

Although $s = 0$ is the minimum value of *s* in the above analysis, $s = 40$ is the one we must choose. This is because any value of *s* lower than 40 will result in a negative value in x_1, which is not allowed. Therefore, we set our lowest allowable value of *s* at $s = 40$.

Plugging this into the equation $t = s + 30$, we get $t = 40 + 30$. Therefore, $\boldsymbol{x_6 = t = 70}$ is the minimum flow along AC.

7. (1 point)

There exist two non-zero matrices A and B such that $A \cdot B$ *is the zero matrix.*

This is a good one to memorize, as it is asked a lot on exams. Here's a quick proof:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

TRUE!

For any two 3x3 matrices A and B, we have $(A+B)^2 = A^2 + 2AB + B^2$.

Sure, polynomials expand this way, but matrices don't…. let's see why!

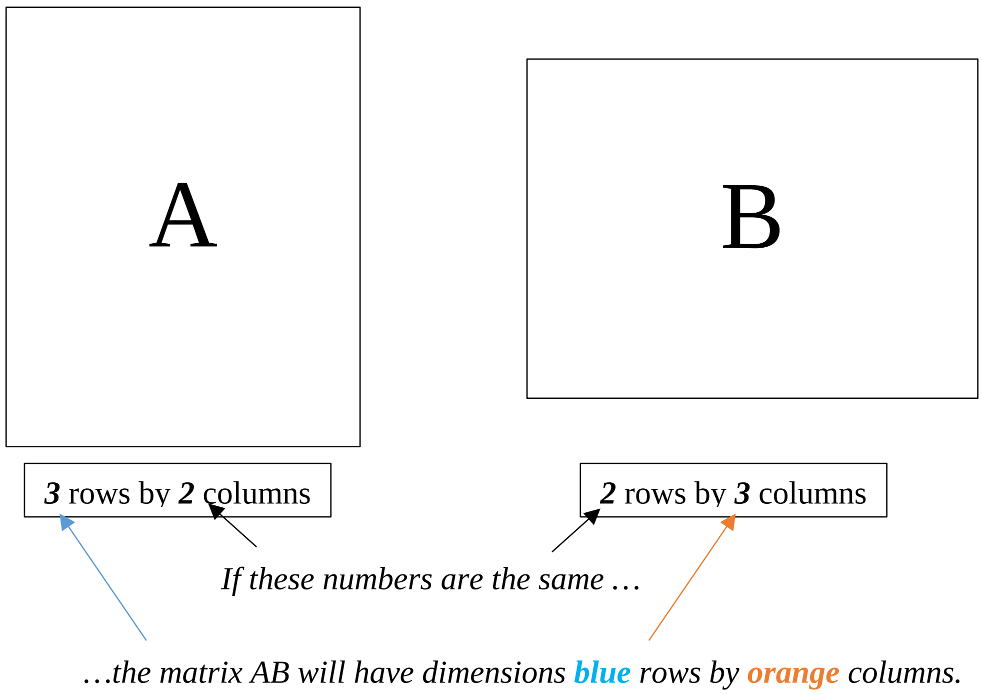
$$\begin{aligned}(A+B)^2 &= (A+B)(A+B) \\ &= AA + AB + BA + BB \\ &= A^2 + AB + BA + B^2\end{aligned}$$

Recall that AB is *most likely* not equal to BA because the order of multiplication in matrices matter.

Therefore, FALSE.

Multiplying a 3x2-matrix A by a 2x3-matrix B, one gets a 3x3 matrix AB.

Let's check!



TRUE.

I'm sorry if you're colour blind ☹.

For any two matrices A and B, we have AB = BA.

As discussed earlier…

FALSE.

8. (1 point)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \end{bmatrix}$$

B is a 3x5 matrix. You can perform this computation using "block form". If you forget, you can make a dummy 3x5 matrix and fill it in with variables like I have here. Either works.

A is 3x3.

B is 3x5.

Therefore AB will be 3x5.

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \end{bmatrix}$$

$$= \begin{bmatrix} a & b & c & d & e \\ a+k & b+l & c+m & d+n & e+o \\ a+f+k & b+g+l & c+h+m & d+i+n & e+j+o \end{bmatrix}$$

The second row of AB is the same as the sum of the *first* and *third* rows of B.

Answer: D

MAT 1341

Fall 2017

Midterm #1, Version #2

1. (1 point)

To find the vectors that are perpendicular to *both* (-2,2,10) and (2,1,2), we will take the cross product of these vectors. ***Any multiple*** of the resulting vector is then perpendicular to the original two.

$$u = (-2, 2, 10)$$
$$v = (2, 1, 2)$$

$$u \times v = (-2, 2, 10) \times (2, 1, 2) = \begin{vmatrix} i & j & k \\ -2 & 2 & 10 \\ 2 & 1 & 2 \end{vmatrix} = (-6, 24, -6)$$

Of the answers, only *C* is a multiple of this vector.

(Can't see it? Divide the resulting vector by -6, and then multiply by *t*. Don't get fooled by F. Stupid F.).

Answer: C

2. (1 point) First, we will calculate the projection of *u* along *v*:

$$\begin{aligned}
proj_v u &= \frac{u \cdot v}{v \cdot v} v \\
&= \frac{(2,-1,1)\cdot(1,1,2)}{(1,1,2)\cdot(1,1,2)}(1,1,2) \\
&= \frac{2-1+2}{1+1+4}(1,1,2) \\
&= \frac{3}{6}(1,1,2) \\
&= \frac{1}{2}(1,1,2) \\
&= \left(\frac{1}{2},\frac{1}{2},1\right)
\end{aligned}$$

The result is a vector. Now, we want to determine the length of the vector. In other words, its magnitude. For this, we use the Pythagorean Theorem:

$$\begin{aligned}
\|proj_v u\| &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (1)^2} \\
&= \sqrt{\frac{1}{4}+\frac{1}{4}+1} \\
&= \sqrt{\frac{1}{2}+1} \\
&= \sqrt{\frac{1}{2}+\frac{2}{2}} \\
&= \sqrt{\frac{3}{2}} \\
&= \frac{\sqrt{3}}{\sqrt{2}}
\end{aligned}$$

Hmmm… doesn't match any of the possible answers… let's try a stupid trick…

Multiply the top and bottom by $\sqrt{2}$ (this technique is often used to "get rid of" rationals in the denominator):

$$\begin{aligned}
\|proj_v u\| &= \frac{\sqrt{3}}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} \\
&= \frac{\sqrt{6}}{2}
\end{aligned}$$

Aha!

Answer: D

3. (1 point)

Is it possible that a system of linear equations with coefficients in R has exactly 3 solutions.

There are only 3 possibilities for the number of solutions that a linear system can have: a unique solution (in other words, exactly one solution), infinite solutions, or no solutions. Exactly 3 solutions is not in that list.

Therefore, FALSE.

A homogeneous system of linear equations is always consistent.

In this class "consistent" is a synonym for "homogeneous". Therefore, if a system is consistent, it must be homogeneous.

TRUE.

There exists a linear system of four equations such that its coefficient matrix has rank 6.

Let's attempt to draw this out… The system has to have four equations and a rank (number of leading 1's once row-reduced) of 6. We are not told how many unknowns it has. (It doesn't actually matter to answer the question. Choose a number, any number. I'll pick 5.)

$$\begin{bmatrix} \underline{1} & 0 & 0 & 0 & 2 \\ 0 & \underline{1} & 0 & 0 & 4 \\ 0 & 0 & \underline{1} & 0 & 0 \\ 0 & 0 & 0 & \underline{1} & 5 \end{bmatrix}$$

No matter how you draw it, the maximum number of leading 1's a matrix with 4 rows (i.e. 4 equations) can have is *4*. So, its max rank is 4. Ain't gonna be 6.

Therefore, FALSE.

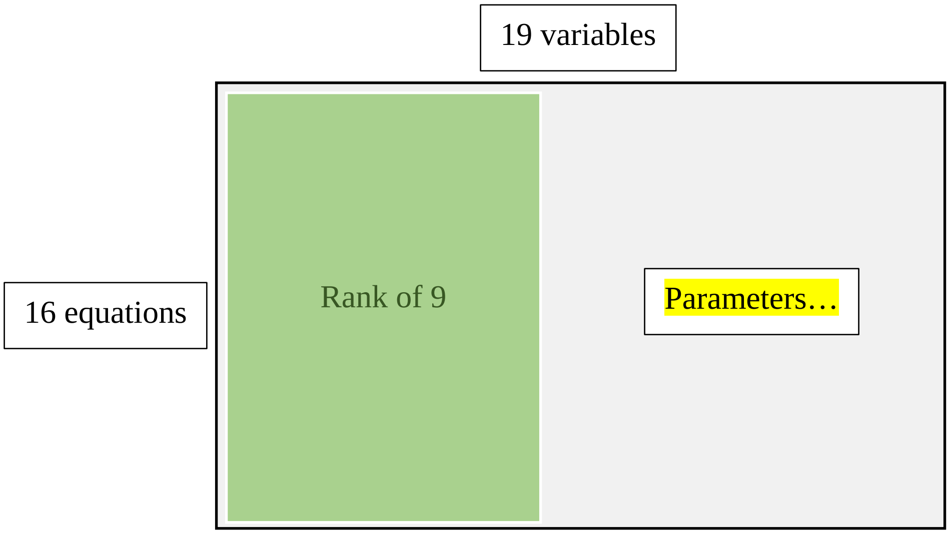
A homogeneous system of linear equations can have a unique solution.

It is not specified the number of equations nor the number of unknowns… only that the system is homogeneous. Therefore, there are two possibilities for the number of solutions: unique or infinite. (Homogeneous systems NEVER have no solutions).

Therefore, TRUE.

4. (1 point)

Let's draw this out.



Strictly based on the "geometry" of this diagram, we can figure out the # of parameters, which is exactly how we'll do it!

$$\begin{aligned} Rank + Parameters &= 19 \\ Parameters &= 19 - Rank \\ &= 19 - 9 \\ &= 10 \end{aligned}$$

<u>Answer: E</u>

5. (5 points)

$$\left[\begin{array}{ccc|c} 2 & -2 & e & f \\ 1 & 0 & 1 & -1 \\ 3 & 1 & 2 & -1 \end{array}\right]$$

a) Let's row reduce the matrix as much as possible.

$$\left[\begin{array}{ccc|c} 2 & -2 & e & f \\ 1 & 0 & 1 & -1 \\ 3 & 1 & 2 & -1 \end{array}\right]$$

$$\xrightarrow{swap\ R_1, R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 2 & -2 & e & f \\ 3 & 1 & 2 & -1 \end{array}\right]$$

$$\xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & -2 & e-2 & f+2 \\ 0 & 1 & -1 & 2 \end{array}\right]$$

$$\xrightarrow{swap\ R_2, R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & -2 & e-2 & f+2 \end{array}\right]$$

$$\xrightarrow{2R_2+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & e-4 & f+6 \end{array}\right]$$

This is as far as we should take the row reduction. If we try to row reduce further… 

Now we have this system:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & e-4 & f+6 \end{array}\right]$$

Focusing on just the coefficient *A* matrix:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & e-4 \end{bmatrix}$$

If $e = 4$, then we end up with $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ and a rank of 2.

If $e \neq 4$, then *A* has a rank of 3.

Focusing on the entire A|b system:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & e-4 & f+6 \end{array}\right]$$

If $e = 4$ and $f = -6$, we end up with $\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ and a rank of 2.

With any other set of numbers, we end up with a rank of 3.

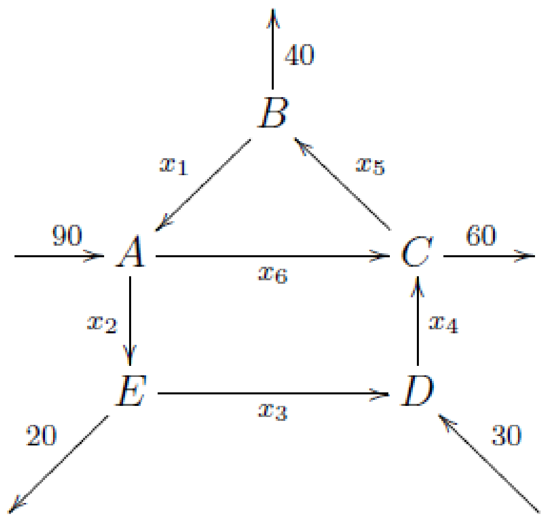
b) For these answers, I am referring back to the table in the course booklet in the section called "***How to determine the number of solutions in a linear system using the rank***". What a convenient section…

i) To get a unique solution, we need rank[A] = rank[A|b] = n. We have a 3x3 matrix, so $n = 3$. For this to happen, we need $e \neq 4$ and f can be any real number ($f \in R$).

ii) To get infinite solutions, we need rank[A] = rank[A|b] < n. So, for this to happen, we need $e = 4,\ f = -6$.

iii) To get no solutions, we need rank[A] < rank[A|b]. For this to happen, we need $e = 4,\ f \neq -6$.

6.



a) We will base the system of linear equations around each of the intersections (A, B, C, D, and E) by writing ***in*** = ***out*** equations for each intersection.

Intersection	***in = out***
A	$90 + x_1 = x_2 + x_6$
B	$x_5 = x_1 + 40$
C	$x_4 + x_6 = x_5 + 60$
D	$30 + x_3 = x_4$
E	$x_2 = 20 + x_3$

Note: If you "clean up" the equations by moving all of the variables to one side and all of the numbers to the other, this is fine. But, it is not necessary to be marked "correct" on the test.

The constraints are ***always the same***: 1. These are one-way streets; 2. You can only have a "whole number" of cars.

Write the constraints in "math language" as follows:

1. One-way streets, therefore $x_i \geq 0$ for $i = 1, 2, \ldots, 6$ (***this number may change***, depending on the question).

2. Only "whole number" of cars, therefore, x_i must be an integer; equivalently $x_i \in Z$.

b) The RREF matrix is

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & \boxed{-1} & \boxed{0} & -40 \\ 0 & 1 & 0 & 0 & \boxed{-1} & \boxed{1} & 50 \\ 0 & 0 & 1 & 0 & \boxed{-1} & \boxed{1} & 30 \\ 0 & 0 & 0 & 1 & \boxed{-1} & \boxed{1} & 60 \\ 0 & 0 & 0 & 0 & \boxed{0} & \boxed{0} & 0 \end{array}\right]$$

The general solution is

$$\begin{aligned} x_1 - x_5 &= -40 \\ x_2 - x_5 + x_6 &= 50 \\ x_3 - x_5 + x_6 &= 30 \\ x_4 - x_5 + x_6 &= 60 \\ x_5 &= s \\ x_6 &= t \end{aligned}$$

Rewriting in terms of the parameters s and t…

$$\begin{aligned} x_1 &= -40 + s \\ x_2 &= 50 + s - t \\ x_3 &= 30 + s - t \\ x_4 &= 60 + s - t \\ x_5 &= s \\ x_6 &= t \end{aligned}$$

c) If ED is closed, this is equivalent to saying $x_3 = 0$. Then, we want to determine the minimum flow along AC (which is x_6).

Let's rewrite the general solution using $x_3 = 0$.

$$x_3 = 0 = 30 + s - t \rightarrow s - t = -30$$

Therefore, $t = s + 30$.

Note that $x_6 = t$

If we can determine the restriction on *s*, we can then solve for the restriction on *t*, which in turns solves the question.

OK…

Let's analyze all of the streets and determine the MINIMUM allowable values on *s*. Recall that the one-way streets must always have a minimum flow of 0 cars.

Also, since we already have $s - t = -30$, streets x_2, x_3, and x_4 are already solved.

$$\begin{aligned}
x_1 &= -40 + s \rightarrow s \geq 40 \\
x_2 &= 50 + s - t = 20 \\
x_3 &= 30 + s - t = 0 \\
x_4 &= 60 + s - t = 30 \\
x_5 &= s \rightarrow s \geq 0 \\
x_6 &= t = s + 30 \rightarrow s \geq 0 \quad (\text{s can't be negative})
\end{aligned}$$

Although $s = 0$ is the minimum value of *s* in the above analysis, $s = 40$ is the one we must choose. This is because any value of *s* lower than 40 will result in a negative value in x_1, which is not allowed. Therefore, we set our lowest allowable value of *s* at $s = 40$.

Plugging this into the equation $t = s + 30$, we get $t = 40 + 30$. Therefore, $\boldsymbol{x_6 = t = 70}$ is the minimum flow along AC.

7. (1 point)

For any three 3x3 matrices A, B, and C, we have (AB)C = A(BC).

The matrices are all size-compatible because they are all 3x3, so they can be multiplied. Also, the order of multiplication hasn't changed (i.e. we aren't saying that AB = BA or anything like that). Basically, this is saying that ABC = ABC. Last time I checked, this is a true statement.

TRUE

There exists a non-zero matrix A such that A^2 *is the zero matrix.*

This is a good one to memorize, as it is asked a lot on exams. Here's a quick proof:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

TRUE!

For any two 3x3 matrices A and B, we have $A^2 - B^2 = (A-B)(A+B)$.

Sure, polynomials factor this way, but matrices don't…. let's see why!

Expand the right-hand side…

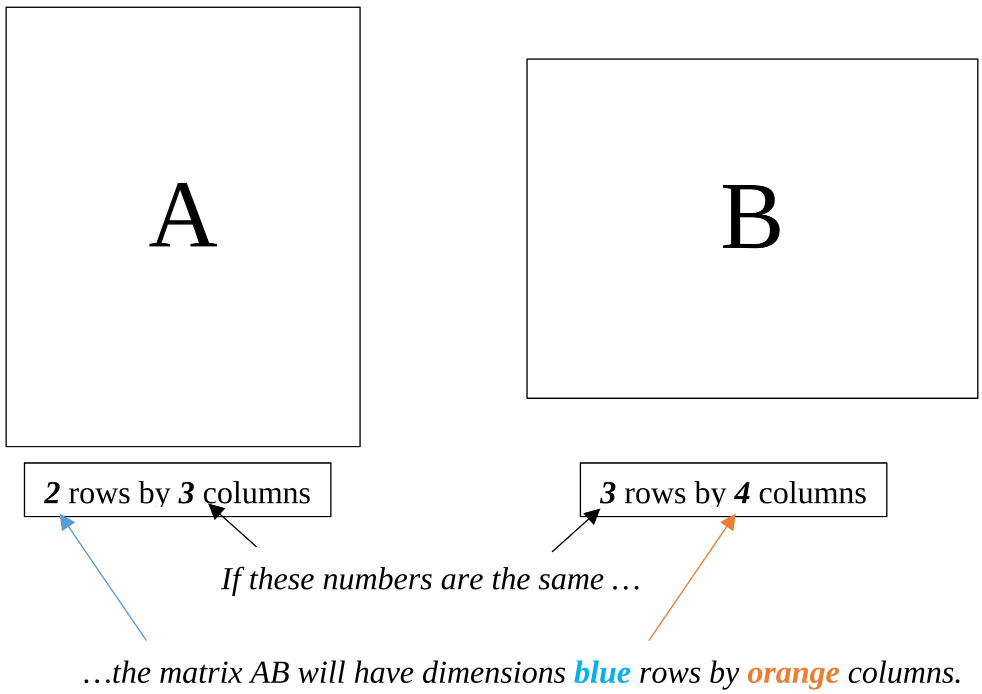
$$\begin{aligned}(A-B)(A+B) &= AA + AB - BA - BB \\ &= A^2 + AB - BA - B^2\end{aligned}$$

Recall that AB is *most likely* not equal to BA because the order of multiplication in matrices matter. So, these two will not cancel out.

Therefore, FALSE.

Multiplying a 2x3-matrix A by a 3x4-matrix B, one gets a 2x4 matrix AB.

Let's check!



TRUE.

I'm sorry if you're colour blind ☹.

8. (1 point)

$$A=\begin{bmatrix}1&1&1\\0&0&1\\1&0&1\end{bmatrix}$$

$$B=\begin{bmatrix}a&b&c&d&e\\f&g&h&i&j\\k&l&m&n&o\end{bmatrix}$$

B is a 3x5 matrix. You can perform this computation using "block form". If you forget, you can make a dummy 3x5 matrix and fill it in with variables like I have here. Either works.

A is 3x3.

B is 3x5.

Therefore AB will be 3x5.

$$AB=\begin{bmatrix}1&1&1\\0&0&1\\1&0&1\end{bmatrix}\begin{bmatrix}a&b&c&d&e\\f&g&h&i&j\\k&l&m&n&o\end{bmatrix}$$
$$=\begin{bmatrix}a+f+k&b+g+l&c+h+m&d+i+n&e+j+o\\k&l&m&n&o\\a+k&b+l&c+m&d+n&e+o\end{bmatrix}$$

The second row of AB is the same as the *third* row of B.

<u>Answer: E</u>