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GNG1105D – Engineering Mechanics

Quiz 5: MIDTERMish EXAMINATION

University of Ottawa

Department of Mechanical Engineering

Fall 2019

Instructions

This exam contains 5 pages including this title page.

You are free to remove sheets from this exam.

You will have 75 min to complete this exam. No extra time will be given.

Hand in only the exam booklet.

This exam is closed book.

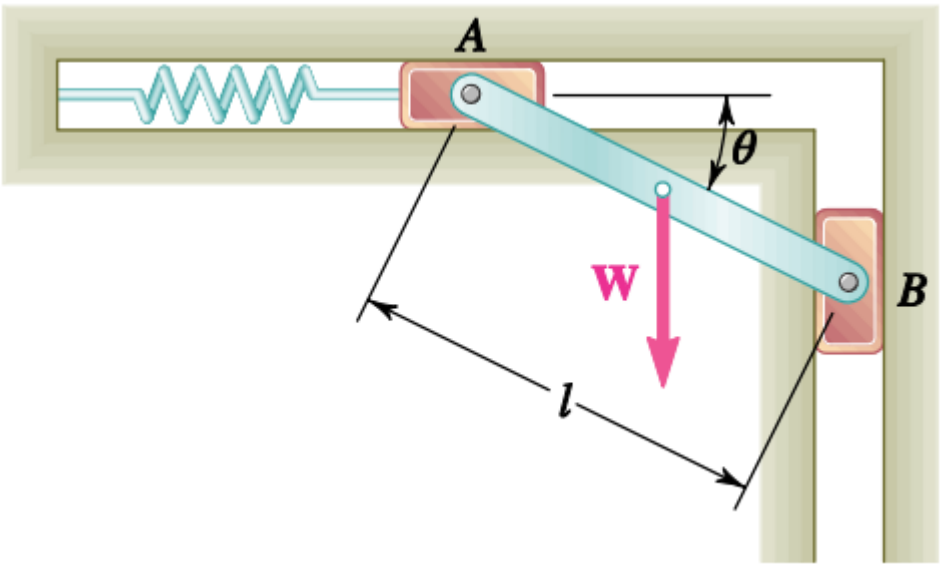
You may use a non-programmable calculator.

You may write in pen or pencil.

Remember units in the final answer.

Question 1:

See the following schematic:



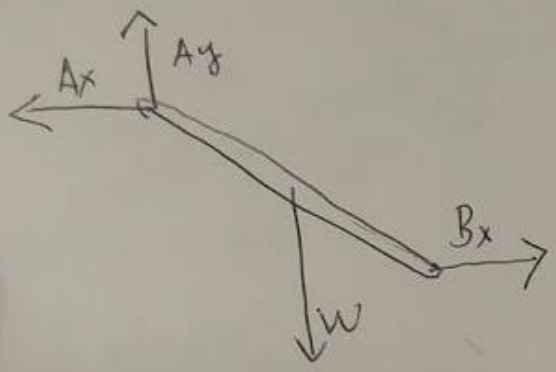
(a) Neglecting the weight of the blocks, draw the free body diagram of rod AB with the reactions in the correct direction.

(b) Is the spring in tension or compression?

(c) Knowing that l=50cm and the weight is 1000N, and at equilibrium the angle θ=50° what is the force in the spring?

Q1

(a) FBD



(b) Tension

(c) $\Sigma \vec{F} = 0$

$\Sigma \vec{M} = 0$

$\Sigma f_x = 0 = B_x - A_x$ ①

$\Sigma f_y = 0 = A_y - W$ ②

$\circlearrowleft + \Sigma M_A = 0 = -\frac{1}{2} l \cos\theta (W) + l \sin\theta (B_x)$ ③

From ③ →

$$B_x = \frac{\frac{1}{2} l \cos\theta (w)}{l \sin\theta} = \frac{\frac{1}{2}(0.5)\cos(50)(1000)}{(0.5)\sin(50)}$$

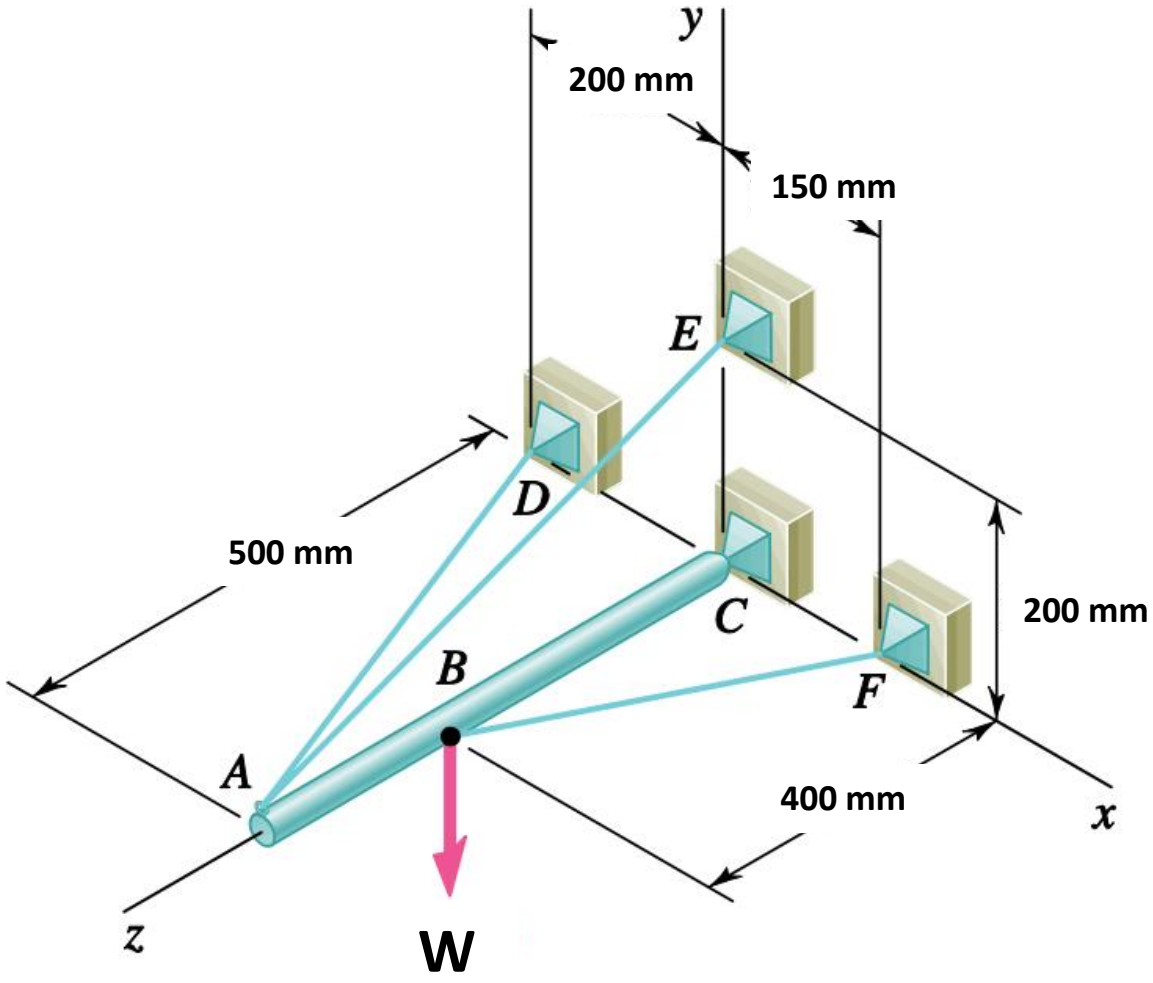
$$= 419 \text{ N}$$

From ①

$A_x = B_x$

$\boxed{A_x = 419 \text{N}}$ ← Tension in spring

Question 2:



In this set up, the pole ABC is held by a ball-and-socket joint at C and Cable DAE passes around a frictionless pulley at A.

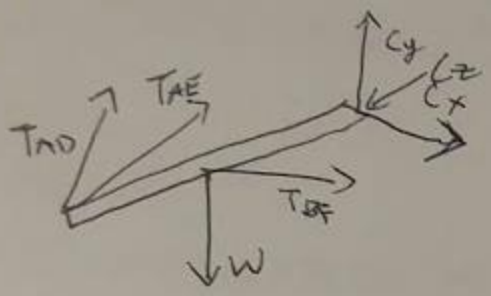
(a) Draw the FBD of the pole ABC.

(b) Define the vectors $\overrightarrow{W}$, $\overrightarrow{T_{AD}}$, $\overrightarrow{T_{AE}}$ and $\overrightarrow{T_{BF}}$ in components ($\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$).

(c) Knowing W=1000N, solve for the tension in the cables and the reaction at C.

Q2

(a) FBD



(b) $\vec{W} = W\vec{\lambda}_W$

$= -W\hat{j}$

$\vec{T}_{AD} = T_{AD}\vec{\lambda}_{AD}$

$\vec{\lambda}_{AD} = \frac{\vec{AD}}{|AD|} = \frac{-0.2\hat{i} + 0\hat{j} - 0.5\hat{k}}{\sqrt{0.2^2 + 0.5^2}}$

$= -0.371\hat{i} + 0\hat{j} - 0.928\hat{k}$

$\vec{T}_{AD} = -0.371 T_{AD}\hat{i} + 0\hat{j} - 0.928 T_{AD}\hat{k}$

$\vec{T}_{AE} = T_{AE}\vec{\lambda}_{AE}$

$\vec{\lambda}_{AE} = \frac{\vec{AE}}{|AE|} = \frac{0\hat{i} + 0.2\hat{j} - 0.5\hat{k}}{\sqrt{0.2^2 + 0.5^2}}$

$= 0\hat{i} + 0.371\hat{j} - 0.928\hat{k}$

$\vec{T}_{AE} = 0\hat{i} + 0.371 T_{AE}\hat{i} - 0.928 T_{AE}\hat{k}$

$\vec{T}_{BF} = T_{BF}\vec{\lambda}_{BF}$

$\vec{\lambda}_{BF} = \dfrac{\vec{BF}}{|BF|} = \dfrac{0.15\hat{i} + 0\hat{j} - 0.4\hat{k}}{\sqrt{(0.15^2 + 0.4^2)}}$

$= 0.351\hat{i} + 0\hat{j} - 0.936\hat{k}$

$\boxed{\vec{T}_{BF} = 0.351T_{BF}\hat{i} + 0\hat{j} - 0.936T_{BF}\hat{k}}$

(c) $\Sigma\vec{F} = 0$

$\Sigma\vec{M} = 0$

$\Sigma f_x = 0 = -0.371T_{AD} + 0.351T_{BF} + C_x$ (1)

$\Sigma f_y = 0 = 0.371T_{AE} - W + C_y$ (2)

$\Sigma f_z = 0 = -0.928T_{AD} - 0.928T_{AE} - 0.936T_{BF} + C_z$ (3)

$\Sigma\vec{M}_E = 0 = \vec{r}_{AB} \times \vec{W} + \vec{r}_{AB} \times \vec{T}_{BF} + \vec{r}_{AC} \times \vec{T}_{AD} + \vec{r}_{AC} \times \vec{T}_{AE}$

$\vec{r}_{AB} = 0\hat{i} + 0\hat{j} + 0.4\hat{k}$

$\vec{r}_{AC} = 0\hat{i} + 0\hat{j} + 0.5\hat{k}$

$\vec{r}_{A/B} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0.4 \\ 0 & -W & 0 \end{vmatrix} = 0.4W\hat{i} + 0\hat{j} + 0\hat{k}$

$\vec{r}_{AB} \times \vec{T}_{BF} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0.4 \\ 0.351T_{BF} & 0 & -0.936T_{BF} \end{vmatrix} = 0\hat{i} + (0.4)(0.351)T_{BF}\hat{j} + 0\hat{k}$

$\vec{r}_{AC} \times \vec{T}_{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0.5 \\ -0.371T_{AD} & 0 & -0.928T_{AD} \end{vmatrix} = 0\hat{i} + 0.5(0.371T_{AD})\hat{j} + 0\hat{k}$

$$\vec{r}_{A/C} \times \vec{T}_{AE} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0.5 \\ 0 & 0.371 T_{AE} & -0.428 T_{AE} \end{vmatrix} = -0.5(0.371 T_{AE})\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\Sigma M_{A_x} = 0 = 0.4W - 0.5(0.371) T_{AE} \quad (4)$$

$$\Sigma M_{A_y} = 0 = 0.4(0.351) T_{BF} - 0.5(0.371 T_{AD}) \quad (5)$$

$$\Sigma M_{A_z} = 0 = 0$$

→ Need another eqn → $T_{AD} = T_{AE}$ (6) ← single cable around frictionless pulley

From (4) →

$$T_{AE} = \frac{0.4W}{0.5(0.371)} = \frac{0.4(1000)}{0.5(0.371)} = 2156.3\ N$$

$$\boxed{T_{AE} = 2156.3\ N}$$

From (5)

$$\boxed{T_{AD} = 2156.3\ N}$$

From (5)

$$T_{BF} = \frac{0.5(0.371) T_{AD}}{0.4(0.351)} = \frac{0.5(0.371)(2156.3)}{0.4(0.351)}$$

$$\boxed{T_{BF} = 284.89\ N}$$

from ①

$C_x = 0{,}371\, T_{AD} + 0{,}351\, T_{BF}$

$C_x = 699{,}9\ N$

from ②

$C_y = W - 0{,}371\, T_{AE}$

$C_y = 200{,}0\ N$

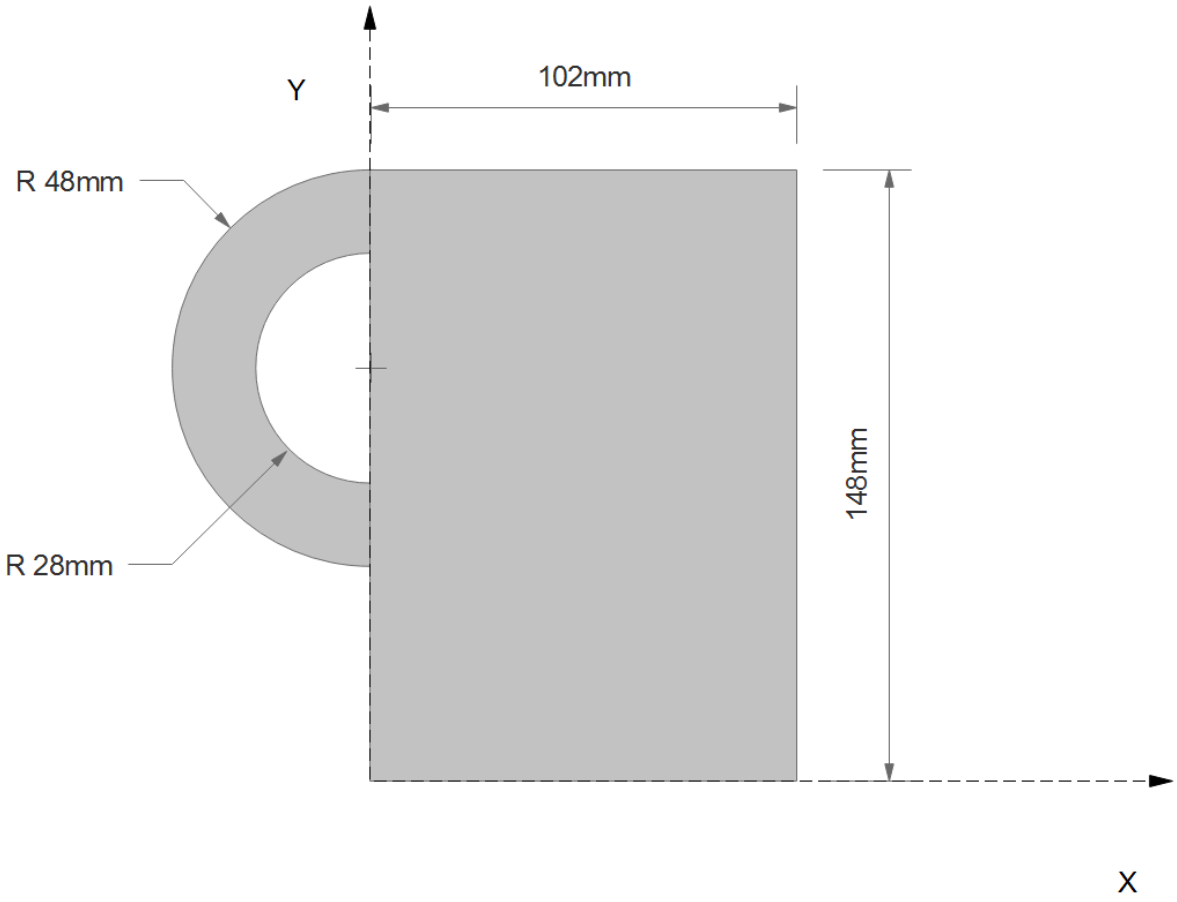
from ③

$C_z = 0{,}928\, T_{AD} + 0{,}928\, T_{AE} + 0{,}934\, T_{BF}$

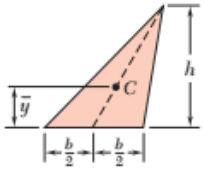
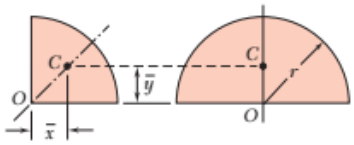
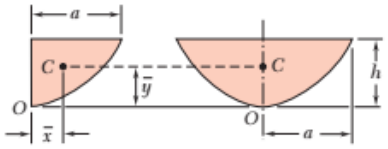
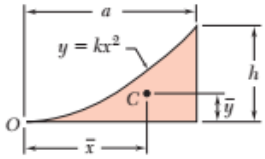
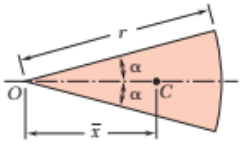
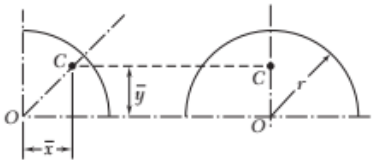
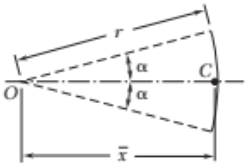
$C_z = 4268{,}7\ N$

Question 3:

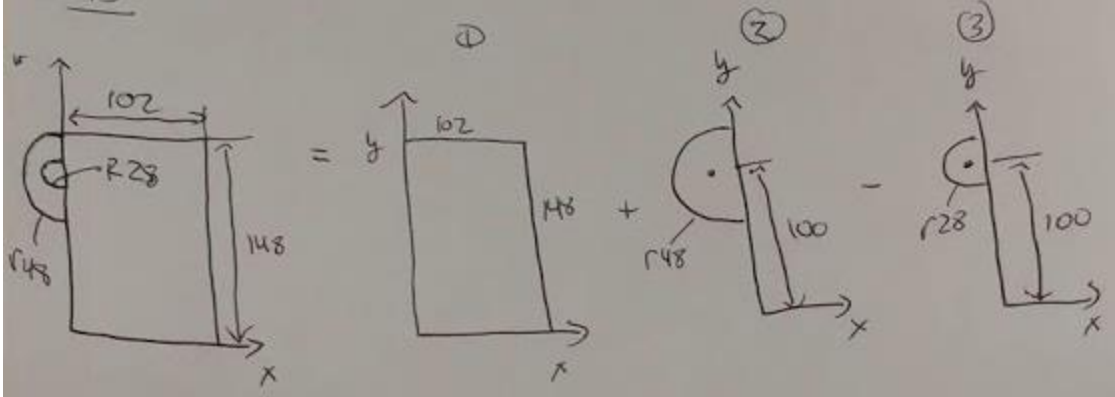
Assuming uniform thickness and density, find the location of the centroid of the following shape. Use a table (as shown in class) and the coordinate system shown for full points.



Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r\sin\alpha}{\alpha}$	0	$2\alpha r$

Q3



Component	Area (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}A$ (mm^3)	$\bar{y}A$ (mm^3)
1	$102 \times 148 = 15096$	51	74	769896	1117104
2	$\frac{\pi r^2}{2} = \frac{\pi (48)^2}{2} = 3619.1$	$-\frac{4r}{3\pi} = -20.37$	100	-73727.7	361910
3	$\frac{\pi r^2}{2} = \frac{\pi (28)^2}{2} = -1231.5$	$-\frac{4r}{3\pi} = -\frac{4(28)}{3\pi} = -11.88$	100	14630.22	-123150
Σ	17483.6	/	/	710798.5	1355864

$\bar{X} \Sigma A = \Sigma \bar{x} A$

$\bar{X} = \frac{710798.5}{17483.6} = 40.65\,mm$

$\bar{Y} \Sigma A = \Sigma \bar{y} A$

$\bar{Y} = \frac{1355864}{17483.6} = 77.55\,mm$