Sauder School of Business **COMM 371**
Finance Division **Gonzalo Morales**

Marked Problem Set 1

Solutions must be **typed**. The work has to be done in groups of 3 to 5 students (include the name of each member at the top of the document). Please submit the solution on **October 10th** at **HA-884F**.

1. You are trying to determine your standard of living after retirement. You make the following assumptions. First, you will earn $200,000 for each of the next 35 years, save 30% of that amount, and retire after 35 years. Second, savings are annual and the first payment towards your savings is one year from today. Third, term structure will be flat at 5% (Annual APR) forever.

   (a) *[3 marks]* Compute how much money you will have saved at the end of the 35th year.

   (b) *[3 marks]* What is the amount you can consume during each of your retirement years? Assume that there are 20 retirement years, that consumption takes place at the end of each retirement year, and that you consume the same amount each retirement year. (Note: If you have not computed the answer to part (a), denote this answer by X and do part (b).)

2. You are holding a 3-year bond with coupon rate 10%. Coupon payments are annual and par value is 100. Spot rates are: $r_1 = 5\%$, $r_2 = 6\%$, $r_3 = 6.5\%$ (Annual APRs).

   (a) *[4 marks]* Determine the coupon bond's price and YTM.

   (b) *[4 marks]* What YTM is needed for the coupon bond to be trading at par?

   (c) *[4 marks]* Calculate the prices of zero coupon bonds with maturities of 1, 2, and 3 years.

   (d) *[3 marks]* Construct a portfolio of zero coupon bonds of maturities of 1, 2, and 3 years to replicate coupon bond (i.e. state the quantity of each zero coupon bond you need to buy).

3. You observe prices for the following bonds:

Bond	Coupon rate (%)	Maturity	Price
X	6	6 months	100.98
Y	8	1 year	103.59

(a) *[14 marks]* Coupon payments are semiannual. Determine the 6-month spot rate ($r_{0.5}$) and the 1-year spot rate (r_1), both expressed as APRs with semiannual compounding.

4. Bonds A and B have 3-year maturity. Bond A has coupon rate 2% and trades at 92.62. Bond B has coupon rate 10% and trades at 114.60. A zero-coupon bond with 3-year maturity trades at 87.13.

   (a) *[14 marks]* Compute the three-year spot rate. Compute the one- and two-year rates under the assumption that they are equal.

   (b) *[14 marks]* Suppose that one year from today (just after bonds A and B pay the first coupon payments) the term structure becomes flat at 7%. Compute the one-year return of an investor in each of Bonds A and B. (That is, the investor buys a bond today, and sells it one-year from today just after bonds A and B pay the first coupon payments.) Compare the two returns and explain the intuition behind the comparison.

   (c) *[14 marks]* Suppose that the investor holds Bonds A and B until maturity, investing the coupons at the relevant spot rates. Suppose that one year from today (just after bonds A and B pay the first coupon payments) the term structure becomes flat at 7%. Suppose also that the term structure remains flat at 7% until the bonds mature. Compute the annualized three-year return of the investor in each of the bonds. Compare the two returns and explain the intuition behind the comparison.

5. You just sold to a client a custom-made 31-year bond (so called *a BC-bond*) which pays a single coupon of $ 1M 30 years from now and $ 2M at maturity (i.e., 31 years from now). You would like to hedge this liability. Assume that the term structure is currently flat at 5%.

   (a) *[7 marks]* Your first idea is to synthetically replicate the liability. Construct the hedging portfolio consisting of the following bonds paying annual coupons:

Bond	Coupon rate (%)	Maturity
A	0	31 years
B	4	30 years
C	6	30 years

   The par values of the three bonds are $ 100.

   (b) *[8 marks]* You consider instead duration-based hedging. Assume that you have 10- and 15-year zero-coupon bonds available, with par values $ 100. Construct the hedging portfolio of the two bonds that has the same market value and the same interest-rate sensitivity (measured using the duration model) as your liability.

(c) *[4 marks]* Suppose the interest rate fell to 4.8%. Would your hedging portfolio computed in part (b) remain the same? If not, explain why not and comment on the composition of the new hedging portfolio relative to the old one. (No computations are necessary.)

(d) *[4 marks]* What are the advantages of synthetic replication (option described in part (a)) relative to approximate hedging (option described in part (b))? Why might one use approximate hedging instead?