Additional Practice for COMM371

1. The four following portfolios are traded at the same time.

Portfolio	Expected Return	Standard Deviation
I	8%	10%
J	16%	20%
K	15%	25%
L	25%	38%

Which one of them does not lie on the efficient frontier?

A) Portfolio I
B) Portfolio J
C) Portfolio K
D) Portfolio L
E) None of the above

2. You have $500,000 available to invest in a risk free security and a stock (you can be long or short on both securities) for a one year period. The risk-free rate is 8% p.a.. The expected return on a risky portfolio is 16% for a period of one year. If you wish to earn an expected 22% return for the one-year period, you should
A) invest $125,000 in the risk-free asset
B) invest $375,000 in the risk-free asset
C) borrow $125,000
D) borrow $375,000
E) None of the above

3. The term structure is flat at 4% (APR annual compounding) and will remain unchanged for the foreseeable future. Consider a three years bond with face value equal to $100 and an annual coupon of $5. A forward contract is written on this bond and has a maturity of two years. Assuming the forward transaction is executed just after the second bond's coupon gets paid, the no arbitrage forward price is closest to

A) 94.5
B) 102.2
C) 92.56
D) 92
E) None of the above

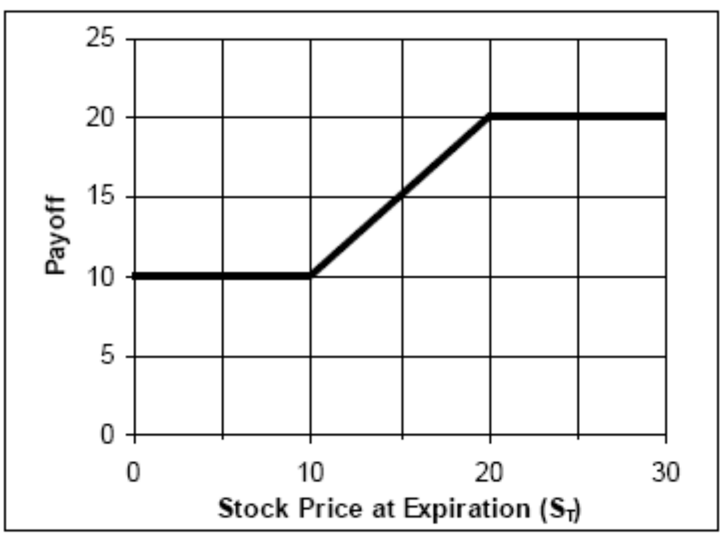
4. Assume that a typical technology stock has a standard deviation of returns equal to 50% per year, and that all technology stocks returns are uncorrelated with each other. If the typical blue-chip stock has a standard deviation of 10% per year, then what is the minimum number of different technology stocks you would have to include in an equally weighted portfolio to make the tech stock portfolio safer (in terms of standard deviation) than holding a single blue-chip stock?

A) 20
B) 30
C) 35
D) 25
E) None of the above

5. A triple A publicly traded firm (very good credit quality) has a positive beta. No dividend is scheduled for the next month. Which of its securities has the highest systematic risk?

A) Its common stocks
B) Its bonds
C) A portfolio that contains its common stock and a European put on it (at the money, one month maturity).
D) Its European put options (at the money, one month maturity)
E) Its European call options (at the money, one month maturity)

6. With the stock at $15 per share, an investor desires the following payoff structure.



Assuming all options are European, this can be achieved by

A) Long the stock, long a put (X=20), short a call (X=10).
B) Buy a bond with face value=10, short a call(X=10), long a call(X=20)
***C) Buy a* bond with face value=20, long a put(X=10), short a put(X=20).**
D) Buy a Bond with face value=20, short 2 puts (X=10), short 2 puts (X=20)
E) None of the above

7. The S&P index is currently at $750. The one-year interest rate is 6% (APR annually compounded) and the S&P index pays $15 dividend in the year end. The price on an index forward contract with a 1-year maturity (just after the dividend gets paid) is $770. An arbitrageur can

A) realize a profit at maturity of $10 by borrowing $750, buying the index, and entering into the futures contract short.
B)* realize a profit at maturity of $10 by shorting the index, investing $750 and entering into the futures contract long. **
C) realize a profit at maturity of $40 by shorting the index, investing $750 and entering into the futures contract long.
D) Realize a profit a maturity of $20 by buying the index and selling it at maturity (when forward-spot convergence drives the index to $770).
E) Realize a profit at maturity of $30 by borrowing $750, buying the index, and entering into the futures contract short.

Answer: By fwd-spot parity we should have F=750*1.06-15=780. F is undervalued relative to the spot by 10: sell the spot and go long the future.

8. Portfolio X consists of 300 shares of stock and 280 puts on that stock. Portfolio Y consists of 150 shares of stock. The put delta is Δ=-0.5. Which portfolio has a higher dollar exposure to a change in stock price?

A) Portfolio Y
B) Portfolio X
C) The two portfolios have the same exposure.
D) X if the stock price increases and Y if it decreases.
E) None of the above

Problem 2.

Assume that the risk free return is 1%, the S&P500 expected return is 12% and the S&P500 volatility is 25% (for a one year horizon). Suppose that you are a pension fund manager and that your investment goal is to have an expected return of 15% (for a one year horizon).

(a) Under the CAPM assumptions, what are the optimal weights in the risk free security and the S&P500 (the market) of your portfolio in order to achieve that goal?

We may use the formula

$$E[r_p] = \sum_{i=1}^{2} w_i \, E[r_i] = w_f \, r_f + w_M \, E[r_M],$$

and the fact that $w_f = (1 - w_M)$ to obtain the optimal weight of the market portfolio in your portfolio:

$$w_M = \frac{E[r_p] - r_f}{E[r_M] - r_f} \; .$$

Since $E[r_p] = 0.15, r_f = 0.01,$ and $E[r_M] = 0.12,$ we have $w_M = \underline{127.27\%}$ and $w_f = \underline{-27.27\%}$.

(b) What is the standard deviation of the portfolio described in a. ?

Observe that the standard deviation of your portfolio is:

$$\sigma_p = w_M \sigma_M = 1.2727 \times 0.25 = \underline{31.82\%}.$$

(c) Suppose that the CAPM holds. Does a portfolio that only invests in large stocks lie:
(1) above the SML, (2) on the SML, or (3) below the SML? No explanation is required for full credit.

If the CAPM holds, then all securities (and portfolios) are fairly priced and thus lie on the SML.

(d) Suppose a hedge fund can deliver a return that has a beta of 1.1 and a standard deviation of 30%. Also assume that you are convinced that the hedge fund can generate an alpha of 3%. Based on this you decided to give the hedge fund a weight of 20%. What weights should you choose for the risk free security and the S&P500, so that your expected return is still 15%?

Answer: It is easy to check that the weights should be 1 in the S&P 500 (more specifically 0.9981) and -0.2 in the bond (more specifically -0.19818).

(e) Compute the standard deviation of the portfolio in part (d), and show that it smaller than that of the portfolio in part (a)-(b). Briefly explain why this result obtains.

Answer. We need to calculate first the covariance of the hedge fund return with the S&P500 return. This is Beta*var(S&P)=1.1*0.25^2=0.06875.The variance of the portfolio is 0.2^2*0.3^2+ 0.99^2*0.25^2+2*0.2*0.99*0.06875=0.0036+0.0622+0.02745=0.0933. This gives a stdev=0.3054<0.3182. This is the benefit of the diversification.

Problem 3 (Portfolio choice)

You decided to invest $200M in the stocks of two companies: A and B. The expected returns and standard deviations of returns for the two stocks are as follows:

Stocks	Expected return	Standard deviation
A	8%	12%
B	13%	20%

(a) What must be the correlation of the return of stock A and the return of stock B if you must invest $164M in Stock A in order to form the global minimum variance portfolio (among all portfolios that combine stock A and stock B)?

Answer: We see that w=0.82 and we invert the the formula

$$w_1 = \frac{\sigma_2^2 - \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

and obtain rho= 0.3.

*w= 0.82 in Stock A (we invest $164M). The expected return is 8%*0.82 + 13%*0.12 =8.9% and the the stdv is 11.45%) (apply the formulae of the variance).*

(b) Assume now that, in addition to stock A and stock B, it is possible to invest in a risk free security with an interest rate of R_f =5%. The resulting tangency portfolio has an expected return of 11%. Assume again that you have $200M and that you want the highest expected return, provided that the standard deviation of your portfolio is 12%. Compute the amounts that you should invest is stock A, stock B and the risk free security as well as the resulting expected return.

Answer: First, we find the weight of the tangency portfoliofrom the expected return formula and compute the resulting stdev (14.2%).You must invest 16% ($32M) in the risk free asset and 84% ($168M) in the tangency portfolio meaning that you have to invest16% in the risk free asset, 50.4% ($100.8M) in A and 33.6% ($67.2M) in B. The resulting expected return is 10.4% (higher than 8%).

(c) Briefly sketch a risk return (σ,Er) diagram positioning the risk free security, stock A, stock B, the tangency portfolio, the portfolio formed in question (a) and the portfolio formed in question (b).

Problem 4 (Options)

IBM stock is currently selling for $100 and no dividend is currently scheduled for the next year. The 1-year European call option on the IBM stock with an exercise price of $105 is currently selling for $12. The annualized risk-free interest rate is 2.5% (APR annually compounded so that the present value of a payment of $105 in one year is PV(105) =105/1.025 = $102.439).
Please notice that the questions of this problem can be done independently.

a) Assuming there are no dividends paid on IBM, compute the price of a 1 one year European put option on the stock with exercise price equal to $105.

Answer: Using put-call parity we get:

P=C-S+K/1.025=12-100+105/1.025=$14.44.

b) Imagine that the European put described in question a. is selling in the market for $13. Is there an arbitrage opportunity? If so, execute a strategy to take advantage of the arbitrage by using the put, the call, the stock and the bond. Clearly state your positions and cash flows in an arbitrage table as well as the arbitrage profit.

Answer:

From put-call parity, the strategy is to buy the cheap put and sell the replicating portfolio, which consists of selling the call, buy the stock and borrowing cash.

	Today	Expiration of Options Contracts	
		S_T<X	S_T>X
Buy Put	-13	105-S_T	0
Sell a Call	+12	0	105- S_T
Buy the Stock	-100	S_T	S_T
Borrow	+102.4	-105	-105

The future net payoff from the position in the put and the replicating portfolio is 0, regardless of the future price of the stock. However, the payoff today is $1.04.

We can also gear the strategy to payoff in the future, as follows:

	Today	Expiration of Options Contracts	
		S_T<X	S_T>X
Buy Put	-13	105-S_T	0
Sell a Call	+12	0	105- S_T
Buy the Stock	-100	S_T	S_T
Borrow	+101	-103.557	-103.557

The future profit is 1.44.

c) A friend of yours wants to purchase a forward contract on IBM with a maturity of 1 year. Unfortunately, his bank told him that no such contract is available and he asks for your help. You can only buy (or sell) the above call and put options as well as buy (or sell) the above risk free bond. Can you replicate the cash flows of the forward contract with these options and the bond? Describe precisely the replication portfolio. What is the implied forward price? Assume that the put is fairly priced (see question a.).

Answer: Long call and short the put. You will end up with the amount 14.44-12=2.44. Since there is no upfront payment with a forward, this amount must be invested in the risk free bond which amounts to 2.44*1.025=$2.5. The implied forward price is the break-even price of the position, that is 105-2.5 =$102.5.

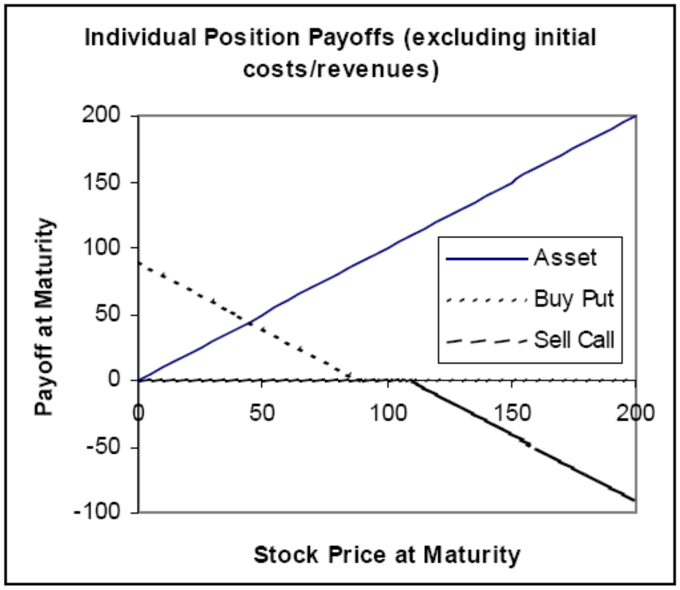
d) Imagine now that you are unable to short-sell the IBM stock for a period of one year. By using the European put, the European call and the risk free bond described above, state what actions you must take to replicate a short position in the IBM stock. Use an arbitrage table to support your argument.

Answer: To replicate shorting a stock, we need to buy a put, sell an otherwise identical call and borrow the present value of the strike price. This is seen in the table.

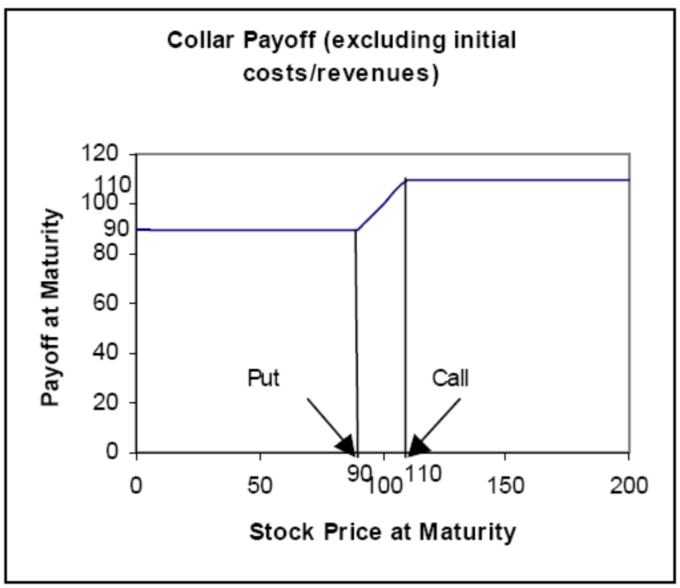
	Today	Expiration of Options Contracts	
		$S_T<X$	$S_T>X$
Short stock	+S	$-S_T$	$-S_T$
Buy a Put	-P	$X-S_T$	0
Sell a Call	+C	0	$X-S_T$
Borrow	X^*e^{-rT}	-X	-X

e) A "collar" is a combined position in which the investor:
• buys the underlying asset,
• buys an out-of-the-money European put option
• sells an out-of-the-money European call option, where both options have the same maturity (1 year).
Assume that the current stock price is $100, the strike price of the put option is $90 and the strike price of the call option is $110. Draw the payoff diagram for a collar position, (recall that payoff diagrams ***exclude*** the initial costs (or revenues) from the purchase or sale of the asset and options). In order to have full marks, clearly label in your diagram both axes, the current asset price, the strike prices of the two options as well as the level of the payoffs on the y-axis. The diagram has to be nicely presented and easy to read.

Answer: The payoff diagrams of the individual positions are:



The payoff of the collar position (adding the individual positions together) is



Problem 5. A stock currently trades for 35. In one year it will be 50 or 15. If it is 50 one year from now, then it will be either 70 or 35 one year after that (i.e. two years from now), and if it is 15 one year from now, then it will be either 35 or 10 one year after that. The one-year risk-free rate is 5% (and will be next year, too) APR annually compounded. In this problem, you are asked to use a two period binomial tree model to replicate a put option on this stock, expiring in two years, with strike price 35. The present time is t=0, the intermediate time is t=1 and the maturity date is t=2.

a. Calculate the hedge ratio (Δ) and the amount to borrow/lend with the bond (B) as well as the price of the put in each of the three nodes of the binomial tree (the nodes at time t=0 and t=1). Assume that the put option is European.

Answer: *The option is worthless at the $50 dollar node, so* $\Delta=0$ *at that node.*

At the period 1, $15 node, then the option can expire either worthlessly should the stock reach 35 or at $15. Therefore, the hedge ratio at the $15 period 1 node is,

$35\,\Delta_1 + 1.05B_1 = 0$

$10\,\Delta_1 + 1.05B_1 = 25$

This implies that $\Delta_1 = -\mathbf{1}$. *Plug that back in, we get* $10(-1)+1.05B_1=25$, *so* $B1 = 35/1.05 = 33.33$, *so the option value is* $(-1)(15)+33.33 = 18.33$

With this in mind, the initial synthetic relation is given by,

$50\,\Delta + 1.05B = 0$

$15\,\Delta + 1.05B = 18.33$

This implies that the initial hedge ratio is n = ***-0.52381.***

B= 24.94331, P = 6.6099

b. Calculate the delta and the time value of the European put at the node (t=1, S=15) and interpret the results. (a 2 or 3 lines answer should be sufficient)

Delta=-1 and time value is 1.67. Therefore early exercise is optimal.

c. Now assume that the put option is American. Calculate the premium of the American put in each node and explain the optimal exercise policy as well as the hedge ratio (Delta) and the amount to borrow/lend at each node.

The American put must be exercise at d because 20>18.33 and thus P=20. At time 0, we have delta=-0.57143, B=27.21 and P=7.21