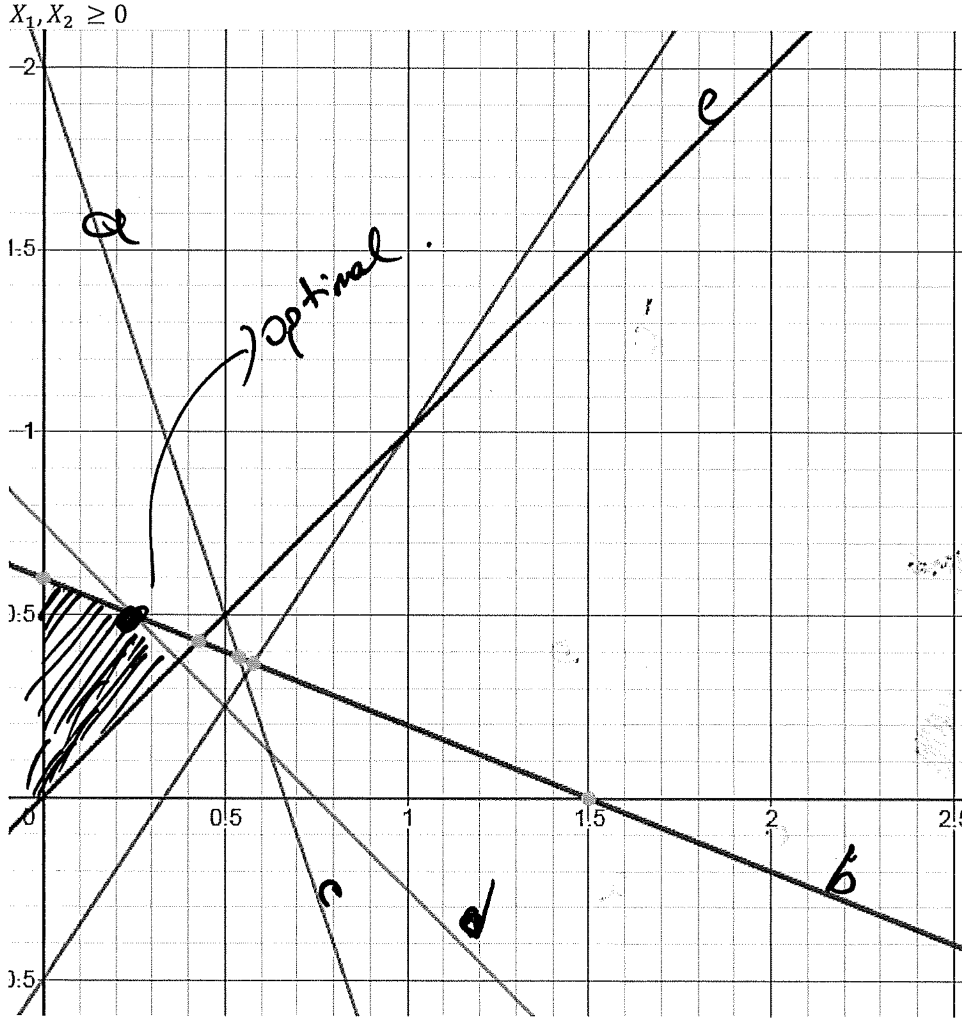
1. As Y_1, Y_2, Y_3 are integer and $Y_1 + Y_2 + Y_3 \leq 1$, this problem reduces to 4 separate cases:

i. Y_1, Y_2, Y_3 are all 0.
ii. $Y_1 = 1$ and other two are 0.
iii. $Y_2 = 1$ and other two are 0.
iv. $Y_3 = 1$ and other two are 0.

Case (i) is trivial. All the right hand sides become 0, therefore there would be only feasible solution with X_1, X_2 are also 0. Objective value in that case is 0.

For case (ii), we have the following problem:

$$\max X_1 + 2X_2$$
$$subject\ to:$$
$$3X_1 + X_2 \leq 2 \quad \text{Constraint a}$$
$$2X_1 + 5X_2 \leq 3 \quad \text{Constraint b}$$
$$4X_1 + 4X_2 \leq 3 \quad \text{Constraint c}$$
$$3X_1 - 2X_2 \leq 1 \quad \text{Constraint d}$$
$$-X_1 + X_2 \leq 0 \quad \text{Constraint e}$$
$$X_1, X_2 \geq 0$$



In this case, $X_1 = 0.25$ and $X_2 = 0.5$ with objective value of 1.25 is optimal when $Y_1 = 1$

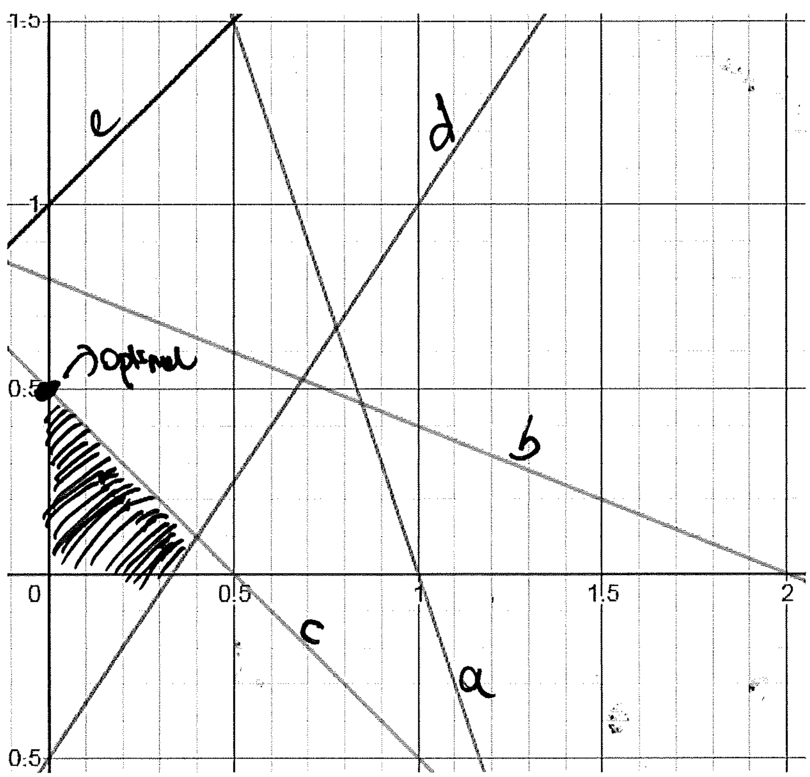
For case (iii), $Y_2 = 1$ and we have the following problem:

$$\begin{array}{lll} \max X_1 + 2X_2 & & \\ subject\ to: & & \\ 3X_1 + X_2 & \leq 1 & \text{Constraint a} \\ 2X_1 + 5X_2 & \leq 2 & \text{Constraint b} \\ 4X_1 + 4X_2 & \leq 5 & \text{Constraint c} \\ 3X_1 - 2X_2 & \leq -5 & \text{Constraint d} \\ -X_1 + X_2 & \leq 0 & \text{Constraint e} \\ X_1, X_2 \geq 0 & & \end{array}$$

This case is infeasible. Even without drawing it can be seen that the intersection of X_2 axis and Constraint d would be at point 2.5 and the constraint requires that $X_2 \geq 2.5$ while such high X_2 values, with nonnegative X_1 violates Constraints a,b,c.

For case (iv), $Y_3 = 1$ and we have the following problem.

$$\begin{array}{lll} \max X_1 + 2X_2 & & \\ subject\ to: & & \\ 3X_1 + X_2 & \leq 3 & \text{Constraint a} \\ 2X_1 + 5X_2 & \leq 4 & \text{Constraint b} \\ 4X_1 + 4X_2 & \leq 2 & \text{Constraint c} \\ 3X_1 - 2X_2 & \leq 1 & \text{Constraint d} \\ -X_1 + X_2 & \leq 1 & \text{Constraint e} \\ X_1, X_2 \geq 0 & & \end{array}$$



In this case, In this case, $X_1 = 0$ and $X_2 = 0.5$ with objective value of 1 is optimal when $Y_3 = 1$

Bringing 4 cases together to pick the best among them, $X_1 = 0.25,\ X_2 = 0.5, Y_1 = 1, Y_2 = 0,\ Y_3 = 0$ with objective value 1.25 is optimal.