Model Solution

1. Let p be the proposition "I am a penguin", q be the proposition "I look good in black", and r be the proposition "I like the cold". Translate the following expressions into English.

   a. $\neg q \vee r$

   I do not look good in black or I like the cold.

   b. $r \wedge \neg p$

   I like the cold but I am not a penguin.

   I like the cold and I am not a penguin.

   c. $\neg q \rightarrow \neg p$

   If I do not look good in black then I am not a penguin.

   I am not a penguin if I do not look good in black.

   I do not look good in black only if I am not a penguin.

   d. $p \leftrightarrow q$

   I am a penguin if and only if I look good in black.

   e. $q \oplus r$

   Either I look good in black or I like the cold, but not both.

2. Translate the following English expressions into logical statements. You must explicitly state what the atomic propositions are (e.g., "Let p be proposition ...") and then show their logical relation.

   a. Either it is a black bear or a grizzly bear.

   Let b be the proposition "it is a black bear."

   Let g be the proposition "it is a grizzly bear."

$$q \oplus r$$

b. If it is a polar bear then it lives in the arctic and eats seals.

Let p be the proposition "it is a polar bear."

Let a be the proposition "it lives in the arctic."

Let s be the proposition "it eats seals."

$$p \rightarrow (a \land s)$$

c. It is a panda bear but it does not eat anything but bamboo.

Let p be the proposition "it is a panda bear."

Let b be the proposition "it eats bamboo."

Let e be the proposition "it eats something else (i.e., other than bamboo)."

$$p \land (b \land \neg e)$$

d. It is a water bear if and only if it is an animal that can survive in outer space.

Let w be the proposition "it is a water bear."

Let a be the proposition "it is an animal."

Let s be the proposition "it can survive in outer space."

$$w \leftrightarrow (a \land s)$$

3. Determine which of the following are True and demonstrate why or why not by performing a reduction.

a. 1 < 2 and 2 < 3 and 3 < 1.

True and True and False

True and False

False

b. $1 < 1$ or $2 < 1$.

False or False

False

c. If $1 < 2$ then either $2 = 3$ or $2 < 3$.

If True then either False or True

If True then True

True

d. $1 = 1$ if and only if either $2 = 1$ or $2 = 2$.

True if and only if either False or True

True if and only if True

True

e. If $4 = 2^2$ and $2 = 1^2$ then your instructor is actually Nyarlathotep.

n.b., it is up to you whether you believe I am Nyarlathotep or not… but the reduction works either way! Let  *be the truth value (i.e., True or False) for whether or not you believe this!*

If $4 = 2^2$ and $2 = 1^2$ then your instructor is actually Nyarlathotep.

If True and False then 

If False then 

n.b., this can be reduced to true because $(F \rightarrow T) = T$ and $(F \rightarrow F) = T$

True

Model Solution

4. Using only the $\neg$ and the $\wedge$ operators, find a logical expression that is equivalent to $p \leftrightarrow \neg(q \wedge \neg r)$. For this question, you do not need to specify "how" you found the equivalent expression because you will show both techniques in questions 5 and 6 below.

$$\neg\big(p \wedge (q \wedge \neg r)\big) \wedge \neg(\neg(q \wedge \neg r) \wedge \neg p)$$

n.b., there are actually several equivalent logical expressions that observe this constraint, but I believe this will probably be the most common response (refer to the solution to 6)

5. Prove that the expression you found for question 4 above is equivalent to the expression $p \leftrightarrow \neg(q \wedge \neg r)$ by using only truth tables. Show all your work and do not skip any steps (i.e., ensure you that you include a new column for every single operation).

n.b., the row numbers are included for your convenience and are not necessary

	p	q	r	$\neg r$	$q \wedge \neg r$	$\neg(q \wedge \neg r)$	$p \leftrightarrow \neg(q \wedge \neg r)$
1	T	T	T	F	F	T	T
2	T	T	F	T	T	F	F
3	T	F	T	F	F	T	T
4	T	F	F	T	F	T	T
5	F	T	T	F	F	T	F
6	F	T	F	T	T	F	T
7	F	F	T	F	F	T	F
8	F	F	F	T	F	T	F

	p	q	r	$\neg p$	$\neg r$	$q \wedge \neg r$	$\neg(q \wedge \neg r)$	$p \wedge (q \wedge \neg r)$ A	$\neg(q \wedge \neg r) \wedge \neg p$ B	$\neg A$	$\neg B$	$\neg A \wedge \neg B$
1	T	T	T	F	F	F	T	F	F	T	T	T
2	T	T	F	F	T	T	F	T	F	F	T	F
3	T	F	T	F	F	F	T	F	F	T	T	T
4	T	F	F	F	T	F	T	F	F	T	T	T
5	F	T	T	T	F	F	T	F	T	T	F	F
6	F	T	F	T	T	T	F	F	F	T	T	T
7	F	F	T	T	F	F	T	F	T	T	F	F
8	F	F	F	T	T	F	T	F	T	T	F	F

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6. Prove that the expression you found for question 4 above is equivalent to the expression $p \leftrightarrow \neg(q \wedge \neg r)$ by using only the logical equivalences. Show all your work and do not skip any steps.

$$p \leftrightarrow \neg(q \wedge \neg r)$$

by the biconditional equivalence

$$= \big(p \rightarrow \neg(q \wedge \neg r)\big) \wedge (\neg(q \wedge \neg r) \rightarrow p)$$

by the implication equivalence

$$= \big(\neg p \vee \neg(q \wedge \neg r)\big) \wedge (\neg(q \wedge \neg r) \rightarrow p)$$

by the implication equivalence

$$= \big(\neg p \vee \neg(q \wedge \neg r)\big) \wedge (\neg\neg(q \wedge \neg r) \vee p)$$

n.b., the following variable assignment is not necessary, but I have included it in this proof here because I believe it will make some of the following steps clearer

let $(q \wedge \neg r) = A$, *substitute*

$$= (\neg p \vee \neg A) \wedge (\neg\neg A \vee p)$$

by double negation

$$= (\neg p \vee \neg A) \wedge (A \vee p)$$

by DeMorgan's Law

$$= \neg(\neg\neg p \wedge \neg\neg A) \wedge (A \vee p)$$

by DeMorgan's Law

$$= \neg(\neg\neg p \wedge \neg\neg A) \wedge \neg(\neg A \wedge \neg p)$$

by double negation

$$= \neg(p \wedge \neg\neg A) \wedge \neg(\neg A \wedge \neg p)$$

by double negation

$$= \neg(p \wedge A) \wedge \neg(\neg A \wedge \neg p)$$

substitute for A

$$= \neg\big(p \wedge (q \wedge \neg r)\big) \wedge \neg(\neg(q \wedge \neg r) \wedge \neg p)$$

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7. Determine if the following expressions are tautologies, contradictions, or contingencies by using truth tables. Show all your work.

a. $(p \leftrightarrow q) \land (\neg p \land q)$

... is a contradiction

	p	q	$\neg p$	$\neg p \land q$	$p \leftrightarrow q$	$(p \leftrightarrow q) \land (\neg p \land q)$
1	T	T	F	F	T	F
2	T	F	F	F	F	F
3	F	T	T	T	F	F
4	F	F	T	F	T	F

b. $\neg(\neg p \land (p \land q))$

... is a tautology

	p	q	$\neg p$	$p \land q$	$\neg p \land (p \land q)$	$\neg(\neg p \land (p \land q))$
1	T	T	F	T	F	T
2	T	F	F	F	F	T
3	F	T	T	F	F	T
4	F	F	T	F	F	T

c. $((q \lor p) \lor (\neg p \land \neg q)) \land (p \lor \neg p)$

... is a tautology

	p	q	$\neg p$	$\neg q$	$\neg p \land \neg q$	$q \lor p$	$(q \lor p) \lor (\neg p \land \neg q)$	$\neg p \lor p$	$((q \lor p) \lor (\neg p \land \neg q)) \land (p \lor \neg p)$
1	T	T	F	F	F	T	T	T	T
2	T	F	F	T	F	T	T	T	T
3	F	T	T	F	F	T	T	T	T
4	F	F	T	T	T	F	T	T	T

d. $\neg(\neg p \lor (\neg p \land \neg q)) \land \neg p$

... is a contradiction

	p	q	$\neg p$	$\neg q$	$\neg p \land \neg q$	$\neg p \lor (\neg p \land \neg q)$	$\neg(\neg p \lor (\neg p \land \neg q))$	$\neg(\neg p \lor (\neg p \land \neg q)) \land \neg p$
1	T	T	F	F	F	F	T	F
2	T	F	F	T	F	F	T	F
3	F	T	T	F	F	T	F	F
4	F	F	T	T	T	T	F	F

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e. $\neg p \rightarrow (q \vee \neg(\neg p \vee \neg q))$

... is a contingency

	p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$	$q \vee \neg(\neg p \vee \neg q)$	$\neg p \rightarrow (q \vee \neg(\neg p \vee \neg q))$
1	T	T	F	F	F	T	T	T
2	T	F	F	T	T	F	F	T
3	F	T	T	F	T	F	T	T
4	F	F	T	T	T	F	F	F

8. Let L(x) be the predicate "x is a lion", G(x) be the predicate "x is a giraffe", and M(x) be the predicate "x eats meat". Translate the following expressions into English. The universe of discourse is all animals.

   n.b., there are many possible translations - I am providing that which I believe "sounds" most like the equivalent expression in conversational English.

   a. $\exists x \left(\neg(G(x) \vee L(x)) \wedge M(x)\right)$

      There is an animal that eats meat and is neither a giraffe nor a lion.

   b. $\forall x \left(M(x) \rightarrow \left(\neg G(x) \wedge (L(x) \vee \neg L(x))\right)\right)$

      Every animal that eats meat is not a giraffe and may or may not be a lion.

9. *This question has been moved to the next assignment – stay tuned!*