



COURSE: CEG2136
Computer Architecture I

Assignment 2

Q1. In a signed binary system of 8 bits ($N=8$):

a) Find the 2's complement representation of the following signed numbers :

$$(+63)_{10} = (00111111)_2$$

63/2	remainder		
31	1	lsb	2^0
15	1		2^1
7	1		2^2
3	1		2^3
1	1		2^4
0	1		2^5

	Sign	2^6	2^5	2^4	2^3	2^2	2^1	2^0
+63=	0	0	1	1	1	1	1	1

Or observe that $63 = 32+16+8+4+2+1$

$$(-115)_{10} = (10001101)_2$$

115/2	remainder		
57	1	lsb	2^0
28	1		2^1
14	0		2^2
7	0		2^3
3	1		2^4
1	1		2^5
0	1	msb	2^6

	Sign	2^6	2^5	2^4	2^3	2^2	2^1	2^0
+115=	0	1	1	1	0	0	1	1

$-115 = 2$'s complement of $(01110011) = 10001101$

- b) Find the 2's complement of the following signed numbers and give your results in decimal, too: $(-63)_{10}$ and $(+115)_{10}$

The 2's complement representation of $(-63)_{10} = (11000001)_2$
 \Rightarrow Do the 2's complement of $(-63)_{10}$ as required in the question
 $=$ 2's complement of $(11000001)_2 = (00111111)_2$ = which is the 2's complement representation of $(+63)_{10}$ (which in fact is the result that we expected)
 The 2's complement representation of $(+115)_{10} = (01110011)$
 \Rightarrow Do the 2's complement of $(+115)_{10} =$ 2's complement of $(01110011) = (10001101)_2 =$ 2's complement representation of $(-115)_{10}$

- c) Perform the following arithmetic operations using the signed-2's complement representation and provide your results in decimal (including intermediary steps), as well:

- 1) Say $A = (+115)_{10}$ and $B = (-63)_{10}$

$$S = A + B = (+115)_{10} + (-63)_{10}$$

	2's complement representation									Base 10
	CY ₈	CY ₇	CY ₆	CY ₅	CY ₄	CY ₃	CY ₂	CY ₁	CY ₀	
CY:	1	1	0	0	0	0	1	1	0	
A +		0	1	1	1	0	0	1	1	115 ₁₀₊
B		1	1	0	0	0	0	0	1	-63 ₁₀
S=A+B		0	0	1	1	0	1	0	0	52 ₁₀
	CY₈ = CY₇ = 1 \Rightarrow NO overflow									

- 2) Say $C = (-115)_{10}$ and $D = (-63)_{10}$

$$X = C - D = (-115)_{10} - (-63)_{10} = (-115)_{10} + [- (-63)]_{10} = (-115)_{10} + (+63)_{10}$$

	2's complement representation									Base 10
	CY ₈	CY ₇	CY ₆	CY ₅	CY ₄	CY ₃	CY ₂	CY ₁	CY ₀	
CY:	0	0	1	1	1	1	1	1	0	
C+		1	0	0	0	1	1	0	1	-115 ₁₀
-D		0	0	1	1	1	1	1	1	63 ₁₀
X=C-D		1	1	0	0	1	1	0	0	-52 ₁₀

As the msb of $X = C - D$ is 1 $\Rightarrow X$ is a negative number ($X = -y$) with a magnitude $y = -(-y) = -(X) = 2$'s complement of $(X) = 2$'s complement of $(11001100) = 00110100 = 52_{10}$
 $\Rightarrow X = -y = -52_{10}$
CY₈ = CY₇ = 0 \Rightarrow NO overflow

Q2. a) Identify the decimal number which is represented next with 32-bit in the IEEE 754 standard: $(1 \mathbf{10001011} \mathbf{111010000000000000000000}) = (?)_{10}$

Sign = 1 \Rightarrow negative ; C = 139, hence Exp = C-127 = 12

1.M = 1.11101 (where M = mantissa)

$$-1.11101 * 2^{12} = -1111010000000 = (-7808)_{10}$$

- b) Represent (221.390625) in the IEEE 754 standard with 32 bits.

$$(221.390625)_{10} = (11011101.011001)_2 = (1.1011101011001) \times 2^7$$

Sign = 0 ; Exp = 7 ; C = 127 + 7 = 134 = $(10000110)_2$

M = **1011101011001**

$$(221.390625)_{10} = (0 \mathbf{10000110} \mathbf{101110101100100000000000})_{\text{IEEE754}}$$

3.1.

$$\begin{aligned}(101110)_2 &= 32 + 8 + 4 + 2 = 46 \\ (1110101)_2 &= 64 + 32 + 16 + 4 + 1 = 117 \\ (110110100)_2 &= 256 + 128 + 32 + 16 + 4 = 436\end{aligned}$$

3.3.

$$\begin{aligned}(1231)_{10} &= 1024 + 128 + 64 + 15 = 2^{10} + 2^7 + 2^6 + 2^3 + 2^2 + 2 + 1 = (10011001111)_2 \\ (673)_{10} &= 512 + 128 + 32 + 1 = 2^9 + 2^7 + 2^5 + 1 = (1010100001)_2 \\ (1998)_{10} &= 1024 + 512 + 256 + 128 + 64 + 8 + 4 + 2 \\ &= 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^3 + 2^2 + 2^1 = (11111001110)_2\end{aligned}$$

3.4

$$\begin{aligned}(7562)_{10} &= (16612)_8 \\ (1938)_{10} &= (792)_{16} \\ (175)_{10} &= (10101111)_2\end{aligned}$$

3.5

$$\begin{aligned}(F3A7C2)_{16} &= (1111\ 0011\ 1010\ 0111\ 1100\ 0010)_2 \\ &= (74723702)_8\end{aligned}$$

3.8

$$\begin{aligned}(295)_{10} &= 256 + 32 + 7 = (100100111)_2 \\ (a) &0000\ 0000\ 0000\ 0001\ 0010\ 0111 \\ (b) &0000\ 0000\ 0000\ 0010\ 1001\ 0101 \\ (c) &10110010\ 00111001\ 00110101\end{aligned}$$

3.10. Decode the following ASCII code:

1001010	1001111	1001000	1001110	0100000	1000100	1001111	1000101
4A	4F	48	4E	20	44	4F	45
J	O	H	N		D	O	E

3.13

	10101110;	10000001	10000000	00000001	00000000.
1's compl	01010001	01111110	01111111;	11111110	11111111
2's compl	01010010	01111111	10000000	11111111	00000000

3.16

$$\begin{array}{r} +42 = 0101010 \\ -42 = 1010110 \\ \hline (+42) 0101010 \\ (-13) 1110011 \\ \hline (+29) 0011101 \end{array} \qquad \begin{array}{r} +13 = 0001101 \\ -13 = 1110011 \\ \hline (-42) 1010110 \\ (+13) 0001101 \\ \hline (-29) 1100011 \end{array}$$

