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1. Evaluate.

[1] (a) $2^{-3} - 3^{-2}$ $\frac{1}{2^3} - \frac{1}{3^2} \rightarrow \frac{1}{8} - \frac{1}{9} \rightarrow \frac{9}{72} - \frac{8}{72} = \frac{1}{72}$

Q1a 1

[1] (b) $64^{2/3}$ $(2^6)^{2/3} \rightarrow 2^4 \rightarrow 16$

Q1b 1

[1] (c) $\sqrt{2}\sqrt{50}$ $\sqrt{2} \times 5\sqrt{2} = 10$

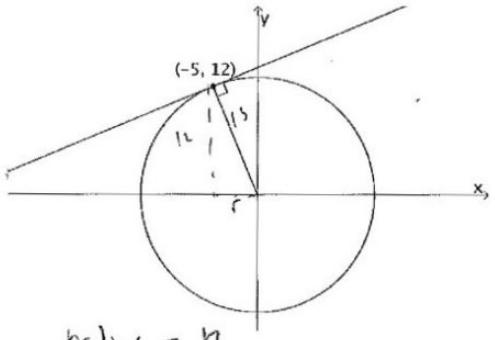
Q1c 1 $\frac{2\sqrt{50}}{\sqrt{2}\sqrt{25}}$ $5\sqrt{2}$

[2] **2.** Solve the inequality $|x - 5| \leq 3$ and express the solution using interval notation.

Q2 1 $|x-5| \leq 3$ $x - 5 \leq 3$ $x \leq 8$ $x - 5 \geq -3$ $x \geq 2$ $2 \leq x \leq 8$

[5] **3.** Find equations for the circle and the line in the figure below.

Q3 0



$-5, 12$

Circle = $(x-13)$

line =

radius = 13

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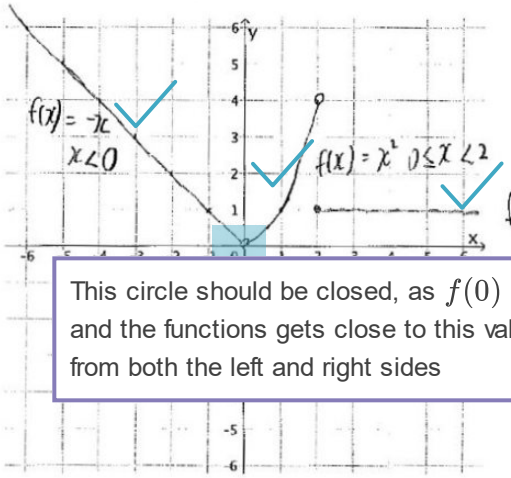
[5] **4.** Sketch the graph of

Q4 2.5

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x < 2 \\ 1 & \text{if } x \ge 2. \end{cases}$$



This circle should be closed, as $f(0) = 0$ and the functions gets close to this value from both the left and right sides

[3] **5.** Determine whether $f(x) = x^3 - x^7$ is even, odd, or neither.

Q5 1

$f(x) = x^3 - x^7$

$f(x) = (-x)^3 - (-x)^7$

$-x^3 + x^7$

it is neither because it isn't the same as the original nor it is the opposite.

[4] **6.** Find the minimum value of the function $g(x) = 2x^2 + 4x - 5$.

Q6 0

$2x^2 + 4x - 5$

$-4 \times 2 \times -5$

$\frac{-4 \pm \sqrt{16+40}}{4}$

$\frac{-4 \pm \sqrt{56}}{4}$

$\frac{-4 \pm 2\sqrt{14}}{4}$

$\frac{-2 \pm \sqrt{14}}{2}$ or $\frac{-2 - \sqrt{14}}{2}$

minimum value = $\frac{-2 - \sqrt{14}}{2}$

$\frac{2(-2 - \sqrt{14})}{4}$ $\quad 2\sqrt{14}$

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[2] **7.** If $T(x) = \dfrac{1}{\sqrt{1+\sqrt{x}}}$, find functions f, g, and h, each simpler than T, such that

$$f \circ g \circ h = T.$$

Q7 2

$h = \sqrt{x}$

$g = \sqrt{1+x}$ ✓

$f = \dfrac{1}{x}$

[3] **8.** (a) If $f(x) = \sqrt{3-x}$, find the inverse function f^{-1} and state its domain.

Q8a 1

$f(x) = \sqrt{3-x}$ →

$x = \sqrt{3-y}$

$x^2 = 3-y$

$x^2 - 3 = -y$

$y = -x^2+3$ ✓

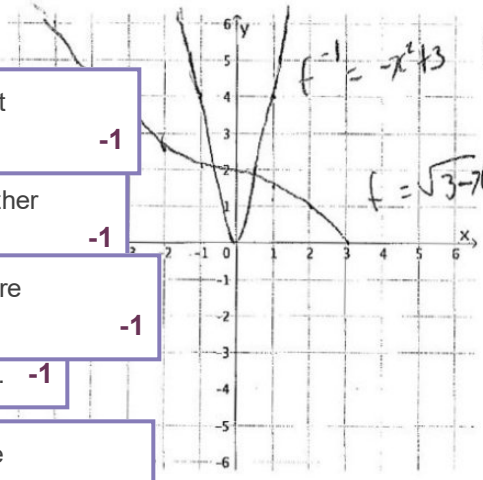
inverse function $= -x^2+3$

domain $= (-\infty, \infty)$

The domain of f^{-1} is the range of f, namely $[0, \infty)$.

[5] (b) Sketch the graphs of f and f^{-1} from part (a) on the same coordinate axes.

Q8b 0



$f^{-1} = -x^2+3$ $(-\infty, \infty)$ as domain

$f = \sqrt{3-x} \to (-\infty, 3]$ as domain

Parabolas not opening in the right direction -1

The symmetry in this picture is either missing or wrong. -1

The positions of the functions are wrong. -1

This is not parabola-shaped. -1

f^{-1} has domain $[0, \infty)$, so the downward parabola should not have its left side. -1

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$(x+3)(x^2-x+3)$

$x^3-x^2+3x+3x^2-3x+9$

x^3+2x^2+9

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Q9 6

9. Find the quotient and remainder when $x^3 + 2x^2 - 10$ is divided by $x + 3$.

$x+3 \overline{)\, x^3+2x^2-10}$

$x^2 - x + 3$

$-x^3 + 3x^2$

$-x^2 - 10$

$-x^2 - 3x$

$3x - 10$

$3x + 9$

-19

quotient $= x^2 - x + 3$

remainder $= -19$

Q10 0

[4] 10. What is the remainder when $P(x) = x^{500} + 6x^{201} - x^2 - 2x + 4$ is divided by $x - 1$?

Use the remainder theorem, evaluate $P(1)$.

remainder $= -3$

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11. Find a polynomial of degree 3 with constant term 12 and zeros $-\frac{1}{2}$, 2, and 3.

Q11 1

Using the factor theorem, the polynomial will be in the form

$$p(x) = k(x + 1/2)(x - 2)(x - 3)$$

for some constant k.

Now, using p(0) = 12, solve for k.

$-(x+\frac{1}{2})(x-2)(x-3)$ 12

$(x+\frac{1}{2})(x^2-5x+6)$

$x^3 - \frac{5}{2}x^2 + \frac{1}{2}x^2 - \frac{5}{2}x + 3$

[6] **12.** Find all x- and y-intercepts and asymptotes for the graph of the function

$$r(x) = \frac{3x - 12}{x + 1}.$$

Q12 6

Vertical asymptote = -1

Horizontal asymptote = 3

x intercept = 4

y intercept = -12

$0 = \frac{3x-12}{x+1}$ $\frac{3(x-4)}{x+1}$ $x \neq -1$

$\frac{3(0)-12}{0+1} = -12$

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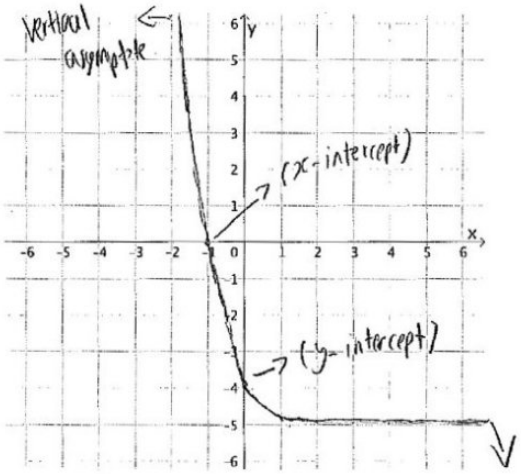
13. Sketch the graph of $g(x) = 5^{-x} - 5$. Label any x- or y-intercepts and asymptotes clearly.

Q13 6



$y = 5^{-x} - 5$

$0 = 5^{-x}$

$5 = 5^{-x}$

5^{-1}

$\frac{1}{5}$ $\frac{25}{5}$ $\frac{-24}{5}$

$\frac{1}{25}$ $\frac{-125}{25}$ $\frac{124}{25}$

14. Evaluate.

[1] (a) $\log_8(1)$

Q14a 0

$= \log_2 2^4 = 4 \log_2$

[1] (b) $e^{2\ln(5)}$

Q14b 0

$= 10$

[1] (c) $\log(\log(10^{100}))$

Q14c 1

$\log(\log_{10} 10^{100})$ $\log_{10} 100 = 2 \log_{10} 10 = 2$

[1] (d) $\log_4\left(\frac{1}{8}\right)$

Q14d 0

$\log_{2^2}(2^{-3}) \rightarrow \log_2 2^{-1} \rightarrow \frac{1}{2}\log_2$

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15. Solve the equation

$$\log(x) + \log(x+1) = \log(12).$$

Q15 4

$\log(x(x+1)) = \log 12$

$\log(x^2+x) = \log(12)$

$x^2+x = 12$

$x^2+x-12 = 0$

$x \quad +4$

$x \quad -3$

$x = -4$ → extraneous, hence it isn't a value

$x = 3$

$x = 3$

[6] **16.** A culture of bacteria is growing exponentially. The culture contains 10,000 bacteria initially, and after an hour the bacteria count is 25,000. Find the doubling period.

Q16 0

$f(x) = 10000^{0.65h}$

$\frac{15000-25000}{25000} = 0.65$ $\frac{15664}{25000} = 65$

65% ↑

The model is $P(t) = P_0 2^{t/T_d}$ where T_d is the doubling period.

$20000 = 10000$

doubling period : 39 minutes

$0.65h\sqrt{20000} = 10000$

$h = 39'$

$= 20000$

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3x180
2180
x3
540

135
4|540
4
14
12
20

7180
-9x'
1620

810
2|1620

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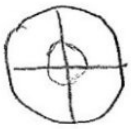
17. Find the exact value.



[2] (a) $\sin\left(\frac{3\pi}{4}\right)$ = sin 135° → $\frac{\sqrt{2}}{2}$ ✓

Q17a 2

[2] (b) $\tan\left(-\frac{\pi}{3}\right)$ = $-\sqrt{3}$ ✓

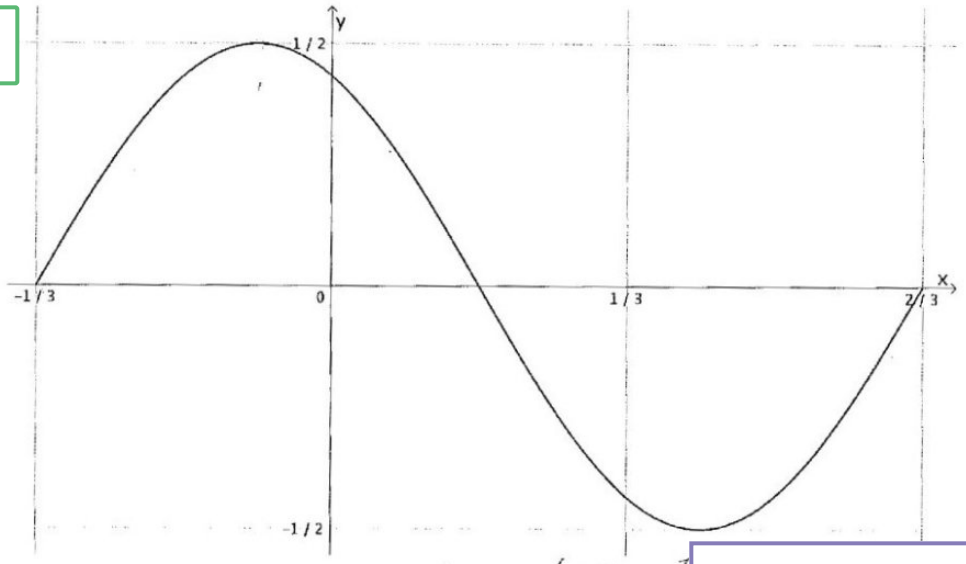
Q17b 2

[2] (c) $\cos\left(\frac{9\pi}{2}\right)$ = 0 ✓

Q17c 2

[4] **18.** The graph of one period of a function of the form $y = a\sin(k(x-b))$ or $y = a\cos(k(x-b))$ is shown. Determine the function.

Q18 2.5



2/3 + 1/3 = 1

$y = \frac{1}{2}\cos\left(2\pi - \right.$

$y = \frac{1}{2}\cos\left(2\pi\,(x\right.$

If you use cos, there is a leftward shift by $\frac{1}{12}$, so this should be $\left(x - \frac{1}{12}\right)$

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$\frac{1}{4\pi}$ 3120 × 4 = 10 7

3 | 720 = 240, 6, 12

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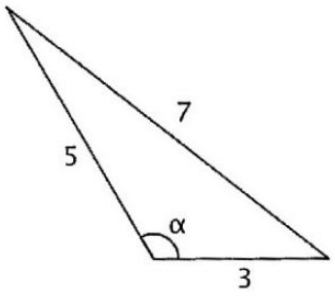
19. (a) Find the measure of angle α in the triangle shown below.

Q19a 5



The angle is $\frac{2\pi}{3}$ or 120°. **-1**

$4\pi/3$

$7^2 = 5^2 + 3^2 - 2 \times 5 \times 3 \cos\alpha$

$2\pi/3$

$49 = 25 + 9 - 30\cos\alpha$

$15 = -30\cos\alpha$

$-\frac{1}{2} = \cos\alpha$

$\alpha = \cos^{-1}\left(-\frac{1}{2}\right)$ $\pi/3$

[4] (b) Find the area of the triangle in part (a).

Q19b 4

$\frac{1}{2} ab\sin(\alpha)$

$\frac{1}{2} \cdot 15 \cdot \sin\alpha$

$\frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{4} \cdot 15$ $\frac{15\sqrt{3}}{4}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \frac{1}{4} = 1$

$\sin\alpha = \frac{\sqrt{3}}{2}$

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$$12\overline{)180}\quad 15,\ \ 12,\ \ 60$$

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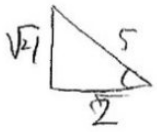
100 - D100

20. Find the exact value of the expression.

[3] (a) $\tan\left(\sin^{-1}\left(\frac{2}{5}\right)\right)$

Q20a 0



$\frac{\sin}{\cos}\left(\sin^{-1}\left(\frac{2}{5}\right)\right)$,

$\frac{1}{\cos}\left(\frac{2}{5}\right)$

$\sec\left(\frac{2}{5}\right)$

$\cos = \frac{5}{2}$

[3] (b) $\sin(2\theta)$ given that $\sec(\theta) = \frac{3}{2}$ and θ is in Quadrant IV.

Q20b 3

$\sin 2\theta = 2\sin\theta\cos\theta$ ✓

$\frac{1}{\cos(\theta)} = \frac{3}{2}$

$\cos\theta = \frac{2}{3}$ ✓

$2 \times -\frac{\sqrt{5}}{3} \times \frac{2}{3}$

$\frac{-4\sqrt{5}}{9}$ ✓

$\frac{4}{9} + \sin^2\theta = 1$

$\sin^2\theta = \frac{5}{9}$ $= \frac{\sqrt{5}}{3}$ $-\frac{\sqrt{5}}{3}$

space ... the IV quadrant ... it is

[4] (c) $\frac{1}{2}\cos\left(\frac{\pi}{12}\right) + \frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{12}\right)$

Q20c 4

$\frac{1}{2}\cos(\pi/4 - \pi/6) + \frac{\sqrt{3}}{2}\sin(\pi/4 - \pi/6)$

$\frac{1}{2}\left((\cos\pi/4 \cdot \cos\pi/6) + (\sin\pi/4 \cdot \sin\pi/6)\right) + \frac{\sqrt{3}}{2}\left((\sin\pi/4 \cdot \cos\pi/6) - (\sin\pi/6 \cdot \cos\pi/4)\right)$

$\frac{1}{2}\left(\left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right)\right) + \frac{\sqrt{3}}{2}\left(\left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right)\right)$

$\frac{1}{2}\left(\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right)$

$\frac{\sqrt{6}+\sqrt{2}}{8} + \frac{\sqrt{18} - \sqrt{6}}{8}$ $\frac{\sqrt{6}+\sqrt{2}+3\sqrt{2}-\sqrt{6}}{8} = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2}$ ✓

answer

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