Student No.:______________________________

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GINA CODY SCHOOL OF ENGINEERING AND COMPUTER SCIENCE

CONCORDIA UNIVERSITY

MIDTERM EXAMINATION	
COURSE: ENGR 311	TRANSFORM CALCULUS
INSTRUCTOR: M. ESHAGHI	MAX. MARKS: 100
DATE: 29 OCTOBER 2019	DURATION: 75 MINUTES

- *Write directly on this question booklet. Show your work in enough details and clearly to qualify for partial marks*
- *YOU CAN FIND FORMULA SHEET IN THE LAST PAGE*
- *YOU CAN DETACH THE FORMULA SHEET*
- *TOTAL NUMBER OF PAGES: 7*

Question #	1:	
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Question #1 (2~~0~~5 Marks)

Solve following system of equations and find $x(t)$ and $y(t)$.

$$\begin{cases} \frac{dx}{dt} - 2x - y = 0 \\ \frac{dy}{dt} - 3x - 4y = 0 \end{cases} \qquad x(0) = 1, \quad y(0) = 0$$

$$\begin{cases} \mathcal{L}\{\frac{dx}{dt}\} - 2\mathcal{L}\{x\} - \mathcal{L}\{y\} = 0 \\ \mathcal{L}\{\frac{dy}{dt}\} - 3\mathcal{L}\{x\} - 4\mathcal{L}\{y\} = 0 \end{cases} \Rightarrow \begin{cases} sX(s) - x(0) - 2X(s) - Y(s) = 0 \\ sY(s) - y(0) - 3X(s) - 4Y(s) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (s-2)X(s) = 1 + Y(s) \\ -3X(s) = -(s-4)Y(s) \end{cases} \Rightarrow X(s) = Y(s)\frac{s-4}{3}$$

$$(s-2)\left(Y(s)\frac{s-4}{3}\right) - Y(s) = 1 \Rightarrow Y(s) = \frac{3}{(s-5)(s-1)}$$

$$X(s) = \frac{s-4}{(s-5)(s-1)}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{-3/4}{s-1} + \frac{3/4}{s-5}\right\} = -\frac{3}{4}e^{t} + \frac{3}{4}e^{5t}$$

$$x(t) = \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{3/4}{s-1} + \frac{1/4}{s-5}\right\} = \frac{3}{4}e^{t} + \frac{1}{4}e^{5t}$$

Question #2 (25 Marks)

a) Solve following initial value problems and find $y(t)$

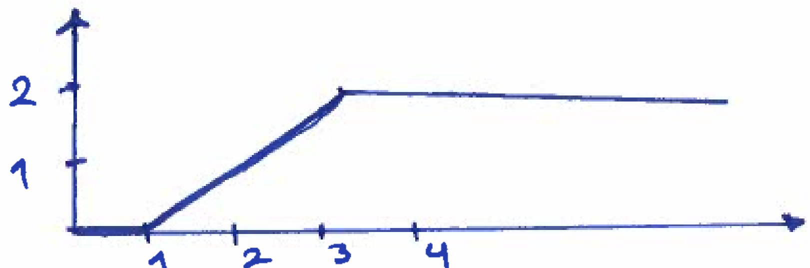
$$y'' = \delta(t-1) - \delta(t-3), \qquad y(0) = 0, \; y'(0) = 0,$$

Graph the function in the time range of [0,6].

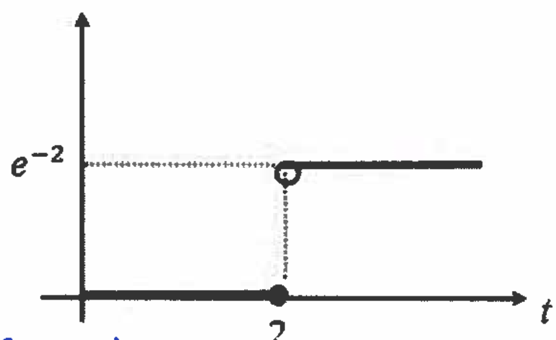
$$s^2 Y - s\,y(0) - y'(0) = e^{-s} - e^{-3s}$$

$$Y(s) = \frac{e^{-s}}{s^2} - \frac{e^{-3s}}{s^2}$$

$$y(t) = \mathcal{L}\{Y(s)\} = (t-1)u(t-1) - (t-3)u(t-3)$$



b) The graph of the function $g(t) = \int_0^t e^{\alpha t}\delta(t-t_0)dt, \quad 0 \le t < \infty$ is shown. Determine the constants α and t_0.



$$\mathcal{L}\{g(t)\} = \frac{\mathcal{L}\, e^{\alpha t}\delta(t-t_0)}{s} = \frac{e^{-t_0(s-\alpha)}}{s} = \frac{e^{t_0\alpha}\, e^{-t_0 s}}{s}$$

$$\mathcal{L}^{-1}\left\{e^{t_0\alpha}\frac{e^{-t_0 s}}{s}\right\} = e^{t_0\alpha}u(t-t_0)$$

$$g(t) = e^{-2}u(t-2) \longrightarrow \begin{array}{l}\alpha = -1 \\ t_0 = 2\end{array}$$

Question #3 (20 Marks)

Find Laplace transform of following functions.

a) $f(t) = \int_0^t \sin^2(5\tau)d\tau$

$$\mathcal{L}\{f(t)\} = \frac{\mathcal{L}\{\sin^2 5t\}}{s} = \frac{\frac{1}{2}\mathcal{L}\{(1-\cos 10t)\}}{s}$$

$$= \frac{1}{2} \frac{1/s - \frac{s}{s^2+100}}{s} = \frac{1}{2}\left(\frac{1}{s^2} - \frac{1}{s^2+100}\right)$$

b) $f(t) = t\int_0^t e^{2(\tau-t)} \cos(5\tau)\, d\tau$

$$\mathcal{L}\{f(t)\} = (-1)\frac{d}{ds}\mathcal{L}\{e^{-2t}\}\mathcal{L}\{\cos 5t\}$$

$$= -\frac{d}{ds} \frac{1}{s+2} \frac{s}{s^2+25}$$

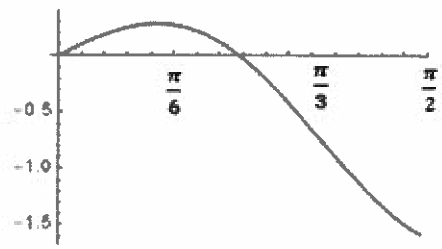
Question #4 (~~25~~ 30 Marks)

For the given function:

$$f(x) = \begin{cases} x\cos(2x) & 0 \le x < \pi/2 \\ -1 & \pi/2 \le x < \pi \end{cases}$$

a) Find coefficient b_2 in the Fourier Sine series expansion.
b) Plot the Fourier Sine series in the range of $[-2\pi, 2\pi]$
c) To what values will the Fourier Sine Series converge at $x = \frac{21\pi}{2}$, $x = 6\pi$ and $x = \frac{7\pi}{4}$

Note: Graph of function $x\cos(2x)$ on the range of $\left[0, \frac{\pi}{2}\right]$ is as follows



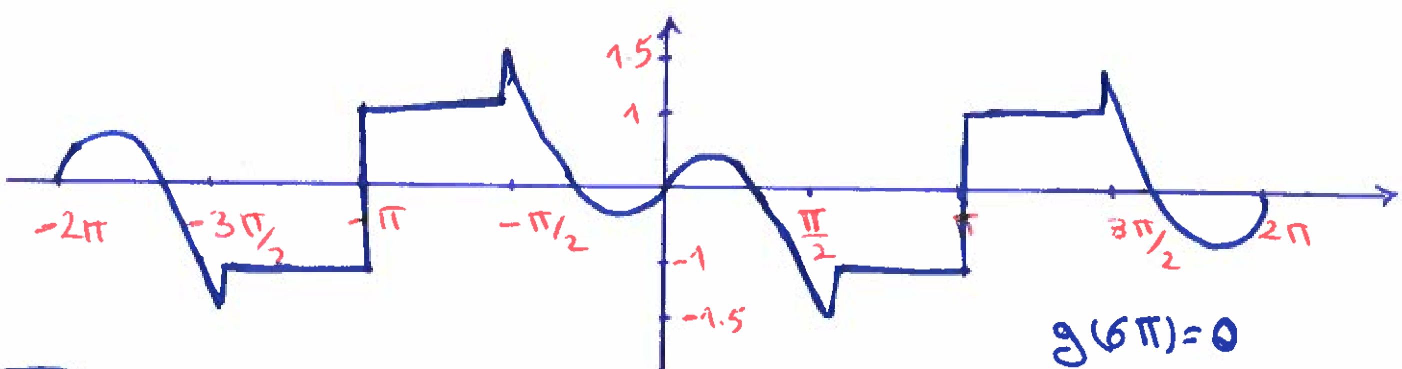
$P = \pi$

$$b_2 = \frac{2}{\pi}\int_0^{\pi} f(x)\sin 2x\,dx$$

$$b_2 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} x\sin 2x\cos 2x\,dx + \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi}(-1)\sin 2x\,dx$$

$$b_2 = \frac{1}{\pi}\int_0^{\pi/2} x\sin 4x\,dx - \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi}\sin 2x\,dx$$

$$b_2 = -\frac{1}{8} + \frac{2}{\pi}$$



$T = 2\pi$

$g(6\pi) = 0$

$g(7\pi/4) = -0 = g(-\pi/4)$

$g(21\pi/2) = g(21\pi/2 - 10\pi) = g(\pi/2) = \frac{-1-1.5}{2} = -1.25$