MATH 1007 F, TEST 1

Time: 50 Minutes, Total Mark: 40

Name: ..... Student #: .....

Solution

Question 1 *Find the natural domain of the following functions. (Show steps of your work)*

a) *[3]* $f(x) = x - 2 + \sqrt{x-2} + \dfrac{1}{(x-2)^2}$

$D(f) = D(x-2) \cap D(\sqrt{x-2}) \cap D\left(\dfrac{1}{(x-2)^2}\right)$

$= (-\infty, \infty) \cap [2, \infty) \cap \{(-\infty, 2) \cup (2, \infty)\}$ → 2 marks

$= (2, \infty)$ → 1 mark

b) *[3]* $f(x) = \dfrac{2\sqrt{x} - 3}{2 - \sqrt{x^2 - 4}}$

$D(2\sqrt{x} - 3) \cap D(2 - \sqrt{x^2-4}) = [0, \infty) \cap \{(-\infty, -2] \cup [2, \infty)\}$

excluding $2 - \sqrt{x^2-4} = 0 \Rightarrow x = \pm\sqrt{8}$ $(= [2, \infty))$ → 1.5 mark

→ (0.5) marks

$D(f) = [2, \infty) - \{\sqrt{8}\}$ → (1 mark)

Question 2 *Which one of the following functions is even and which one is odd. (Show steps of your work)*

a) *[3]* $f(x) = \dfrac{x^3 \sin(x)}{\cos(x)}$

$f(-x) = \dfrac{(-x)^3 \sin(-x)}{\cos(-x)} = \dfrac{(-x^3)(-\sin x)}{\cos x}$

$= \dfrac{x^3 \sin x}{\cos x} = f(x)$ → 2.5 marks

so, $f(x)$ is even → (0.5) mark

b) *[3]* $f(x) = \sin x + \dfrac{x}{x^2 - 1}$

$f(-x) = \sin(-x) + \dfrac{-x}{(-x)^2 - 1}$

$= -\sin x - \dfrac{x}{x^2 - 1}$

$= -\left(\sin x + \dfrac{x}{x^2-1}\right) = -f(x)$ → 2.5 marks

so, $f(x)$ is odd → 0.5 mark

Question 3 *[8] Let* $f(x) = \sqrt{\frac{x}{x-1}}$ *and* $g(x) = \frac{x^2}{x^2-1}$. *Find* $(fog)(x)$.

$(fog)(x) = f(g(x)) = f\left(\frac{x^2}{x^2-1}\right)$

5 marks ← $= \sqrt{\frac{\frac{x^2}{x^2-1}}{\frac{x^2}{x^2-1} - 1}} = \sqrt{x^2}$ or $= |x|$ → both are correct answers!

3 marks

Question 4 **a)** *[6] Find* $\tan(x)$ *when* $\sin(x) = -\frac{1}{3}$ *and* $x \in [\pi, \frac{3\pi}{2}]$

$\cos^2 x = 1 - \sin^2 x$

2 marks ← $= 1 - \frac{1}{9} = \frac{8}{9}$

2 marks → $\cos x = -\sqrt{\frac{8}{9}}$ or $-\frac{\sqrt{8}}{3}$

2 marks ← $\tan x = \frac{\sin x}{\cos x} = \frac{-1/3}{-\sqrt{8}/3} = \frac{1}{\sqrt{8}}$

b) *[6] Find* $\tan(x)$ *when* $\sin^3(x + 2\pi) = 27\cos^3(x + 2\pi)$

2 marks ← $\sin^3 x = 27\cos^3 x$

2 marks ← $\sin x = 3\cos x$

2 marks ← $\tan x = \frac{\sin x}{\cos x} = 3$.

Question 5 *Using the rules of exponents, simplify the functions below as much as possible.*

a) *[4]* $\left((1-x)\sqrt{\cos(x)}\right)^4 \left(\frac{1}{(x-1)^2\sqrt{1-\sin^2(x)}}\right)^2$

3 marks ← $(1-x)^4 \cos^2 x \times \frac{1}{(x-1)^4 \cos^2 x}$

1 mark ← $= 1$

b) *[4]* $2^{2x}\, 8^{-2x}\, 4^{x+1}$.

$2^{2x}\left(2^3\right)^{-2x}\left(2^2\right)^{x+1}$ → 1 mark

$= 2^{2x-6x+2x+2}$ → 1 mark

$= 2^{2x-6x+2x+2}$ → 1 mark

$= 2^{-2x+2}$ → 1 mark.