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1. Determine whether the sequence $a_n = \frac{\ln n}{\sqrt{n+1}}$ converges or diverges; if it converges, find its limit L

A. Convergent, L = 0.5 B. Convergent, L = 0 C. Convergent, L = ∞ D. Divergent
E. Convergent, L = -0.5 F. Divergent, L = 0 G. none of the above

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln n}{\sqrt{n+1}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2\sqrt{n+1}}}$$

$$= \lim_{n\to\infty} \frac{2\sqrt{(n+1)}}{n}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n+1}}$$

$$= 0$$

$\therefore$ a_n is convergent

$\sqrt{n+1} = (n+1)^{\frac{1}{2}}$

$(\sqrt{n+1})' = \frac{1}{2}\cdot(n+1)^{-\frac{1}{2}} = \frac{1}{2\sqrt{n+1}}$

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2. Write a formula for the n-th entry of the sequence $a_1 = -1, a_2 = \frac{3}{4}, a_3 = -\frac{9}{16}, a_4 = \frac{27}{64}, \cdots$ and the sum of $s = \sum_{n=1}^{\infty} a_n$.

A. $a_n = (-1)^{n-1}\frac{3^n}{4^n}, s = -\frac{4}{7}$ B. $a_n = (-1)^n\frac{3^n}{4^{n-1}}, s = -\frac{12}{7}$ C. $a_n = (-1)^n\frac{3^{n-1}}{4^{n-1}}, s = -\frac{7}{4}$
D. $a_n = (-1)^n\frac{3^n}{4^n}, s = -\frac{3}{7}$ E. $a_n = (-1)^n\frac{3^n}{4^n}, s = -\frac{7}{4}$ F. $a_n = (-1)^n\frac{3^{n-1}}{4^{n-1}}, s = -\frac{4}{7}$
G. none of the above

$a_n = (-1)^n \frac{3^{n-1}}{4^{n-1}}$

a_n is a geometric series with $a_1 = -1$, $r = -\frac{3}{4}$

it converges to $\frac{a_1}{1-r} = \frac{-1}{1+\frac{3}{4}} = -\frac{4}{7}$

3. Which of the following series is absolute convergent? I) $\sum_{n=1}^{\infty}\frac{n+4}{n^3+1}$ II) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ III) $\sum_{n=1}^{\infty}\frac{\cos n}{n!}$

A. I B. I and II C. I and III D. All of them E. II and III F. III G. none of the above

I) $0 \le \frac{n+4}{n^3+1} \le \frac{5n}{n^3}$ for all n

for $\sum_{n=1}^{\infty}\frac{5}{n^2}$ since it is a p-series with $p=2>1$ it is convergent

by the Comparison Test, $\sum_{n=1}^{\infty}\frac{n+4}{n^3+1}$ is convergent since all terms are greater than 0

it is also absolute convergent.

II) $\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{n}\right| = \sum_{n=1}^{\infty}\frac{1}{n}$ since it is a p-series with $p=1\le 1$

$\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{n}\right|$ is divergent $\therefore \sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ is not absolute convergent

III) $0 \le \left|\frac{\cos n}{n!}\right| \le \frac{1}{n!}$ $\lim_{n\to\infty}\left|\frac{\frac{1}{(n+1)!}}{\frac{1}{n!}}\right| = \lim_{n\to\infty}\frac{1}{n+1} = 0 < 1$

$\sum_{n=1}^{\infty}\frac{1}{n!}$ is absolute convergent by Ratio Test Hence $\sum_{n=1}^{\infty}\frac{\cos n}{n!}$ is absolute convergent by comparison Test

4. What **best** describes the series $\sum_{n=1}^{\infty}\frac{(-1)^n e^n \sin n}{n!}$

A. Conditional convergent B. Absolute convergent C. Alternating absolute convergent

D. absolute divergent E. Alternating divergent F. Alternating conditionally convergent

G. none of the above

$$\left|\frac{(-1)^n e^n \sin n}{n!}\right| \le \frac{e^n}{n!}$$

for $\sum_{n=1}^{\infty}\frac{e^n}{n!}$, $\lim_{n\to\infty}\left|\frac{e^{n+1}/(n+1)!}{e^n/n!}\right| = \lim_{n\to\infty}\left|\frac{e^{n+1}\cdot n!}{e^n\cdot(n+1)!}\right|$

$$= \lim_{n\to\infty}\frac{e}{n+1}$$

$$= 0 < 1$$

$\therefore \sum_{n=1}^{\infty}\frac{e^n}{n!}$ is absolute convergent by Ratio Test

since $\left|\frac{(-1)^n e^n \sin n}{n!}\right| \le \frac{e^n}{n!}$ for all n

$\sum_{n=1}^{\infty}\frac{(-1)^n e^n \sin n}{n!}$ is absolute convergent by Comparison Test

5. How many terms do you need to estimate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+4}$ so that the error is within 0.005?

A. 10 B. 11 C. 12 D. 13 E. 14 F. 15 G. none of the above

$b_n = \frac{1}{n^2+4}$

from ASET we have $|s - s_n| \leq b_{n+1}$

to obtain $|s - s_n| \leq 0.005$, we need to find n for which $b_{n+1} \leq 0.005$

find $b_{n+1} = \frac{1}{(n+1)^2+4} = \frac{1}{n^2+2n+5} \leq \frac{5}{1000}$

$\frac{1}{n^2+2n+5} \leq \frac{1}{200}$

$n^2 + 2n+5 \geq 200$

$(n+1)^2 \geq 196.$

$n \geq 13$

we need to generate at least 13 terms from 1 to 13

6. Estimate the error bound when you use the first 30 terms to approximate series

$$\overset{a_1}{1} + \overset{a_2}{\frac{1}{2\sqrt{2}}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \cdots$$

A. 0

B. $\sqrt{31}$

C. $\frac{2}{\sqrt{31}}$

D. $\frac{1}{\sqrt{31}}$

E. $\frac{2}{3\sqrt{31}}$

F. $\frac{2}{\sqrt{30}}$

G. none of the above

$a_n = \frac{1}{n\sqrt{n}}$ from $n=1$ to ∞

Let $f(x) = \frac{1}{x\sqrt{x}} = x^{-\frac{3}{2}}$. $f(x)$ is continuous and positive

$f'(x) = -\frac{3}{2} x^{-\frac{5}{2}} < 0$

it is decreasing

$|s - s_n| \leq \int_{30}^{\infty} x^{-\frac{3}{2}} dx$

$= \lim_{t\to\infty} \int_{30}^{t} x^{-\frac{3}{2}} dx$

$= \lim_{t\to\infty} -2x^{-\frac{1}{2}} \Big]_{30}^{t}$

$= \lim_{t\to\infty} -\frac{2}{\sqrt{t}} + \frac{2}{\sqrt{30}}$

$= \frac{2}{\sqrt{30}}$