**1.** Determine whether the sequence $a_n = \ln(3n^2+3) - \ln(n^2+1)$ converges or diverges; if it converges, find its limit $L$.

A. converges, $L = 3$ B. diverges C. converges, $L = \ln(-3)3$ D. converges, $L = \ln(3)$
E. converges, $L = -\ln 3$ F. diverges, $L = \ln 3$ G. converges, $L = \infty$ H. none of the above

$$\lim_{n\to\infty} \ln(3n^2+3) - \ln(n^2+1) = \lim_{n\to\infty} \ln\left(\frac{3n^2+3}{n^2+1}\right)$$

$$= \ln\left(\underbrace{\lim_{n\to\infty} \frac{3n^2+3}{n^2+1}}\right)$$

$$\lim_{n\to\infty} \frac{(3n^2+3)\frac{1}{n^2}}{(n^2+1)\frac{1}{n^2}} = \lim_{n\to\infty} \frac{3 + \cancel{\frac{3}{n^2}} \to 0}{1 + \cancel{\frac{1}{n^2}} \to 0} = 3$$

$$= \ln(3)$$

**2.** Write a formula for the $n$-th entry of the sequence $a_1 = -1/3, a_2 = 2/9, a_3 = -4/27, a_4 = 8/81, \ldots$ and find $S = \sum_{n=1}^{\infty} a_n$.

A. $a_n = (-1)^{n-1}2^n/3^n, S = 2/5$ B. $a_n = (-1)^n 2^{n-1}/3^n, S = 1/5$ C. $a_n = -2^n/3^n, S = 1/5$
D. $a_n = (-1)^{n-1}2^n/3^n, S = 1/5$ E. $a_n = (-1)^{n-1}2^{n-1}/3^n, S = 1/5$ F. none of the above

$a = \frac{1}{2}\left(\frac{2}{3}\right)^n$

$a_1 = -\frac{1}{3}$ $a_2 = \frac{2}{9}$ $a_3 = -\frac{4}{27}$ $a_4 = \frac{8}{81}$

numerator $(-1)^n$ $1 \to \underbrace{2 \to 4 \to 8}$
$2^0$ $2^1$ $2^2$ $2^3$

denominator $3 \to 9 \to 27 \to 81$
$3^1$ $3^2$ $3^3$ $3^4$
$3^n$

$(-1)^n 2^{n-1}$

$$a_n = \frac{(-1)^n 2^{n-1}}{3^n}$$

$2^{n-1} = 2^{-1} \cdot 2^n$

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{(-1)^n 2^{n-1}}{3^n}$$ alternating, $|a_n| = \frac{2^{n-1}}{3^n} = \frac{1}{2}\left(\frac{2}{3}\right)^n \Rightarrow$ decreasing

$\hookrightarrow = \frac{1}{2} \frac{2^n}{3^n}$

geometric series w/
$a = \frac{1}{2}$ $r = -\frac{2}{3}$

$\frac{5}{3}$

$$\text{sum} = \frac{a}{1-r} = \frac{1/2}{1+2/3} = -\frac{1/2}{5/2} = -\frac{1}{2}\cdot\frac{2}{5} = -\frac{1}{5}$$

**3.** Which of the following series converge? (I) $\sum_{k=1}^{\infty} \frac{k \sin k}{1+k^3}$, (II) $\sum_{n=0}^{\infty}(-1)^n \frac{n}{n+2}$ (III) $\sum_{n=1}^{\infty} \frac{2n^2+n}{3n^3-2}$

~~A. all these series~~ ~~B. (II) and (III)~~ C. none of these series ~~D. (I) and (III)~~ E. (I) only
~~F. (II) only~~ ~~G. (III) only~~ H. none of the above

I) $\sum_{k=1}^{\infty} \frac{k\sin k}{1+k^3}$ compare to $\frac{k}{k^3}$ b/c $-1 \le \sin k \le 1$ and adding a # in denom will lower value.

$\frac{k \sin k}{1+k^3} \le \frac{k}{k^3} = \frac{1}{k^2}$ general comparison of $\frac{1}{k^2}$, p-series w/ $p=2$, $p>1 \Rightarrow$ convergent.

$\Rightarrow$ convergent

II) $\sum_{n=0}^{\infty} \frac{(-1)^n n}{n+2}$ alternating, $a_{n+1} \not\le a_n \rightarrow a_1 \not\le a_0$ $\frac{1}{3} \not\le 0$ divergent by alternating test

ratio test $\lim_{n\to\infty} \frac{\frac{n+1}{n+1+2}}{\frac{n}{n+2}} = \lim_{n\to\infty} \frac{n+1}{n+3} \cdot \frac{n+2}{n} = \lim_{n\to\infty} \frac{n^2+3n+3}{n^2+3} = 1$

III) $\sum_{n=1}^{\infty} \frac{2n^2+n}{3n^3-2}$ +ve, decreasing, continuous compare to $\frac{2n^2}{3n^3} = \frac{2}{3n} = \frac{2}{3} \cdot \frac{1}{n}$ p-series $p \le 1$ $\Rightarrow$ divergent

$\lim_{n\to\infty} \frac{2n^2+n}{3n^3-2} = \lim_{n\to\infty} \frac{\cancel{\frac{2}{n}} + \cancel{\frac{1}{n^2}}}{3 - \cancel{\frac{2}{n^3}}} = 0$

$\frac{2n^2+n}{3n^3-2} \ge \underbrace{\frac{2}{3}\frac{1}{n}}_{\text{divergent}}$ $\Rightarrow$ both divergent.

**4.** Choose one of the following descriptions of the series $\sum_{n=1}^{\infty}(-1)^n \frac{3^n \sin n}{4^n - n^2}$

A. absolutely divergent B. geometric divergent C. alternating conditionally convergent
D. alternating absolutely convergent E. absolutely conditionally convergent F. alternating divergent
G. absolutely convergent H. geometric convergent I. none of the above

$\sum_{n=1}^{\infty} (-1)^n \frac{3^n \sin n}{4^n - n^2}$ abs. value $\left|\frac{3^n \sin n}{4^n - n^2}\right|$ compare to $\frac{3^n}{4^n - n^2}$

↳ root test $\lim_{n\to\infty} \sqrt[n]{|a_n|}$

$= \lim_{n\to\infty} \left(\left|\frac{3^n \sin n}{4^n - n^2}\right|\right)^{1/n} = \lim_{n\to\infty} \left(\frac{3^n (\sin n)}{4^n\left(1 - \frac{n^2}{4^n}\right)}\right)^{1/n} = \lim_{n\to\infty} \frac{3}{4}\left(\frac{|\sin n|}{1 - \frac{n^2}{4^n}}\right)^{1/n}$

$= \frac{3}{4} \lim_{n\to\infty} \left(\left|\frac{\sin n}{\underbrace{1 - \frac{n^2}{4^n}}_{\text{not zero}}}\right|\right)^{1/n \to 0} = \frac{3}{4} \lim_{n\to\infty} 1 = \frac{3}{4}$ $L < 1 \Rightarrow$ absolutely convergent

squeeze thm

not zero $\Rightarrow$ full term $\Rightarrow 0$

$\frac{1}{1} = 1 \quad \frac{1}{0.1} = 10 \quad \frac{1}{0.01} = 100$



**5.** How many terms $n$ of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ should one take so that the error $R_n$ be less than 0.01?

A. 10 B. 11 C. $\infty$ D. 15 E. 14 F. 8 G. 7 H. none of the above

$\frac{1}{n^3} \to$ convergent p-series

$$\int_n^{\infty} \frac{1}{x^3}\,dx = \lim_{t\to\infty} \int_n^t x^{-3}\,dx = \lim_{t\to\infty}\left[-\frac{1}{2}\cdot\frac{1}{x^2}\right]_n^t = \frac{1}{2n^2}$$

$R_n \leq 0.01$

$\frac{1}{2n^2} \leq 0.01$

$100 \leq 2n^2$

$\sqrt{50} \leq \sqrt{n^2}$

$\sim 7.1 \leq n$

$\Rightarrow$ must be over $7 \Rightarrow 8$

**6.** Estimate the error when the series $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \ldots$ is approximated by its first 50 terms.

A. $\frac{1}{50}$ B. $-\frac{2}{101}$ C. $\frac{2}{103}$ D. $\frac{1}{103}$ E. 0 F. $\frac{2}{101^2}$ G. none of the above

$(-1)^{n+1}\frac{2}{2n+1}$

denominator: 3 5 7 9

$2n+1$

$= 3 \quad 5 \quad 7 \quad 9$

alternating, decreasing $\to$ shown above

$\hookrightarrow$ numerator constant + denominator increasing.

$\Rightarrow a_{n+1} < a_n$

$|R_n| \leq |a_{n+1}|$

$|a_{n+1}| = \frac{2}{2(n+1)+1} \qquad n = 50 \Rightarrow \frac{2}{2(51)+1} = \frac{2}{103}$

$R_n \leq \frac{2}{103}$

Error : $\frac{2}{103}$

extra page for calculations (please remove it when submitting the test!)