

MAST218 MIDTERM EXAM. OCTOBER 22, 2019, 8:45am–10:00am.

Closed book exam. Department approved calculators (SHARP EL-531/CASIO FX-300MS).

1. Consider the parametric curve C given by

$$x = t^3 + 1, \quad y = 2t - t^2.$$

(a) For which value(s) of t is the tangent to C **horizontal**?

(b) Find all values of t for which C is concave **down**.

(c) Find the **area** under the curve C and above the x -axis.

[11 marks]

2. Find the **arc length** of the parametric curve given by

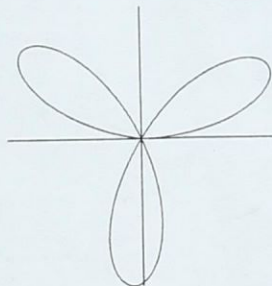
$$x = t - \sin t, \quad y = 1 - \cos t, \quad 0 \leq t \leq 2\pi.$$

[6 marks]

3. Find the **area** of one petal of the rose given by the polar equation $r = 2 \sin(\pi\theta)$.

(This polar equation defines a rose with infinitely many petals, each of the same area. Below is a plot of part of the curve, showing three of these petals.)

[5 marks]



4. Consider the following polar equation of a conic:

$$r = \frac{24}{4 + 12 \cos \theta}$$

- (a) Is this conic a parabola, an ellipse, or a hyperbola? Justify your answer.
- (b) Give a **polar** equation for the directrix of this conic.

[5 marks]

5. Find an equation for the plane passing through the points

$$(0, 1, -2), \quad (1, 2, -3), \quad \text{and} \quad (0, -2, 0).$$

Write your answer in the form $ax + by + cz + d = 0$.

[6 marks]

6. Consider the twisted cubic curve given by the vector function

$$\vec{r}(t) = \langle t, t^2, t^3 \rangle.$$

Give **symmetric equations** for the tangent line to this curve at the point $(2, 4, 8)$.

[4 marks]

END OF EXAM