2.4 Cumulative Damage & Life Prediction Concepts

Practical examples of variable amplitude fatigue

(a) Blades in gas turbine engines:
low amplitude, high frequency vibration during operation
+ small number of start-up and shut-down fatigue cycles.

(b) Rotor & bearings of turbo-generators:
subjected to overload during each start-up.

(c) Wings of aircrafts:
- variable loads due to gust on lower wing skin.
- wing tension skins near landing gear.

e.g

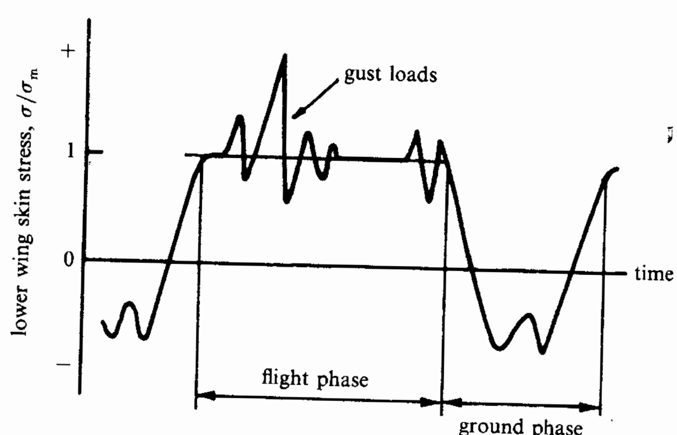


Fig. 10.1. Typical stress–time plot for the lower wing–skin of a transport aircraft. (After de Jonge & Nederveen, 1980.)

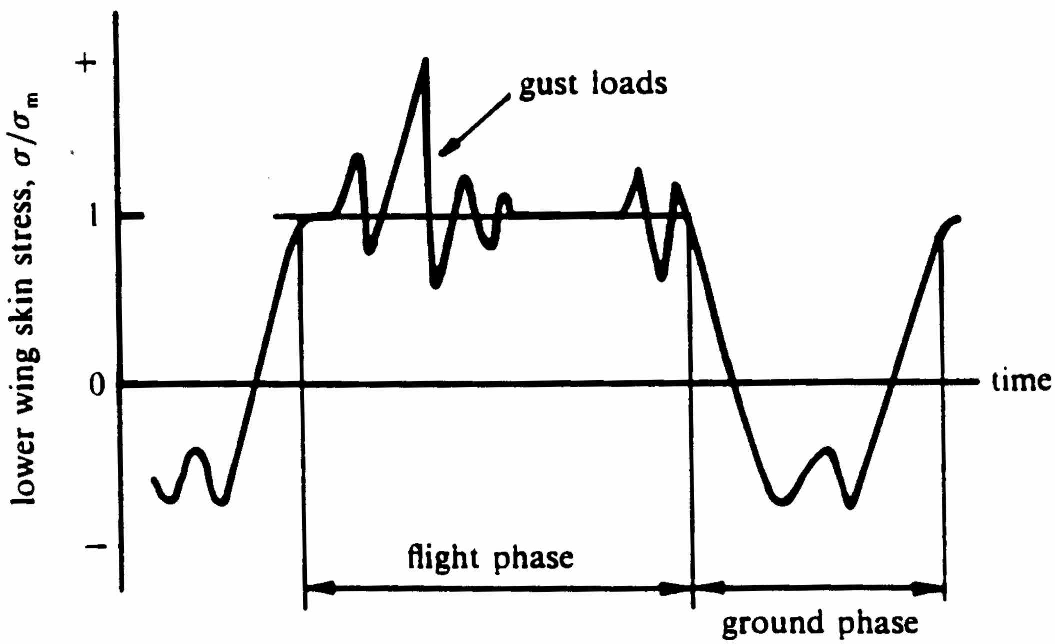


Fig. 10.1. Typical stress–time plot for the lower wing-skin of a transport aircraft. (After de Jonge & Nederveen, 1980.)

2.4.1 Cycle Counting

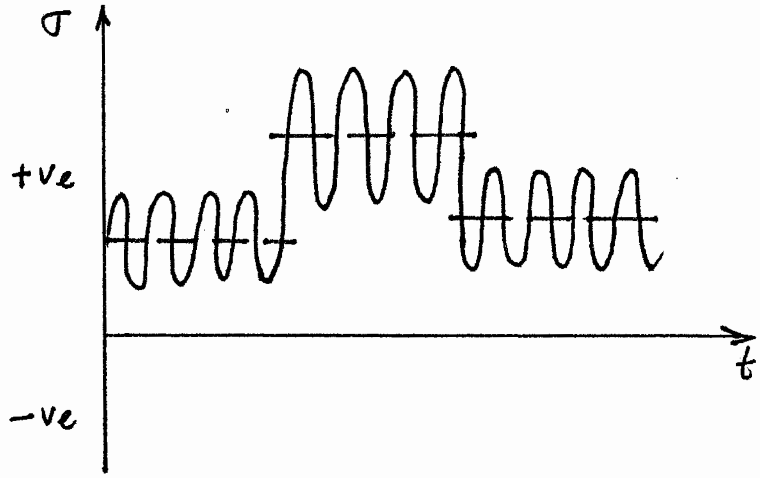
- Cycle counting methods — reduce the random loading spectra into a series of discrete events which can be analysed using the laboratory data obtained for constant amplitude fatigue loads.

- Several counting methods (established via "trial and error" and "educated guess"; empirical); used with varying degree of "success".

  - Level crossing counting
  - Peak counting
  - Simple range counting
  - Racetrack counting
  - Hayes method
  - Rainflow counting (ASTM standard) *

2.4.2 Cumulative Damage Predictions

- Various blocks of constant amplitude cyclic stresses. →



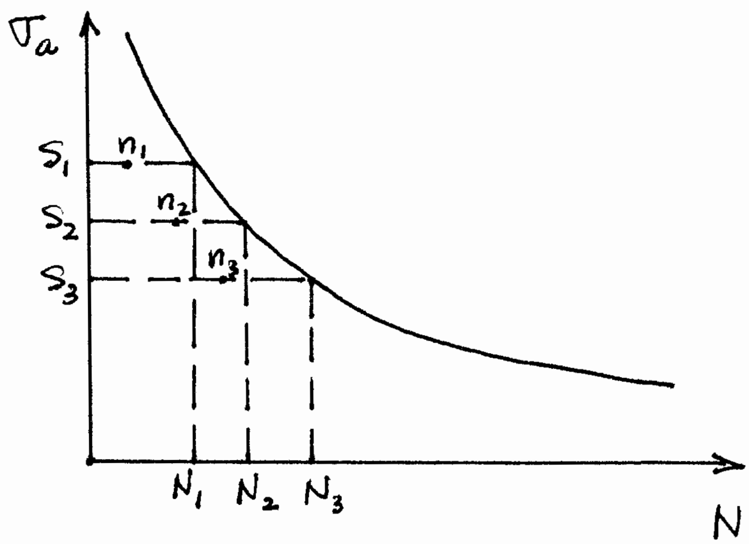
- Linear Damage Rule: (Palmgren – Miner)

— Miner's Hypothesis.

$\frac{n}{N}$ = cycle ratio.

n – no. of cycles at stress level S

N – fatigue life (in cycles) at stress level S.



Damage fraction

$$D_i = \frac{n_i}{N_i} \quad \text{for the } i\text{-th block.}$$

Miner's Hypothesis: Failure occurs when

$$\boxed{\sum D_i = \sum \frac{n_i}{N_i} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \cdots = 1} \quad \text{---} (2.7)$$

Assumption: Prior stressing and load sequence do not affect fatigue behaviour. Not true in practice!

e.g. (i) Prior stressing in many metals (strain ageing) shifts S-N curve upwards → σ_L increases. "coaxing".

High Cycle Fatigue

Example:

A plain bar member in a machine for a production process is made of a high strength steel with an ultimate tensile strength, $\sigma_u = 1180$ MPa, and yield stress, $\sigma_y = 960$ MPa.The bar is designed for a 99% probability of survival. The *S-N* fatigue data from experimental tests for 99% probability of survival of this steel under fully reversed cyclic axial stresses are given below:

Stress amplitude, *S* (MPa)	Cycles, *N*
850	4000
750	8200
700	14500
620	30000
550	56300
500	60800
420	220000
350	445000
300	1950000
270	∞

(i) During each period of continuous operation (a duty cycle), a spectrum of fully reverse cyclic loads which gives rise to the following maximum stresses are obtained:

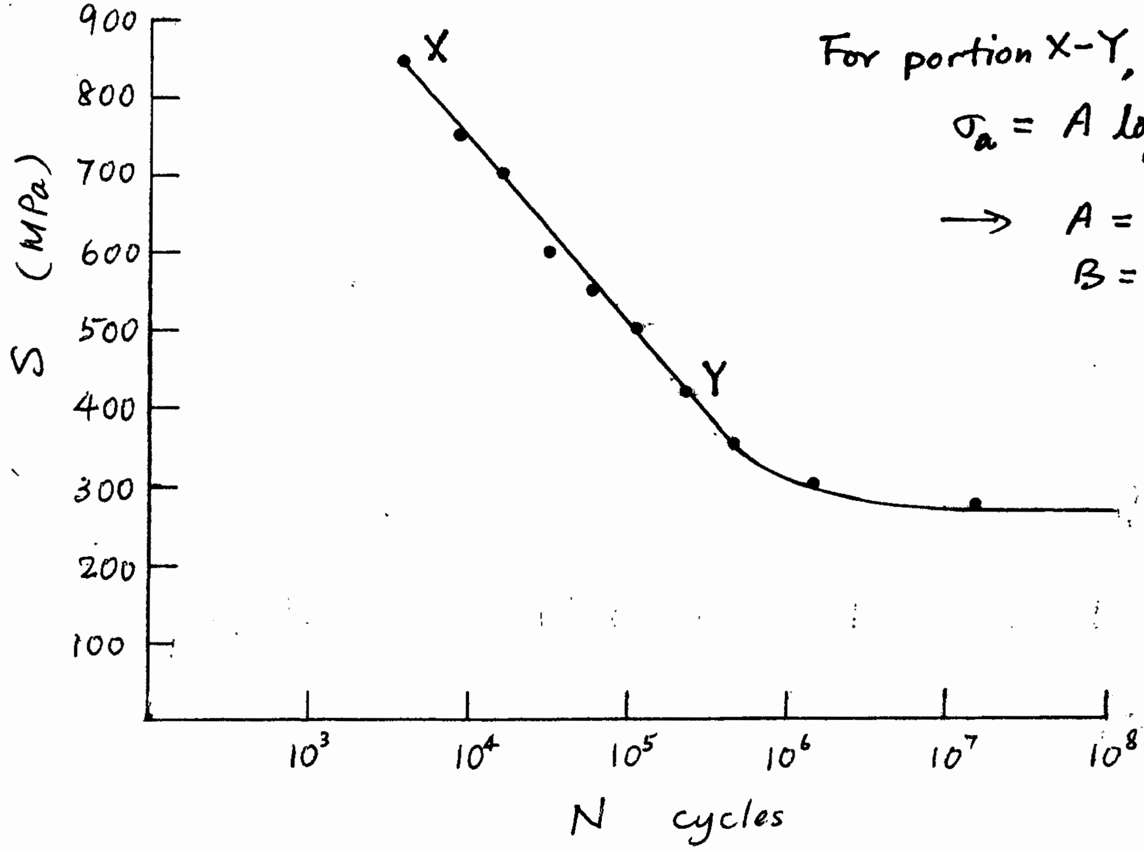
σ_{max}	*n*, cycles
650	800
560	2400
240	60000

After how many duty cycles should this bar component be replaced?

(ii) An identical machine is also used in the manufacture of another line of products. The load spectrum, however, is one of zero-tension cyclic loading with the following maximum stresses in each duty cycle.

σ_{max}	*n*, cycles
750	1000
650	2500
280	90000

What is the corresponding duty cycle interval for the replacement of the bar? Use the Soderberg criterion for the mean stress effects.



For portion X-Y, linear fit

$$\sigma_a = A \log_{10} N + B$$

$\longrightarrow$ $A = -241$

$B = 1706$

Solution:

(i) Fit straight line for the *S-N* data between X and Y (850 MPa - 420 MPa stress amplitude):

$$\sigma_a = A \log_{10} N + B \quad \longrightarrow \quad A = -241 \qquad B = 1706$$

Compile table for Miner's rule:

σ_{max}(MPa)	N_i	n_i	n_i/N_i
650	24084	800	0.037
560	56910	2400	0.042
240	∞	60000	0
		$\sum(n_i/N_i) =$	0.075

Let number of duty interval cycles for replacement of the bar be N_d.
Therefore,

$$N_d\,(0.075) = 1, \quad \text{giving } N_d = 13.33$$

Thus the bar member should be replaced after 13 duty cycles.

(ii) Soderberg criterion: $\dfrac{\sigma_a}{\sigma_N} + \dfrac{\sigma_m}{\sigma_y} = 1$

For the first load cycle $\quad \dfrac{375}{\sigma_N} + \dfrac{375}{960} = 1 \quad \Rightarrow \quad \sigma_N = 615.4\,\text{MPa}.$

For the second load cycle $\quad \dfrac{325}{\sigma_N} + \dfrac{325}{960} = 1 \quad \Rightarrow \quad \sigma_N = 491.3\,\text{MPa}.$

For the third load cycle $\quad \dfrac{140}{\sigma_N} + \dfrac{140}{960} = 1 \quad \Rightarrow \quad \sigma_N = 163.9\,\text{MPa}.$

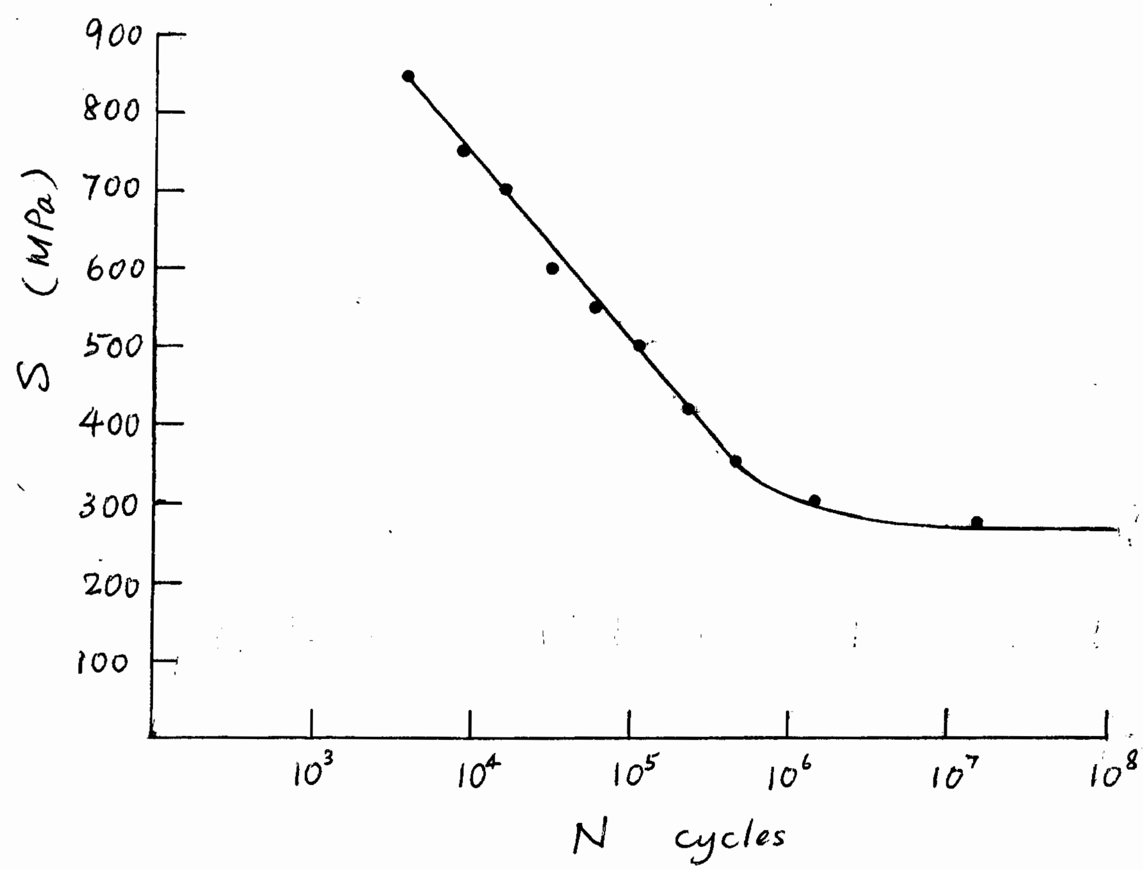
Using Miner's rule again:

σ_a(MPa)	N_i	n_i	n_i/N_i
619.4	33520	1000	0.030
491.3	109711	2500	0.023
163.9	∞	90000	0
		$\sum(n_i/N_i) =$	0.053

Therefore,

$$N_d\,(0.053) = 1, \quad \text{giving } N_d = 19,$$

i.e. the bar should be replaced after 19 duty cycles.


S (MPa)
900
800
700
600
500
400
300
200
100
10^3
10^4
10^5
10^6
10^7
10^8
N cycles

$\sum \frac{n_i}{N_i} < 1 \Rightarrow$ Non-conservative

(ii) In tests with monotonically decreasing stress sequence, $\sum \frac{n_i}{N_i} < 1$; if monotonically increasing stress sequence, $\sum \frac{n_i}{N_i} > 1$. Variations in $\sum \frac{n_i}{N_i}$ — 0.5 to 2.0
Quasi-random tests show $\sum \frac{n_i}{N_i}$ closer to unity.

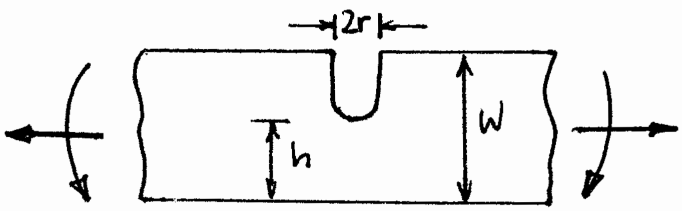
(iii) Residual stresses at notches due to tensile overload retard fatigue growth rate.

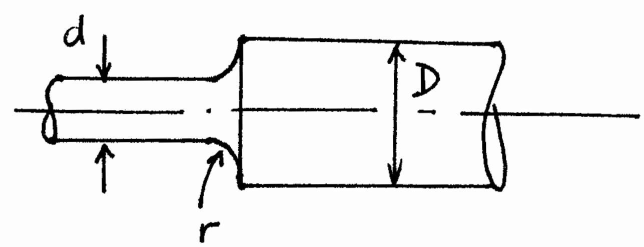
- Miner's rule is popular because of its <u>simplicity</u>.

- Non-linear theories, e.g. by Marco and Starkey; Gatts; Henry; etc. — not necessarily better in practice even though they are often very much more complicated!

2.5 <u>Stress Concentrations</u>

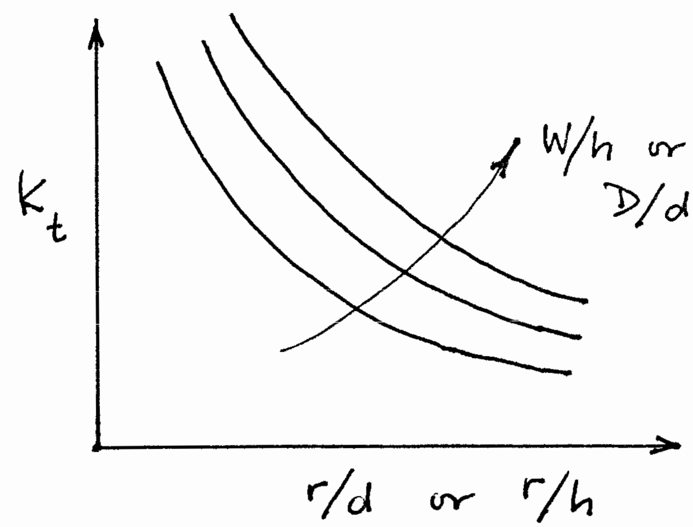
- Static stress (elastic) at geometric discontinuities, e.g.





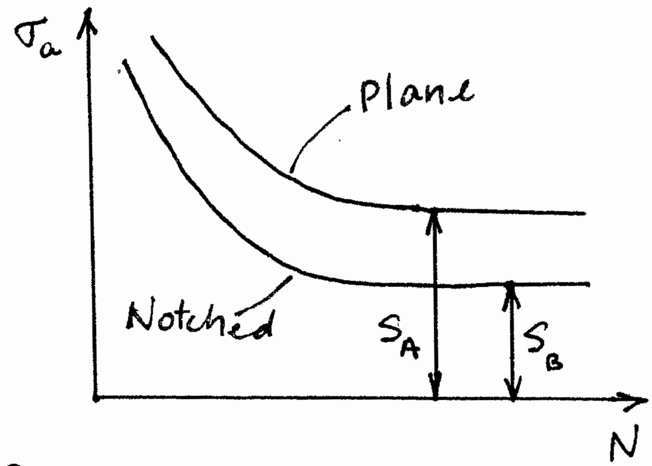
Stress concentration factor, K_t

– dependent on geometry and type of loading only.



- Fatigue strength reduction/notch factor, K_f.

$$\boxed{K_f = \frac{S_A}{S_B}} \quad \text{---(2.8)}$$



K_f — dependent not just on geometry and loading, but also on material.

$\longrightarrow$ Notch sensitivity index, q.

$$\boxed{q = \frac{K_f - 1}{K_t - 1}} \quad \text{---(2.9)} \qquad (0 \leq q \leq 1)$$

$q = 0$, no notch effect i.e. $K_f = 1$
$q = 1$, full notch effect i.e. $K_f = K_t$.

- Estimation of K_f (empirical):

(i) Peterson:

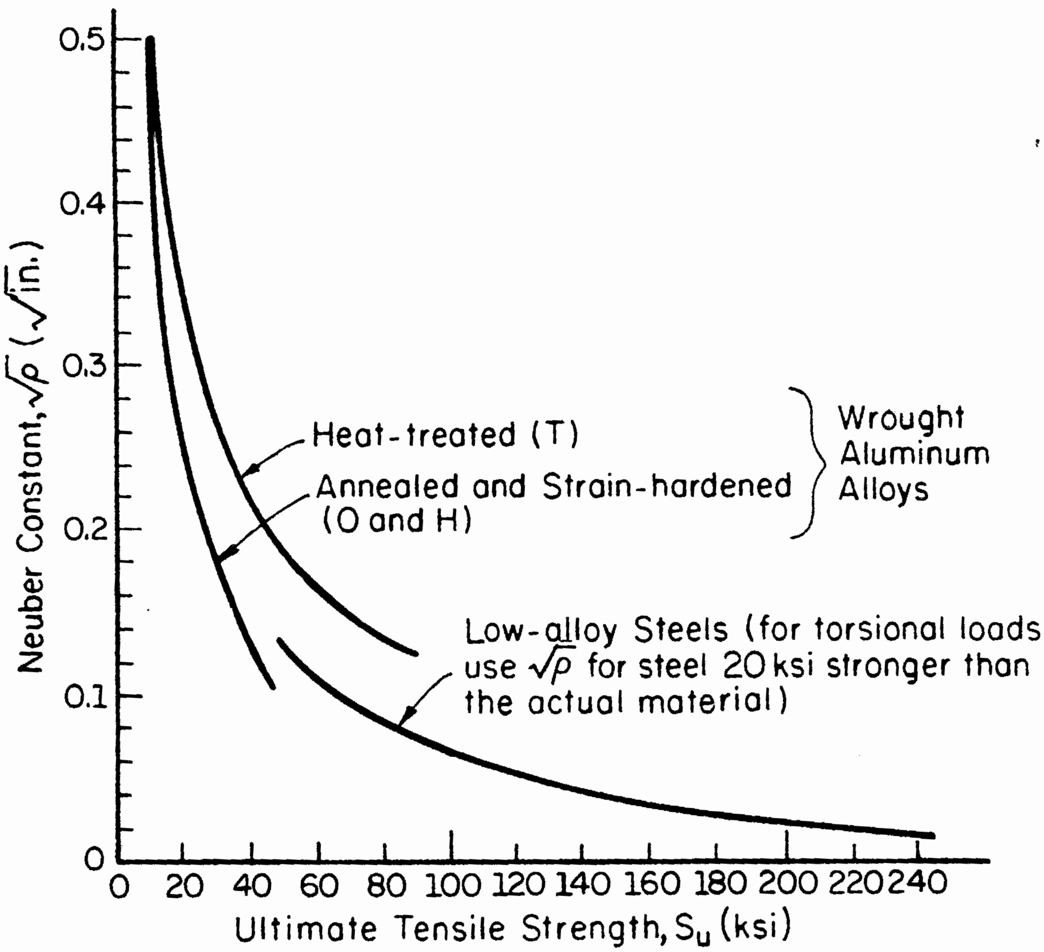
$$\boxed{K_f = 1 + \frac{K_t - 1}{\left(1 + \frac{a}{r}\right)}} \quad \text{----(2.10)}$$

$$\boxed{q = \frac{1}{\left(1 + \frac{a}{r}\right)}} \quad \text{------(2.11)}$$

r — notch root radius

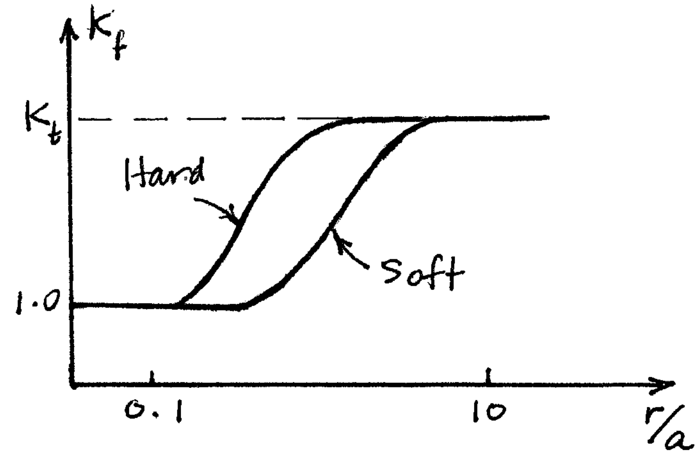
a — material constant; depends on material strength and ductility.

e.g. for annealed (soft) steels $a \simeq 0.25$ mm.
quenched tempered steels $a \simeq 0.06$ mm.
highly hardened steels $a \simeq 0.02$ mm.
Al. alloys $a \simeq 0.07 - 0.20$ mm.



Neuber constants for steel and aluminum. (From Ref. 6.)

Hard steels and "sharp" notches → more notch sensitive.

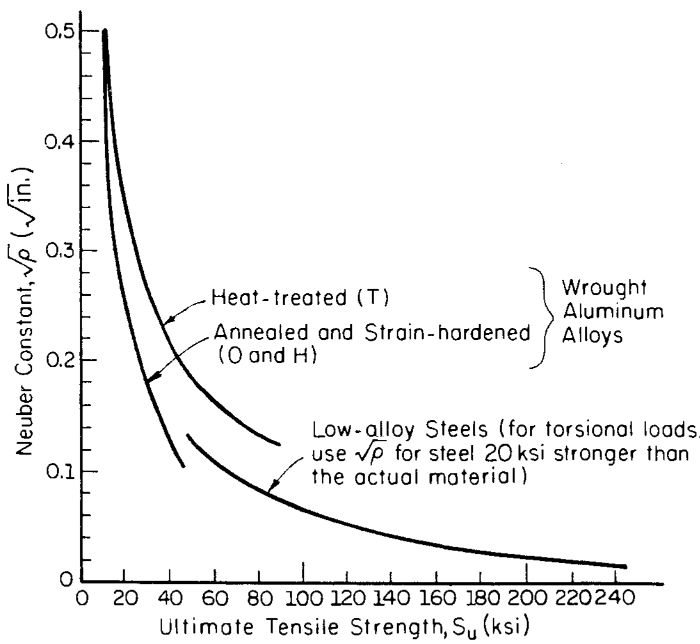


(ii) Neuber:

$$\boxed{K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\rho/r}}} \quad \text{------} \quad (2.12)$$

$$\boxed{q = \frac{1}{1 + \sqrt{\rho/r}}} \quad \text{------} \quad (2.13)$$

ρ — material constant; related to grain size of material.



- Size effect:

Large components are more notch sensitive!

e.g. Two geometrically similar plates, $d_1/W_1 = d_2/W_2$

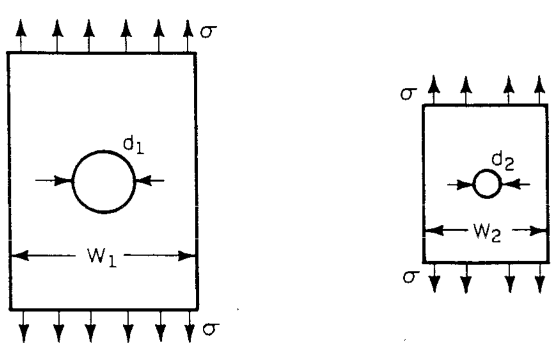


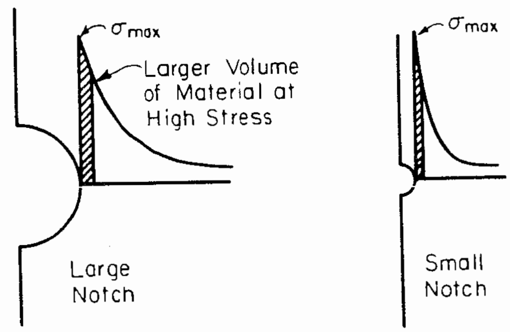
Plate 1 Plate 2

K_t — same for both plates

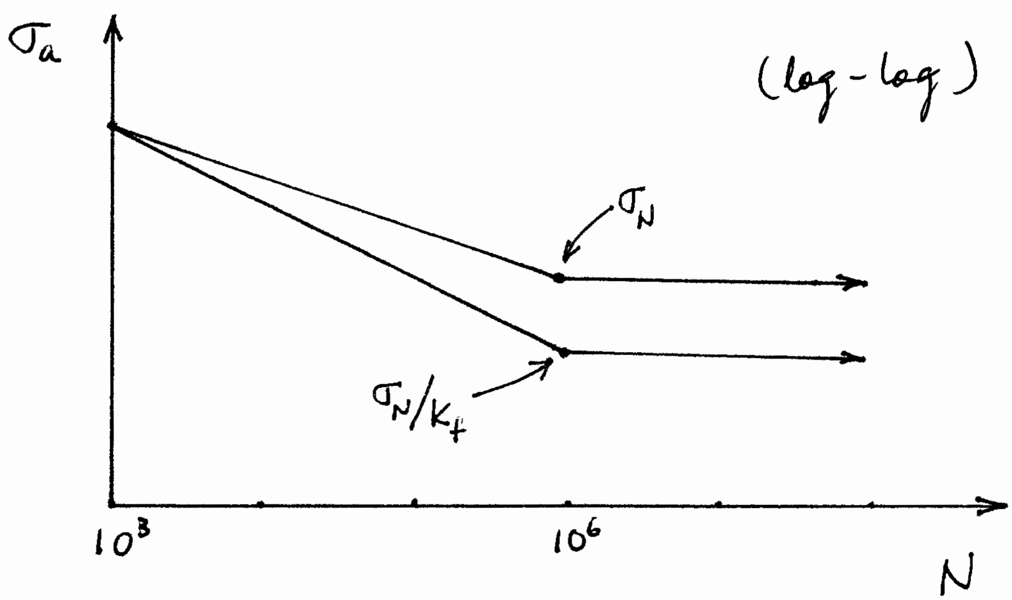
- But K_f is larger for Plate 1 (with larger d_1).

- May be explained by the fact that, in larger components, the volume of material under high stress is greater.
  $\Rightarrow$ greater probability of crack initiation.

- In practice, K_t is generally < 5; plastic flow occurs otherwise $\rightarrow$ linear elastic analysis becomes invalid.



• Modification to S-N curve for notched components:

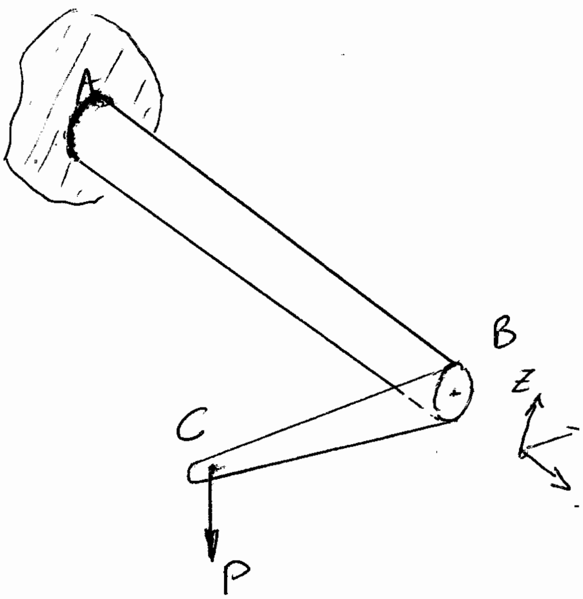


— Conservative estimate!

— Defines S-N curve in the LCF region where stress-life approach may not be valid!

Example:

A shaft AB is clamped at A and carries a radial arm BC. A fluctuating load is applied at C which varies between the peak values of $P = W$ and $\frac{W}{2}$. The dimensions of the shaft are such that the load W produces at A:



Bending stress $\sigma_x = \sigma_0$

Shear stress $\tau_{xy} = \frac{3}{4}\sigma_0$

Find the maximum value of σ_0 for a fatigue life of 10^7 cycles. Assume Soderberg criterion with safety factor (n) of 2.

Given: $\sigma_y = 310$ MPa, $\sigma_L = 205$ MPa

fatigue strength reduction factor $k_f = 1.5$

Solution:

- Find equivalent stress at A for loads W and $\frac{W}{2}$

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

- For $P = W$,

$$\sigma_{1,2} = \frac{1}{2}\sigma_0 \pm \frac{1}{2}\sqrt{\left[\sigma_0^2 + 4\left(\frac{3}{4}\sigma_0\right)^2\right]} \Rightarrow \begin{array}{l} \sigma_1 = 1.4\sigma_0 \\ \sigma_2 = -0.4\sigma_0 \end{array}$$

VonMises equivalent stress

$$\sigma_{eq}^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2$$

$$\therefore \sigma_{eq} = \left[\sigma_0^2\{1.4^2 + (-0.4)^2 - (1.4)(-0.4)\}\right]^{1/2} = 1.64\sigma_0$$

Similarly for $P = \frac{W}{2}$

$$\sigma_{eq} = 0.82\,\sigma_0$$

∴ Fluctuating equivalent stress is:

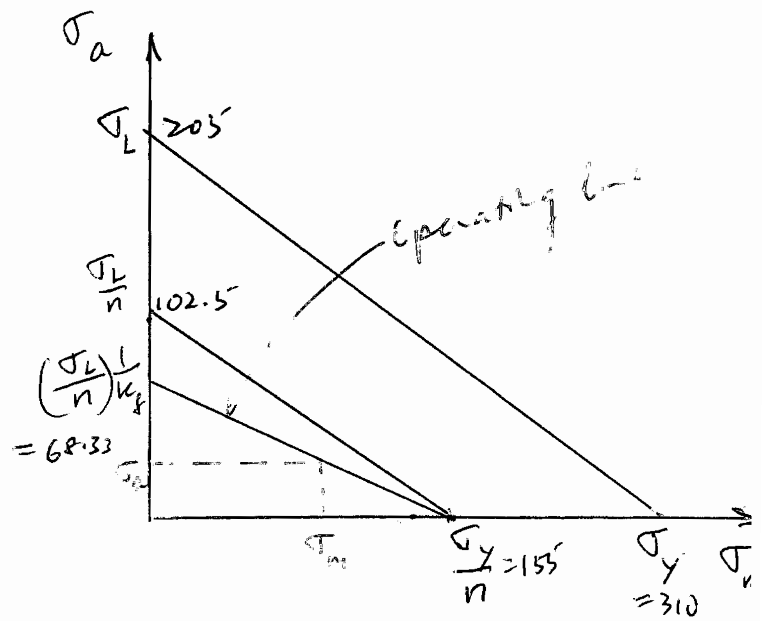
$$\sigma_{eq} = \left[\left(\frac{1.64+0.82}{2}\right) \pm \left(\frac{1.64-0.82}{2}\right)\right]\sigma_0$$

i.e. $\sigma_{eq} = (1.23 \pm 0.41)\,\sigma_0$

Soderberg diagram:

$\sigma_a = 0.41\,\sigma_0$

$\sigma_m = 1.23\,\sigma_0$



Using similar triangles

$$\frac{0.41\,\sigma_0}{155 - 1.23\,\sigma_0} = \frac{68.33}{155}$$

$$\Rightarrow \quad \sigma_0 = \underline{71.8 \text{ MPa}}.$$

2.6 Factors affecting fatigue life

(a) Size effect: Fatigue strengths are generally lower for larger specimens/components than for smaller ones, especially under bending loads.

(b) Surface conditions: Scratches, pits and machining marks add stress concentrations to the ones due to component geometry; particularly important in high strength steels.

(c) Surface treatment: mechanical; plating; chemical/metallurgical.

Mechanical — e.g. cold-rolling, shot-peening, honing, polishing $\rightarrow$ compressive residual stresses on surface; beneficial.
machining, grinding $\rightarrow$ tensile residual stresses $\rightarrow$ harmful.

Plating — Chromium and nickel plating of steels causes as much as 60% decrease in σ_L (due to high tensile residual stresses generated in the plating process).

Chemical/Metallurgical — Diffusion processes such as carburising and nitriding; case hardening; all produce high compressive residual surface stresses due to volume change/phase transformation.

(d) Temperature: σ_L in general decreases with increase in T. Exceptions — materials which undergo metallurgical changes as T increases.

(e) Environment: σ_L decreases with increasing corrosion extent. Very complex topic!