2. High Cycle Fatigue — Stress-Life Approach

- Problems with predicting/preventing failure by fatigue
  - (a) Calculations of life — less accurate than other static strength analysis; can be orders of magnitude in error!
  - (b) Fatigue characteristics of a material must be empirically determined; cannot be deduced from other mechanical properties $\Rightarrow$ full-scale testing
  - (c) Results of different but "identical" tests may differ quite widely $\rightarrow$ often require statistical interpretation

2.1. Fatigue loading

- Constant amplitude loading.

σ_m = mean stress

$$= \left(\frac{\sigma_{max} + \sigma_{min}}{2}\right)$$

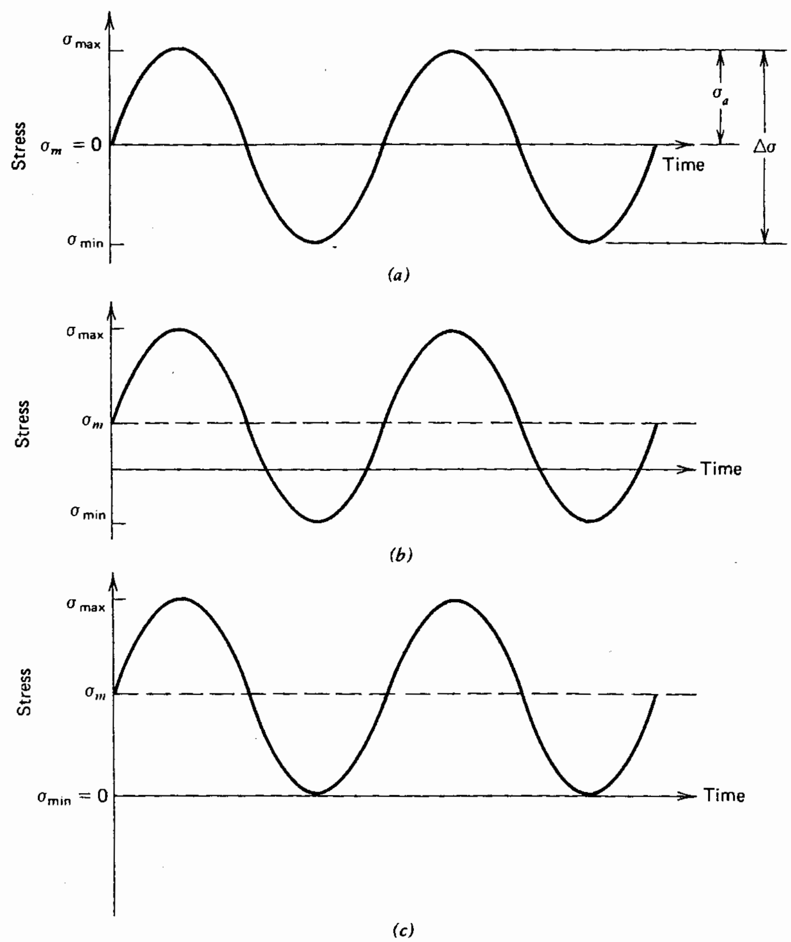
σ_a = alternating stress amplitude

$$= \left(\frac{\sigma_{max} - \sigma_{min}}{2}\right)$$

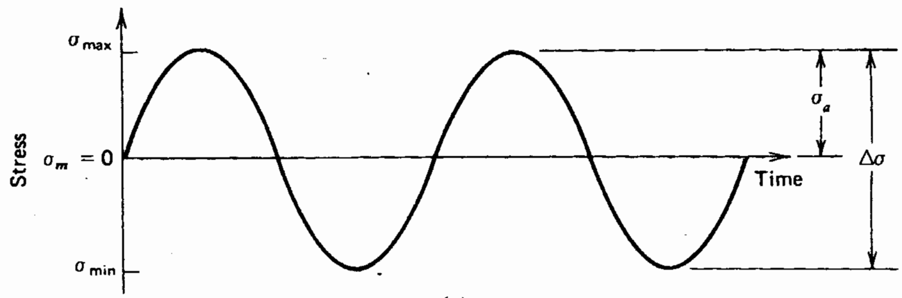
$\Delta\sigma = (\sigma_{max} - \sigma_{min})$ stress range

$R = \left(\frac{\sigma_{min}}{\sigma_{max}}\right)$ stress ratio

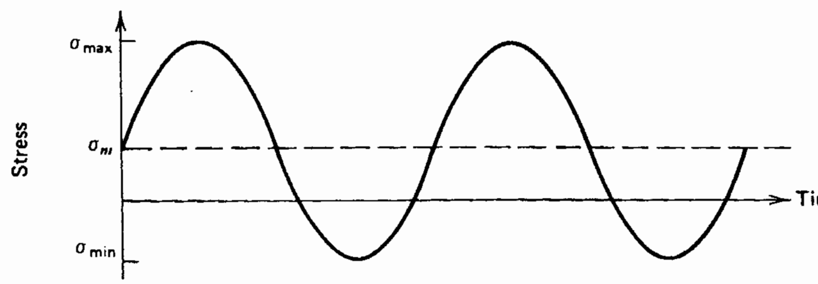
$A = \frac{\sigma_a}{\sigma_m} = \left(\frac{1-R}{1+R}\right)$ amplitude ratio



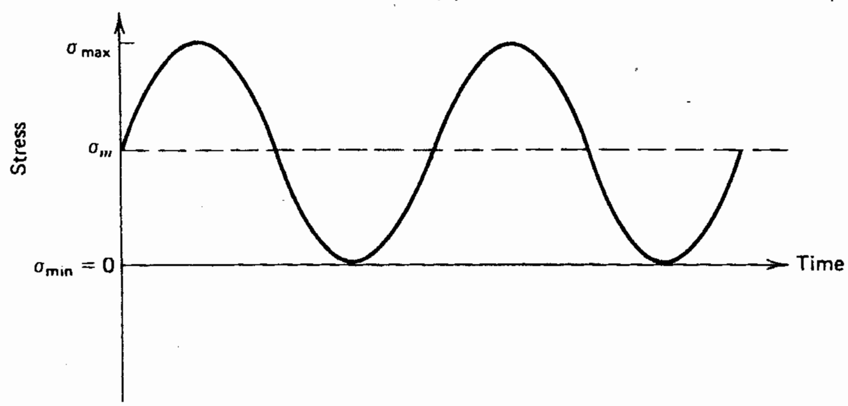
Constant amplitude loading:



(a)



(b)



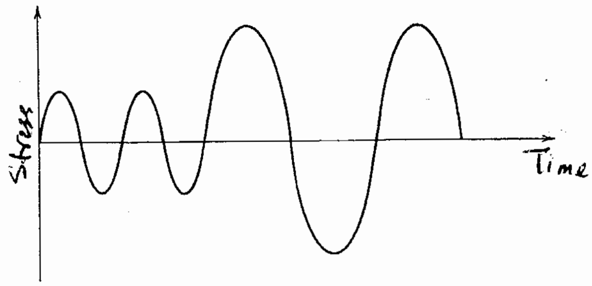
(c)

(a) $\sigma_m = 0$; $R = -1$ (fully reversed loading)

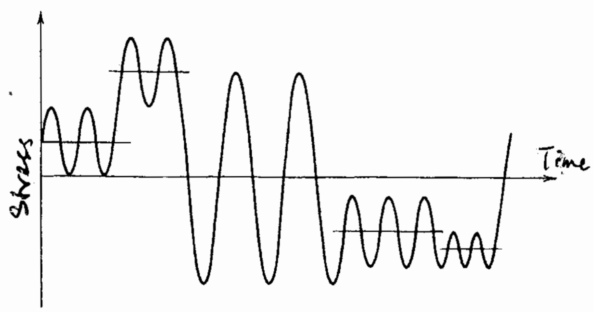
(b) $\sigma_m \neq 0$; $-1 < R < 0$

(c) $\sigma_m \neq 0$; $R = 0$ (zero-tension fatigue)

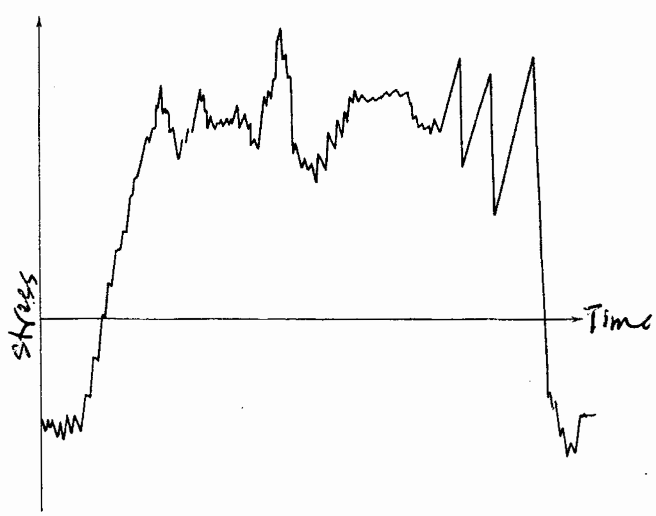
(Note: $R = +1 \Rightarrow$ static load)



Zero mean; changing amplitude.



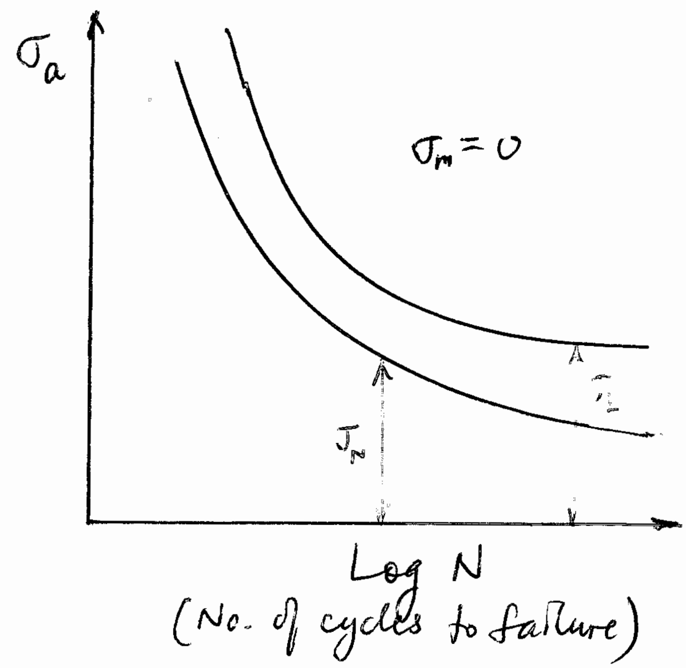
Changing mean and amplitude



Quasi-random stress-time load spectrum.

2.2 Stress-Life (S-N) approach

- Stress-Life plot (S-N curve) on plain / smooth specimens.
- Test method
  — ASTM E466–E468
- Endurance / Fatigue limit:
  σ_L — characteristic of some strain ageing materials e.g. steels, titanium alloys.
  "Infinite" life!

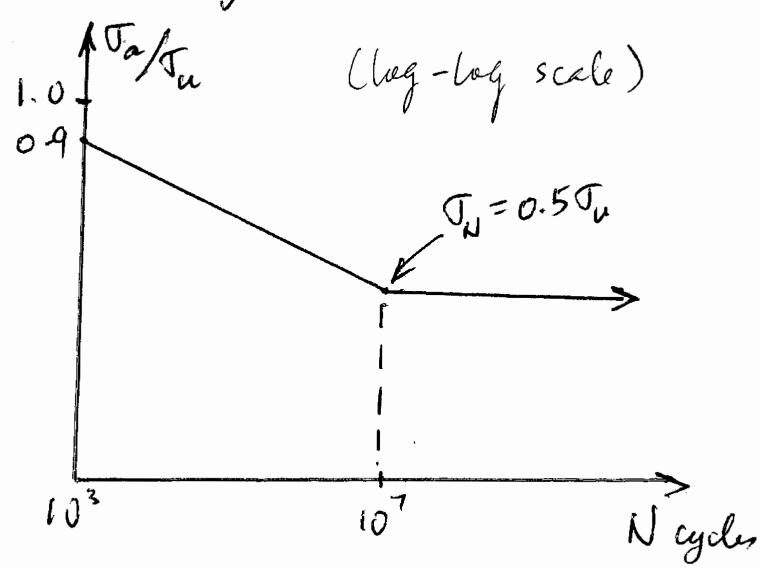


- Most non-ferrous alloys do not have an endurance limit
  → "pseudo" endurance limit — stress amplitude (fatigue strength) at, say, 10^7 cycles to failure.
- Fatigue strength $\sigma_N = \begin{cases} \text{Fatigue limit } \sigma_L \\ \text{or} \\ \text{Fatigue strength @ } 10^7 \text{ cycles to failure} \end{cases}$
- For many metals, if no actual fatigue data are available, estimate:

$\sigma_N = 0.5\,\sigma_u$ @ 10^7 cycles

$\sigma_a = 0.9\,\sigma_u$ @ 10^3 cycles

σ_u — ultimate tensile stress (UTS)



σ_L

Material	Condition	σ_{UTS} (MPa)	σ_y (MPa)	σ_e (MPa)
<u>Aluminum alloys</u>[a]				
1100	Annealed	90	34	34
2024	T3	483	345	138
6061	T6	310	276	97
7075	T7			
<u>Steels</u>[b]				
1015	Annealed	455	275	240
1015	60% Cold-worked	710	605	350
1040	Annealed	670	405	345
4340	Annealed	745	475	340
4340	Q&T† (204°C)	1950	1640	480
4340	Q&T† (538°C)	1260	1170	670
HY140	Q&T† (538°C)	1030	980	480

[a]Endurance limit based on 10^7 cycles. Source: *Structural Alloys Handbook*, Mechanical Properties Data Center, Traverse City, Michigan, 1977.

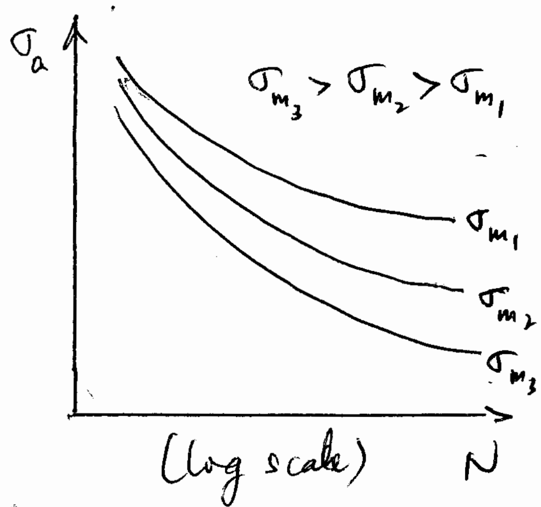
[b]Endurance limit based on 5×10^8 cycles. Source: *Aluminum Standards and Data*, The Aluminum Association, New York, 1976.

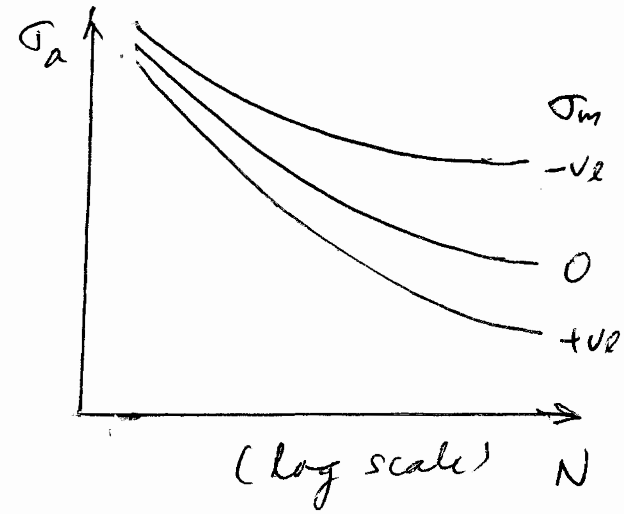
†Refers to quenched and tempered condition; the data within parantheses refer to tempering temperature.

Source: Fatigue of Materials
S. Suresh
Cambridge University Press, 1991

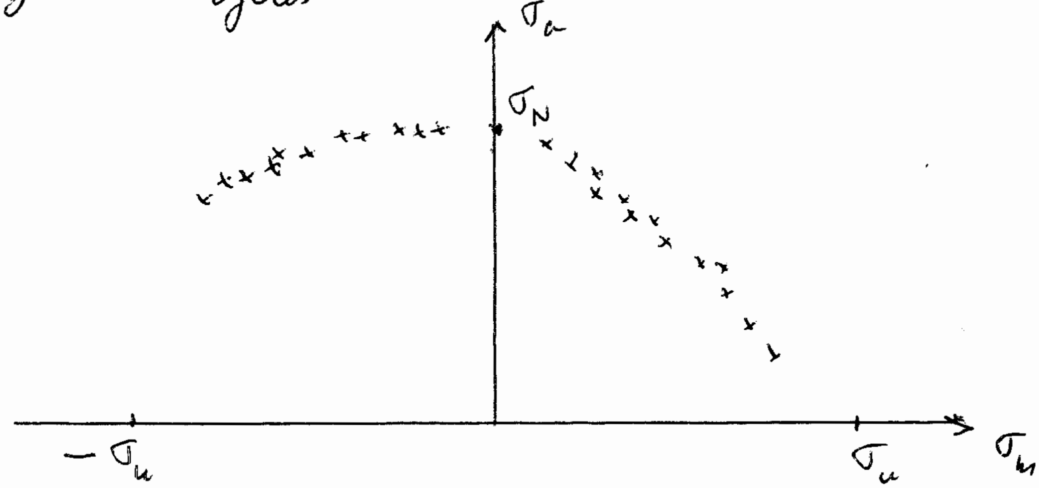
2.2 Mean stress effects

- Generally

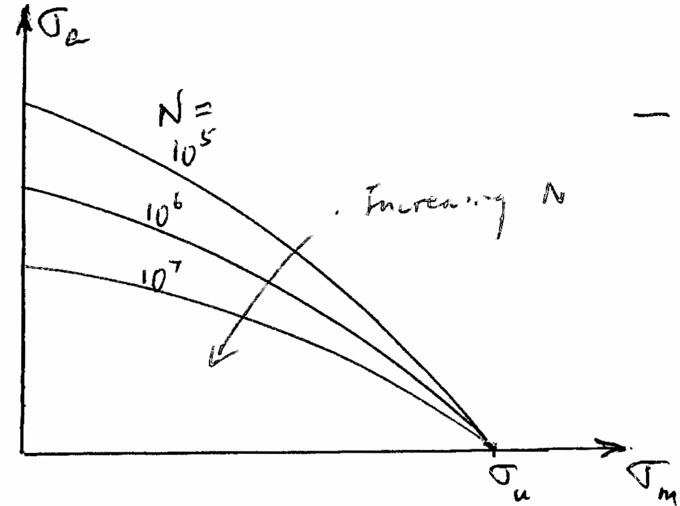




For given N cycles

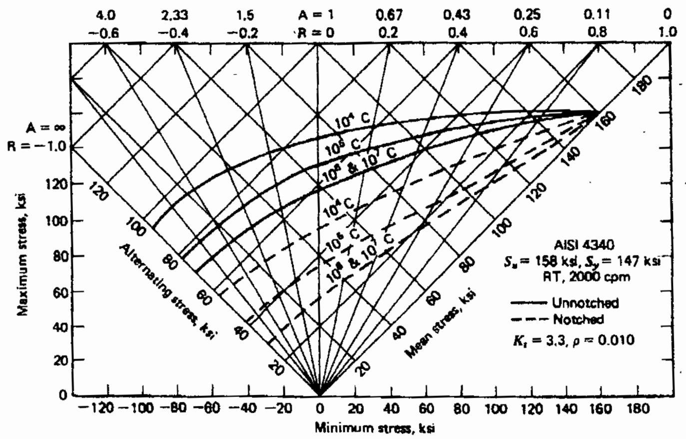


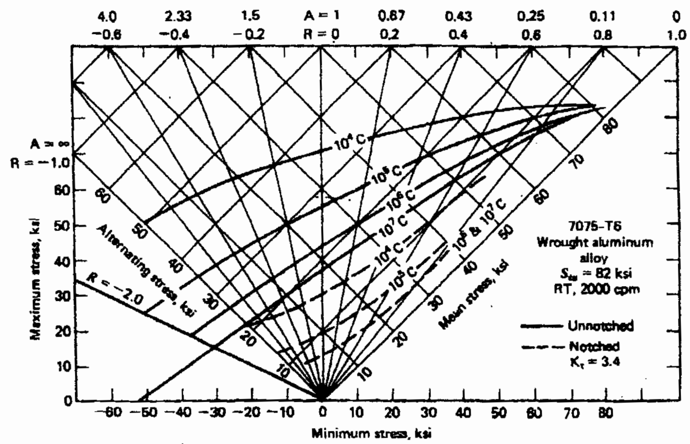
- Data may be presented on "Master Diagram" (Haigh diagram) for the material

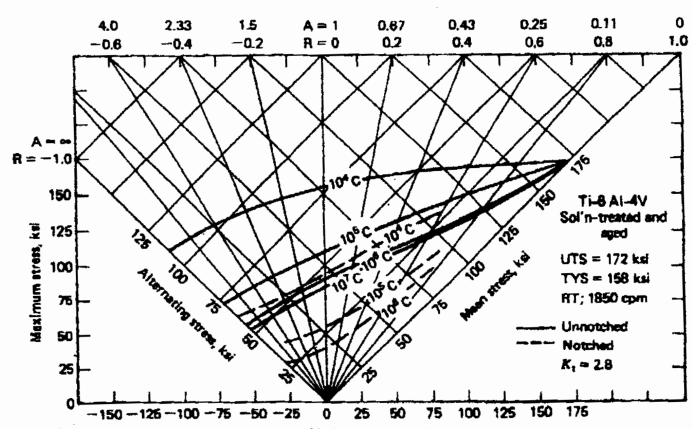


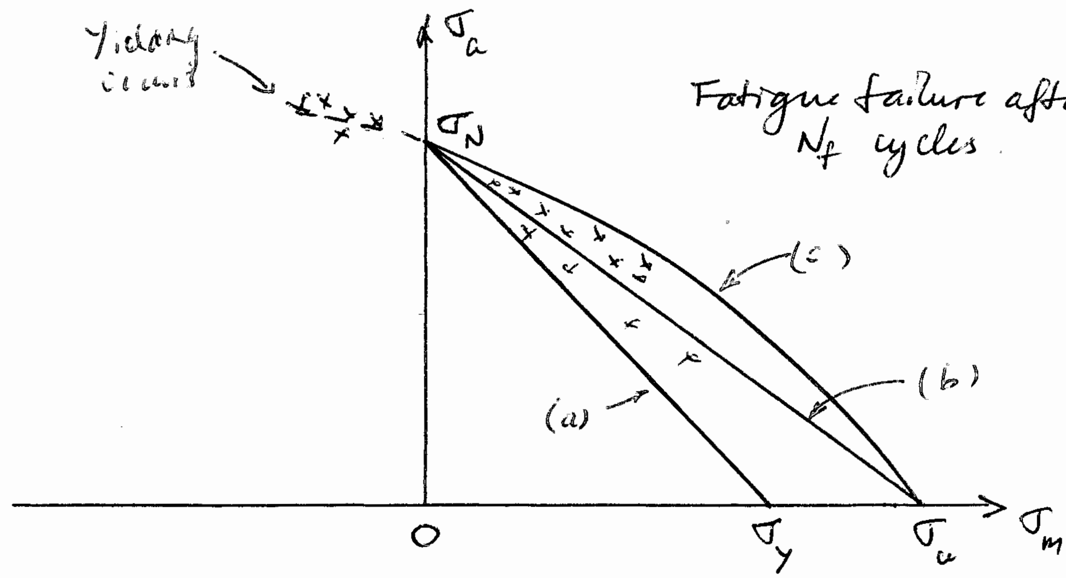
— Expensive to produce however!

Master diagrams (Source: Grover H.J. Fatigue of Aircraft Structures Govt. Printing Office, Washington DC. 1966)









(a) Soderberg (USA, 1930):

$$\boxed{\frac{\sigma_a}{\sigma_N} + \frac{\sigma_m}{\sigma_y} = 1} \qquad (2.1)$$

(b) Goodman (UK, 1899):

$$\boxed{\frac{\sigma_a}{\sigma_N} + \frac{\sigma_m}{\sigma_u} = 1} \qquad (2.2)$$

(c) Gerber (Germany, 1874):

$$\boxed{\frac{\sigma_a}{\sigma_N} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1} \qquad (2.3)$$

- Soderberg criterion — very conservative

  Goodman criterion — generally good for brittle materials but conservative for ductile materials.

  Gerber criterion — generally good for ductile materials

Example(a): A component undergoes a cyclic stress with $\sigma_{max.} = 440$ MPa and $\sigma_{min} = 40$ MPa. It is made from a steel with $\sigma_u = 650$ MPa and endurance limit $\sigma_L = 260$ MPa. Determine the life of the component using Goodman criterion.

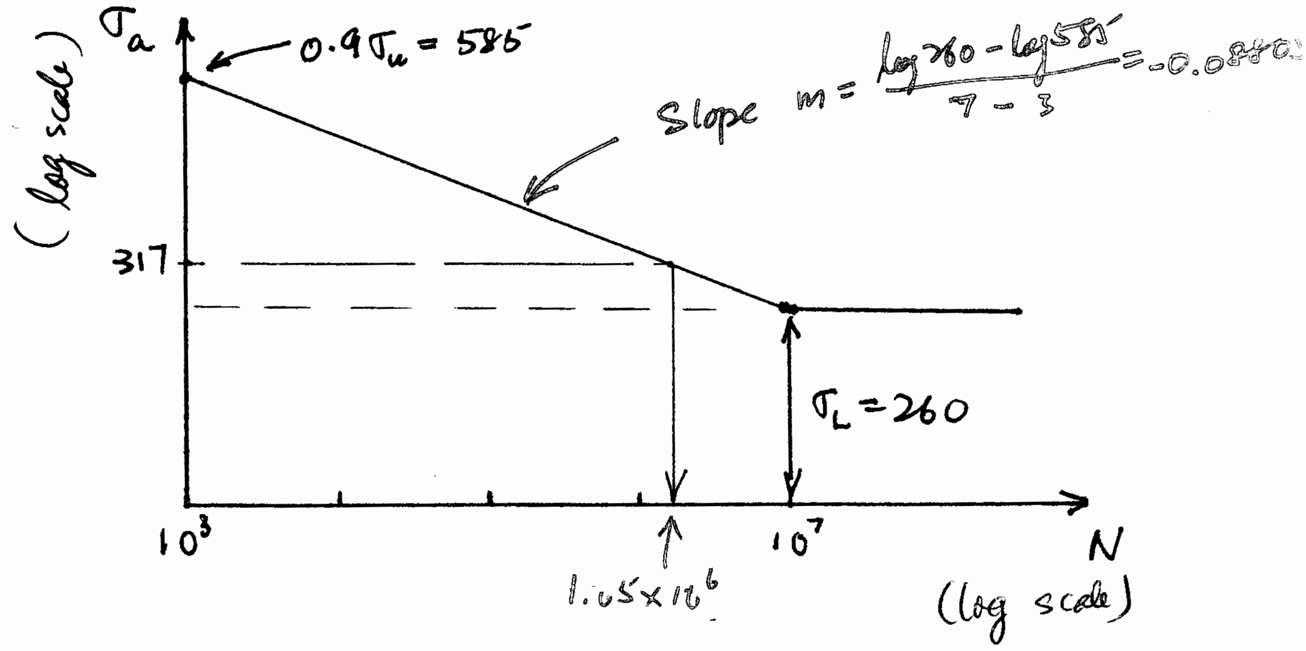
$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{440-40}{2} = 200 \text{ MPa}.$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{440+40}{2} = 240 \text{ MPa}.$$

Goodman: $$\frac{200}{\sigma_N} + \frac{240}{650} = 1$$

$$\therefore \quad \sigma_N = 317 \text{ MPa}.$$

Construct S-N diagram (since not given):



$$\therefore \frac{\log 260 - \log 317}{7 - \log N} = -0.08801 \rightarrow \log N = 6.0223$$

$$\therefore N = \underline{1.05 \times 10^6} \text{ cycl}$$

Example (b)

An aluminium alloy cylindrical bar is a critical component in a structural system and is pre-tensioned to 110 MPa. It is subjected to a cyclic load, reaching a maximum stress of 178 MPa. *S-N* data for fully reversed loading available for this alloy provides the endurance limit (for $N = 10^7$ cycles) to be 55 MPa and the fully reversed stress for 3000 cycles to be 206 MPa. Estimate the life of this component (a) using the Goodman criterion, and (b) the Soderberg criterion.

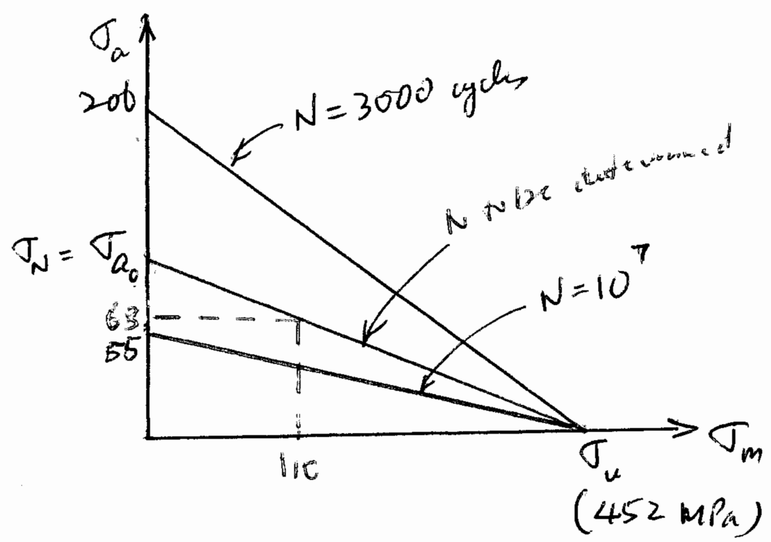
For the alloy: ultimate stress, $\sigma_u = 452$ MPa; yield stress $\sigma_Y = 350$ MPa.

(a) Goodman:

$\sigma_m = 110$ MPa

$\sigma_{max} = 178$ MPa

$\sigma_a = 68$ MPa



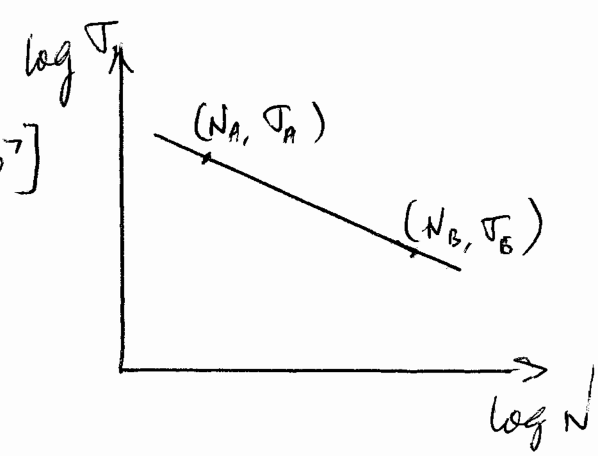
$$\frac{68}{\sigma_N} + \frac{110}{452} = 1 \implies \sigma_N = 89.9 \text{ MPa} = \sigma_{a_0}$$ (corresponding amplitude with $\sigma_m = 0$)

S-N curve (log-log) – straight line

For fully reversed loading (zero mean stress)

$$\log 206 - \log 55 = b\left[\log 3000 - \log 10^7\right]$$

$\therefore \quad b = -0.163$



$\therefore$ For $\sigma_{a_0} = 89.9$ MPa,

$$\log 89.9 - \log 55 = -0.163 [\log N - \log 10^7]$$

$$\Rightarrow \quad N = \underline{490700 \text{ cycles}}.$$

(b) Soderberg criterion:

$$\frac{68}{\sigma_N} + \frac{110}{350} = 1 \Rightarrow \sigma_N = 99.2 \text{ MPa} = \sigma_{a_0}$$

$$\therefore \log 99.2 - \log 55 = -0.163 [\log N - \log 7]$$

$$\Rightarrow \quad N = \underline{268250} \text{ cycles}.$$

2.3. Multiaxial fatigue stresses

- Complex stress state e.g. combined bending and torsion in crankshafts, propeller shafts, etc.

- Concept of equivalent uniaxial fatigue stress
  — based on extension of static yield criteria to fatigue.

- Two commonly used criteria : (i) Tresca's criterion
  (ii) von Mises' criterion.

- Tresca's (Max. Shear stress) Criterion :

$$\boxed{\frac{1}{2}\sigma_{eq.} = \frac{1}{2}(\sigma_1 - \sigma_2) \text{ or } \frac{1}{2}(\sigma_2 - \sigma_3) \text{ or } \frac{1}{2}(\sigma_3 - \sigma_1)} \quad ---(2.4)$$

- von Mises' (Max. Shear Strain Energy) Criterion:

$$\boxed{\sigma_{eq.} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}} \quad ---(2.5)$$

or in 2D,

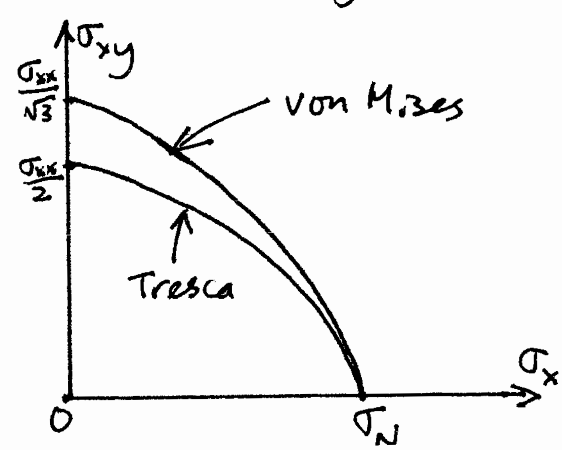
$$\boxed{\sigma_{eq} = \left[\sigma_1^{\,2} - \sigma_1\sigma_2 + \sigma_2^{\,2}\right]^{1/2}} \quad -----(2.6)$$

- Under, say, stress system with $\sigma_{yy} = 0$, σ_{xx} and $\sigma_{xy} \neq 0$,

Tresca : Fatigue strength

$$\sigma_N = \sigma_{eq.} = \sqrt{\sigma_x^{\,2} + 4\sigma_{xy}^{\,2}}$$

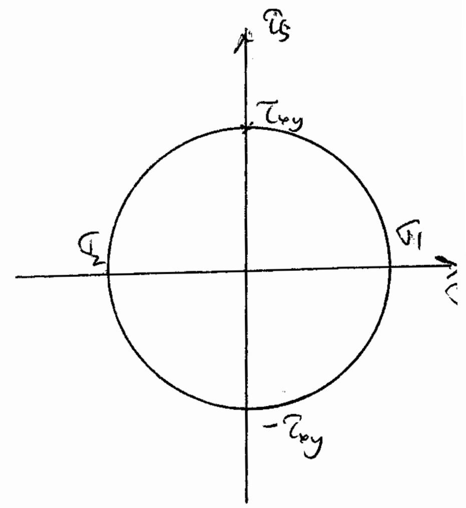
von Mises : $$\sigma_N = \sigma_{eq.} = \sqrt{\sigma_x^{\,2} + 3\sigma_{xy}^{\,2}}$$



Example:
- Cylindrical shaft 0.1 m diameter subjected to completely reversed torque (twisting moment).
- Fatigue strength of material for reversed axial loading $\sigma_N = 290$ MPa for 10^6 cycles life
- Determine the twisting moment that will give a life of 10^6 cycles.
- Use von-Mises criterion.

Solution: Under pure torsion

$$\sigma_1 = +\tau_{xy} \;;\; \sigma_2 = -\tau_{xy}$$



$$\sigma_{eq}^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2$$

$$\therefore \sigma_{eq} = \sqrt{3}\,\tau_{xy} = \sigma_N$$

$\therefore$ Max. allowable shear stress for life of 10^6 cycles

$$\tau_{xy} = \frac{\sigma_N}{\sqrt{3}} = \frac{290}{\sqrt{3}} \text{ MPa.}$$

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow T = \tau_{xy}\frac{J}{r} = \tau_{xy}\frac{\pi d^4}{32}\cdot\frac{1}{(d/2)}$$

$$= \frac{\pi d^3}{16}\tau_{xy}$$

$$= \frac{\pi \times (0.1)^3}{16}\cdot\frac{290}{\sqrt{3}} \times 10^6 \text{ N m}$$

$$= \underline{32.9 \times 10^3} \text{ Nm.}$$

Example: A drive shaft, subjected to an alternating shear stress τ_s and an alternating bending stress σ_b, in phase and are equal in magnitude.
What is the maximum value of σ_b and τ_s if fatigue failure is never to occur?
Given: UTS $\sigma_u = 1400$ MPa; uniaxial cyclic tension tests on the material with stresses varying from 0 up to 800 MPa, give infinite life.
Assume Goodman criterion applies for this material.

Goodman:
$$\frac{\sigma_a}{\sigma_N} + \frac{\sigma_m}{\sigma_u} = 1$$

$$\sigma_m = \frac{0+800}{2} = 400 \text{ MPa}$$

$$\sigma_a = \frac{800-0}{2} = 400 \text{ MPa}$$

$$\therefore \quad \frac{400}{\sigma_N} + \frac{400}{1400} = 1 \Rightarrow \sigma_N = 560 \text{ MPa.}$$

Tresca:
$$\sigma_N = \sigma_{eq.} = \sqrt{\sigma_b^2 + 4\tau_s^2} = \sqrt{5}\,\sigma_b, \text{ since } \sigma_b = \tau_s$$

$$\therefore \quad \sigma_b = \frac{560}{\sqrt{5}} = \underline{250 \text{ MPa.}}$$

or

von Mises:
$$\sigma_N = \sigma_{eq.} = \sqrt{\sigma_b^2 + 3\tau_s^2} = 2\sigma_b$$

$$\therefore \quad \sigma_b = \frac{560}{2} = \underline{280 \text{ MPa.}}$$