FRACTURE MECHANICS

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Chapter 1

Introduction to Fracture Mechanics

Many structural failures of bridges, tanks, pipes, weapons, ships, railways and aerospace structures occurred at stress levels considerably lower than the ultimate strength of the material in the past. These failures could not be explained by the conventional design criteria of that time. Therefore, there was a need for the design criteria which can explain these failures. The experiments performed by A. A. Griffiths, the British physicist, in 1924, led to the conclusion that the strength of real materials is much smaller, typically by two orders of magnitude, than their theoretical strength [1]. He concluded that this discrepancy in strength was due to the presence of small pre-existing flaws or defects, which could greatly reduce the fracture strength. Therefore, catastrophic fracture is due to the unstable propagation of a crack from these pre-existing defects. Griffith's ideas were used in the formulation of a new design philosophy, known as fracture mechanics, as opposed to the use of the conventional failure criteria such as Tresca and von Mises criteria. In short, fracture mechanics is the study of the load-carrying capabilities of structures and components containing defects. Fracture mechanics introduces a parameter, the crack driving force (stress intensity factor), which characterizes the propensity of a crack to extend. On the other hand, the critical value of this parameter (a property of the material) is determined from laboratory tests and is called fracture toughness [2]. The fracture toughness is possibly the most important information for structural design and is used to rank the ability of a material to resist fracture in the same way as yield or ultimate strength ranks the resistance of a material to yield or fracture in the conventional design criteria [1].

Linear Elastic Fracture Mechanics (LEFM) and Elastic Plastic Fracture Mechanics (EPFM) are the two sub-branches of fracture mechanics. LEFM considers the fundamentals of the linear elasticity theory, while EPFM characterizes elastic and plastic behaviours of solids [3]. LEFM is by far the most highly developed and widely applied version of fracture mechanics. Sometimes, the conditions of LEFM are not fulfilled, then EPFM is used which is not as well developed as LEFM.

This report has been prepared to include the basic concepts and underlying principles of fracture mechanics. The first, introductory chapter gives a brief account of Griffith's experiments which gave impetus to the development of fracture mechanics in engineering design. The second chapter studies the stress concentration factor and distribution around a crack tip in cracked bodies. The third chapter covers the basic conditions, principles and governing equations, limitations of LEFM with particular emphasis on life estimation. Finally, the forth chapter describes the basic concepts and fundamentals of EPFM including the crack tip plasticity and its impact on stress distribution, the crack tip opening displacement concept, the J-Integral concept, life estimation based on EPFM and its limitations and dynamic fracture mechanics.

1.1 Griffith's Theory

Before discussing the Griffith's theory and basic concepts further, it is appropriate to pay attention to brief historical developments of fracture mechanics. Fracture mechanics could be dated back to 1924 when A. A. Griffith made a quantitative connection between the strength and the crack size. Then, George Irwin's works made major contributions in 1948 to the development of fracture mechanics when he published his first classic papers which were almost entirely focused on LEFM. In 1968, J. R. Rice presented his J-integral concept and J. W. Hutchinson used the plasticity concept and gave a direct description of the discrete and nonlinear events involved in crack extension [4].

At first, A. A. Griffith developed an approach to put fracture prediction on an analytic basis and showed that the theoretical strength of a defect-free, brittle material exhibiting linear elastic behaviour is given by [3]:

$$\sigma_{th} = \left(\frac{2E\gamma_s}{\lambda_0} \right)^{1/2} \tag{1}$$

where E is Young's modulus, γ_s is the specific surface energy, λ_o is the lattice parameter, and σ_{th} is the theoretical cohesive strength [3]. Then, he performed an experiment on a large plate, containing an elliptical crack with a crack tip radius ρ, which was subjected to a remote and uniform tensile load in the direction of the y-axis and perpendicular to the crack line along the x-axis as shown in Figure 1 [3]. Griffith noted that the specific surface energy resulting from the presence of a crack is increased and the potential energy related to the release of the stored energy as well as the work done by the external loads is decreased [5]. Thus, the total potential energy of the system is given by [3]:

$$U = U_0 - U_a + U_\gamma \tag{2}$$

$$U = U_0 - \frac{\pi\beta a^2 \sigma^2 B}{E} + 2(2aB\gamma_s) \tag{3}$$

$$\frac{dU}{da} = -\frac{2a\pi\beta\sigma^2 B}{E} + 4B\gamma_s \tag{4}$$

with U = Potential energy of the cracked body

U_o = Potential energy of the uncracked body

U_a = Energy due to the presence of a crack

U_γ = Surface energy due to the formation of crack surfaces

a = One-half of the crack length

$4a\beta = 2(2a\beta)$ = Total surface crack area

γ_s = Specific surface area

E = Modulus of elasticity

σ = Applied stress

ν = Poison's ratio

$\beta = 1$ for plane stress and $\beta = 1- \nu^2$ for plane strain

B = Specimen thickness

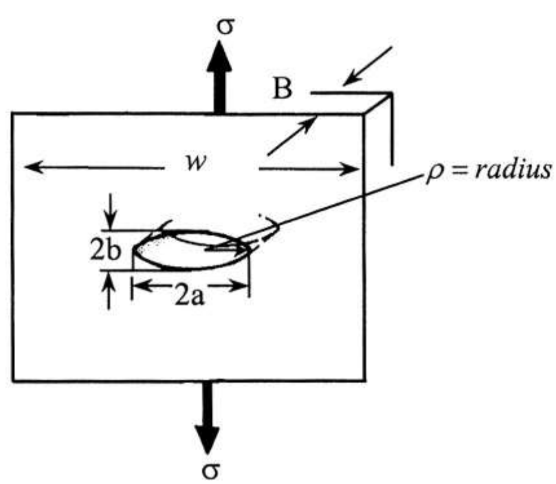

Figure 1: Through thickness elliptical crack in a large plate [3].

Under equilibrium conditions, the first derivative is of significance because the critical crack size or critical stress may be predicted very easily. From equation (4), if *dU/da* = 0, the crack size and total surface energy are, respectively:

$$a = \frac{(2\gamma_s)E}{\pi\beta\sigma^2} \tag{5}$$

$$2\gamma_s = \frac{\pi\beta a\sigma^2}{E} \tag{6}$$

Assuming $\beta = 1$ and rearranging equation (6), gives the critical stress σ_c required for crack propagation:

$$\sigma_c = \left(\frac{2E\gamma_s}{\pi a}\right)^{1/2} \tag{7}$$

This equation (7) is known as the Griffith's equation which was developed on the basis of minimizing the energy of the system, which energetically can be a necessary condition for fracture. This equation applies only to brittle materials which do not deform plastically. Further, the effects of plastic deformation were accounted for by Orowan by simply noting that the effective surface energy (which includes the work of plastic deformation around the fracture surface) can be substituted for the true surface energy in the Griffith's equation. Conceptually, it is given by [3]:

$$\gamma_E = \gamma_S + \gamma_P \tag{8}$$

where γ_p is the work of plastic deformation, γ_s is the true surface energy, and γ_E is the effective surface energy. For metals and polymers, γ_p is much greater than γ_s and to a good approximation [3]:

$$\gamma_P \approx \gamma_E \tag{9}$$

These basic concepts and equations will be further used to explain the stress field around a crack tip and to describe the linear elastic and elastic plastic fracture mechanics theories.

Chapter 2

Stress Concentration and Distribution around a Crack or a Defect

As mentioned previously, all engineering materials contain cracks or defects that can exist in a material due to its composition, as second-phase particles, or can be introduced in a structure during fabrication, as in welds, or can be created during the service life of a component, like fatigue cracks, environment assisted or creep cracks [1]. Moreover, the estimation of the remaining life of a machine or structural components requires the knowledge of the stress concentration and distribution caused by the introduction of cracks in conjunction with their growth condition. Therefore, it is necessary to know about the stress concentration and distribution around a crack tip. Linear elastic fracture mechanics principles require that the material be homogeneous, isotropic, and elastic and can be applied in order to describe stress concentration and distribution around a crack tip. Fundamentals and concepts of linear elastic fracture mechanics will be discussed in the next chapter.

When a remote stress (nominal stress, σ) is applied perpendicular to the crack length/depth in a thin plate, the local stress may be amplified at the tip; the magnitude of this concentrated stress depends on the crack orientation and geometry. Due to their ability to amplify the nominal stress in their vicinity, these flaws are sometimes called stress raisers [2]. The geometry of surface and internal cracks, stress amplification at the crack tip and a schematic stress profile along x-axis is shown in Figure 2 [2]. As indicated by this profile, the magnitude of the localized stress diminishes with distance away from the crack tip. At positions far removed, the stress is equal to the nominal stress σ [2].

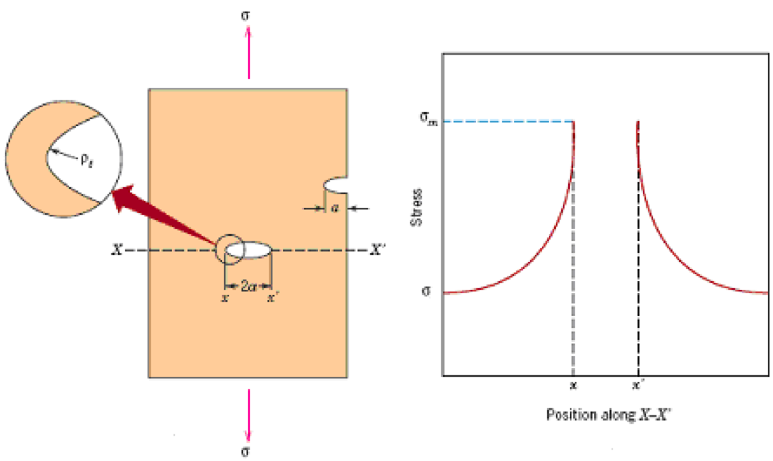

Figure 2: Geometry of surface and internal cracks as well as the resulting stress profile [2].

The equation of an ellipse and the radius of curvature are given by equation (10) and (11), respectively [3]:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{10}$$

$$\rho=\frac{b^2}{a} \tag{11}$$

The resultant maximum stress, σ_m, occuring at the edge of the ellipse (crack tip) is given by [3]:

$$\sigma_m=\left(1+\frac{2a}{b}\right)\sigma \tag{12}$$

The second derivative of the Griffith's energy equation (3) with respect to the crack length yields [3]:

$$\frac{d^2U}{da^2} = -\frac{2\pi\beta\sigma^2 B}{E} \tag{13}$$

If $d^2U/da^2<0$, the system is unstable; consequently, the crack will grow. Combining equations (11) and (12) gives equation (14) in terms of crack tip radius as:

$$\sigma_m = \sigma\left[1 + 2\left(\frac{a}{\rho_t}\right)^{1/2}\right] \tag{14}$$

The ratio of σ_m/σ is denoted as the stress concentration factor (K_t) which is simply a measure of the degree to which an external stress is amplified at the crack tip. Thus, the theoretical stress concentration factor (K_t) becomes:

$$K_t = \frac{\sigma_m}{\sigma} = \left[1 + 2\left(\frac{a}{\rho_t}\right)^{1/2}\right] \tag{15}$$

For a relatively long microcrack that has a small tip radius of curvature, the factor $(a/\rho_t)^{1/2}$ tends to be very large and yields a value of maximum axial stress (σ_m) that is many times the value of σ. Therefore, the equation (15) can be written as:

$$K_t = \frac{\sigma_m}{\sigma} = 2\left(\frac{a}{\rho_t}\right)^{1/2} \tag{16}$$

In the case of a circular hole, $a = b$, $\sigma_m = 3\sigma$, and $K_t = 3$. In the special case of b tending to zero, σ_m tends to infinity; this means a very sharp crack is formed with the radius ρ_t tending to zero. In such a situation, the stress concentration factor (K_t) tends to infinity as well. In fact, microstructural and geometrical discontinuities, such as notches, grooves, and the like, lead to extremely high stress concentration factors [1-3].

Any stress component in the vicinity of the crack tip (see Figure 3) is given by the following equation [5]:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \qquad (17)$$

where σ_{ij} is the stress component of interest; r, θ are polar coordinates with the origin at the crack tip, and $f_{ij}(\theta)$ is a function of θ that depends on the stress component being considered. In this notation, the subscript, i, refers to the plane on which the stress component acts and the subscript, j, refers to the direction of stress application. K is the stress intensity factor (crack driving force) which is a measure of the magnitude of the stress field in the crack tip region [5].

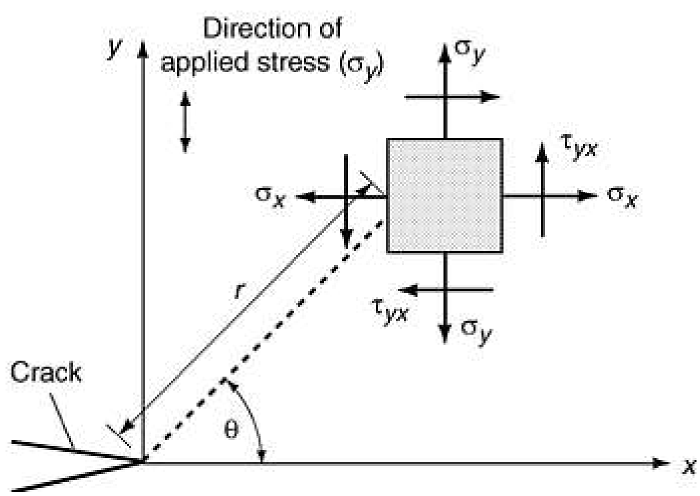

Figure 3: Stress components ahead of the crack tip [5].

It is clear from equation (17) that if r tends to zero, stress tends to infinity, then a singularity state of stress occurs which represents an unrealistic situation because no material is capable of withstanding an infinite stress. Instead, the material will tend to deform plastically near the crack tip and absorb energy in the process. The preceding analysis must be modified to include a small zone of plasticity at the crack tip. The modified stress distribution ahead of the crack in which small plasticity is included is shown schematically in Figure 4 [5]. The extent of the small plastic zone directly ahead of the crack is denoted by R_p. This small plastic zone is very important because energy is absorbed in this region which is responsible for the relatively high fracture toughness of metals and polymers compared to brittle materials such as ceramic [5].

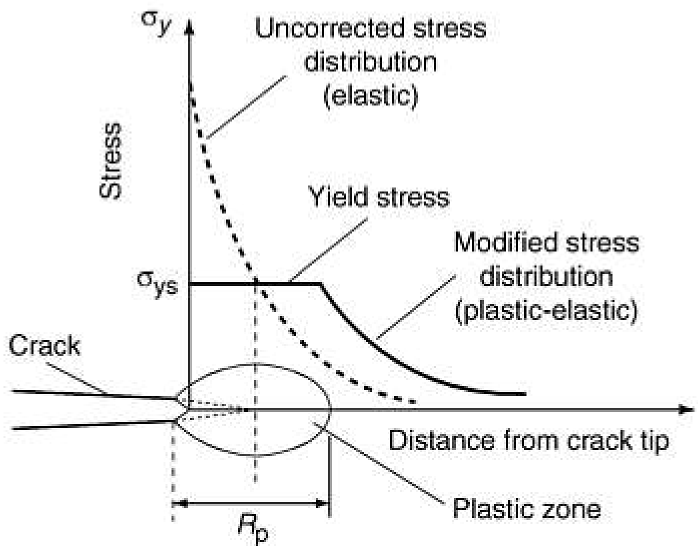

Figure 4: Modified stress distribution ahead of crack tip [5].

The plastic zone size depends on both K and σ_{ys} and is given by [5]:

$$R_p = C\left(\frac{K}{\sigma_{ys}}\right)^2 \tag{18}$$

The constant, C, depends on various factors including thickness and deformation characteristics of the material. It usually has a value between $1/\pi$ and $1/3\pi$. Since the validity of equation (17) is limited to linear elastic bodies, it is only applicable when R_p is much smaller (i.e. R_p <10% of the crack size) than the crack size and other physical dimensions of the body. If the load is increased, K increases which also tends to increase R_p and the material in the plastic zone becomes more severely strained. Because of this direct relationship between K and the strains at the crack tip, it can be concluded that there is a critical value of the stress intensity factor at which fracture will take place [3]. The critical value of the stress intensity factor, known as the fracture toughness, as introduced in chapter 1, expresses the ability of the material to resist fracture in the presence of cracks. The equations listed above and fracture mechanics concepts form the basis of computing fracture stresses and critical crack lengths in life estimation and other applications.

Chapter 3

Linear Elastic Fracture Mechanics

It is very important to briefly describe the conditions, principles and governing equations of LEFM in order to understand the situations under which it works well. This chapter introduces conditions, principles and governing equations of LEFM followed by life estimation based on LEFM and its limitations.

3.1 Conditions, Principles and Governing Equations

Knowledge of loading modes, plane conditions (stress and strain), etc. are necessary in order to understand the conditions, principles and governing equations of LEFM and life prediction based on LEFM. Brief introduction of loading modes and plane conditions are covered in the following paragraphs.

Modes of Loading: Specimens and structural components having flaws or cracks can be loaded to various levels of the stress intensity factors. The three different types of loadings involving displacements of the crack surfaces can be referred to as mode I (opening or tensile mode), mode II (sliding mode) and mode III (tearing mode) as shown in Figure 5 [3]. This is analogous to unflawed components being loaded to various levels of the applied stresses [3]. The way the material is loaded is very important because the mechanical behaviour of a material containing a crack of a specific geometry and size can be predicted by evaluating the stress intensity factors for each of the three forms of loadings.

The stress intensity factor depends linearly on the applied load and is a function of the crack length and the geometrical configuration of the cracked body and can be generally defined as [1, 3]:

$$K_i = f\,(\text{Stress, Crack Geometry, Specimen Configuration}) \qquad (19)$$

where i = I, II, III, stand for mode I, mode II and mode III, respectively.

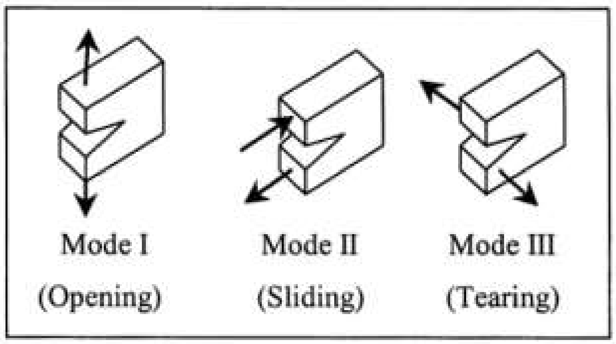

Figure 5: Three modes of the crack surface displacement [3].

In order to determine the static or dynamic fracture stress, the fatigue crack growth rate and corrosion crack growth rate, the parameter K_i can be used. The strain-energy release rate (G_i), known as the crack driving force, is related to the stress intensity factor and the modulus of elasticity by the expression [3]:

$$G_i = \frac{K_i^2}{E'} \qquad (20)$$

with E = Elastic modulus of elasticity (MPa)

ν = Poisson's ratio

$E' = E$ For plane stress (MPa)

$E' = \frac{E}{(1-\nu^2)}$ For plane strain (MPa)

From equation (17), the stresses near the crack tip for mode I loading type (as most commonly used) can be given by [3]:

$$\sigma_x = \frac{K_1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{21}$$

$$\sigma_y = \frac{K_1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{22}$$

$$\tau_{xy} = \frac{K_1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{23}$$

The quantity K_I is the mode I stress intensity factor which expresses the strength of the elastic stress field. The stress intensity factor for infinite (large) specimen dimensions (uncorrected) and finite specimen dimensions (corrected) under mode I can be expressed by equation (24) and (25) respectively [3]:

$$K_I = \sigma\sqrt{\pi a} \quad \text{(Uncorrected)} \tag{24}$$

$$K_I = Y\sigma\sqrt{\pi a} \quad \text{(Corrected)} \tag{25}$$

where $Y = f\,(a/w)$ refers to the crack geometry correction factor and w is the specimen width. K_I in equation (25) is a linear function of the applied stress, σ, and increases with the crack size as can be seen in Figure 6 [3].

By equating the stress intensity factor to fracture toughness, a relationship between applied load, crack size and structure geometry can be obtained that gives the necessary information for structural design [1]. The fracture toughness (K_C) that relates critical stress (σ_c) for crack propagation and crack length (a) is given by [1]:

$$K_C = Y\sigma_C\sqrt{\pi a} \tag{26}$$

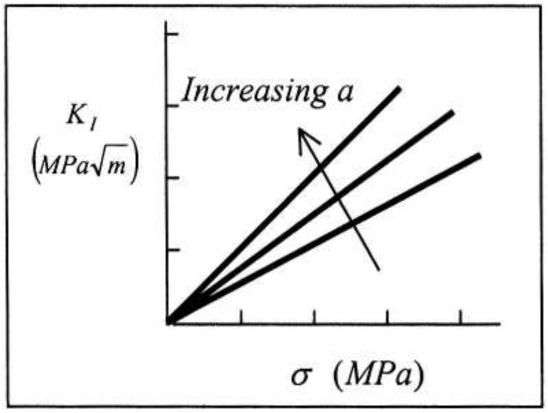

Figure 6: Linear change of K_I as a function of σ [3].

Plane Conditions: Another important parameter for characterizing the mechanical behaviour of either a cracked or a crack-free specimen is its thickness. Thus, there are two plane conditions, i.e. plane stress and plane strain which are described below.

Plane Stress: This is a stress condition used for thin bodies (plates), in which the specimen thickness, *B,* is much smaller than the width, *w*, i.e. $B \ll w$. Thus, there are only two stress components because the plate is too thin to sustain a stress through the thickness. According to plane stress conditions, the stresses are expressed through equation (27) [3]:

$$\sigma_{zz} = \tau_{yz} = \tau_{zx} = 0 \tag{27}$$

For relatively thin specimens, the value of K_C, given by equation (27), depends on specimen thickness.

Plain Strain: When specimen thickness is much larger than the crack dimensions, K_C becomes independent of thickness; under these situations, a condition of plane strain exists which develops a triaxial state of local stress at the crack tip. The value of the fracture toughness (K_C) for this thick specimen situation is known as the plane strain fracture toughness, K_{IC}. The plain strain fracture toughness K_{IC}, for mode I type of loading, can be expressed by [2]:

$$K_{IC} = Y\sigma_c\sqrt{\pi a} \tag{28}$$

The through-thickness stress for plane strain condition is given by [3]:

$$\sigma_{zz} \approx \nu(\sigma_{xx} + \sigma_{yy}) \tag{29}$$

The effect of specimen thickness on fracture toughness (K_C and K_{IC}) is shown in Figure 7. Stress intensity factor or fracture toughness strongly depends on the specimen thickness up to a limiting value as can be seen in Figure 7 [3]. For a thin plate, plane stress condition ($\sigma_z = 0$) governs the fracture process as mentioned above. For a thick plate, plane strain condition ($\sigma_z \neq 0$) prevails in which K_{IC} becomes a material's property. Plane stress and plane strain modes of fracture also differ in terms of the characteristics of the fracture surface. For example, plane stress fracture mode shows a slant fracture (shear lips at approximately 45°) as an indication of partial ductile fracture whereas plane strain fracture mode exhibits a flat fracture surface as a representation of brittle fracture as schematically indicated in Figure 7. A mixed mode fracture surface corresponds to any combination of these modes of fracture [3].

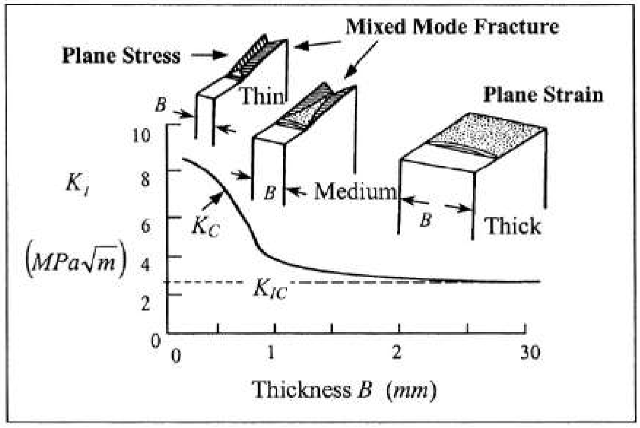

Figure 7: Effect of specimen thickness on fracture toughness (K_C and K_{IC}) [3].

Thus, in order to apply the linear elastic fracture mechanics (LEFM) methodology correctly, the following conditions must be met [1-6].

1. The deformation at the crack tip must be mostly elastic
2. The material must be isotropic and homogeneous at the crack tip
3. There must only be a single well defined crack
4. The plastic zone must be small compared to in-plane dimensions of specimen and structure (crack length, grain size, etc.), which mean that there can only be limited plastic deformation at the crack tip at the time of fracture
5. The crack must propagate in mode I (opening mode)
6. The plane strain conditions must prevail

According to these listed conditions of LEFM, the materials for which the LEFM approach works well are [5]:

1. High strength metallic alloys such as heat treated martensitic steels
2. Precipitation-hardened aluminum alloys used in the aerospace industry
3. Titanium alloys

4. Ni based alloys
5. Ceramic materials which are essentially brittle, even at high temperatures, because their ductility is limited
6. Polymers of limited inelastic deformation

3.2 Life Estimation based on LEFM

Fracture mechanics describes relationships between material properties, stress level, the presence of crack-producing flaws, and crack propagation mechanisms. Pre-existing flaws or cracks (initiated during service) grow with time owing to fatigue, creep, oxidation mechanisms, etc. LEFM principles are most frequently applied to solve crack growth problems and determine the critical size of the crack at which fracture occurs. Figure 8 is an example of a cracked component loaded under cyclic condition [7].

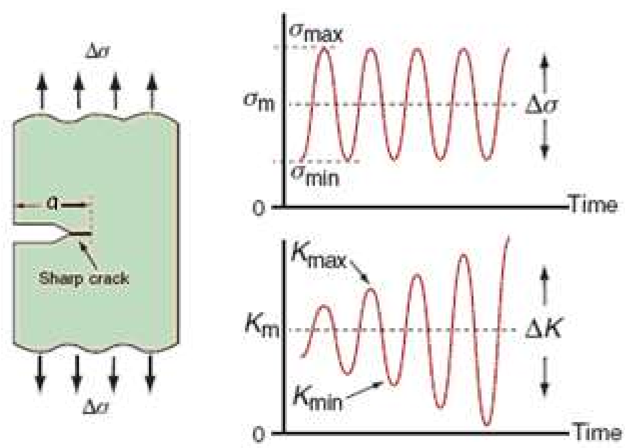

Figure 8: Cyclic loading of a cracked component [7].

The stress intensity factor (K), defined as in equation (25), controls the plastic deformation near the crack tip, and consequently the fracture process. Therefore, the stress intensity factor range, ΔK, can be defined as:

$$\Delta K = K_{max} - K_{min} = Y\Delta\sigma\sqrt{\pi a} \tag{30}$$

As the crack grows and increases in length during service, the stress intensity range ΔK increases with time under constant cyclic stress as can be seen in Figure 8. The crack growth per cycle, *da/dN*, increases with ΔK as illustrated in Figure 9 [1].

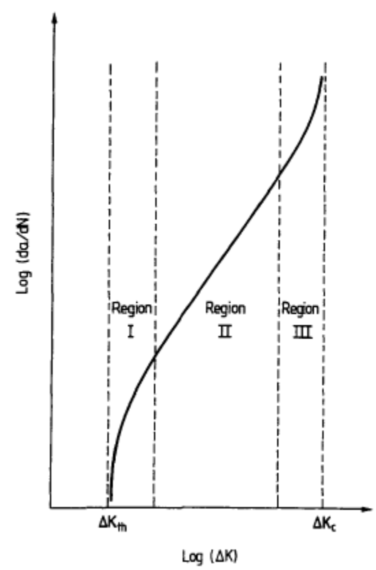

Figure 9: Crack growth rate *da/dN* vs. ΔK in a log-log scale [1].

There are three main stages in the curve:

In the first stage, below the threshold stress intensity factor range (ΔK_{th}), non-propagating cracks exist and there is no observable fatigue crack growth. The second stage is the most important and describes a linear relationship between log(*da/dN*) and log(ΔK) through the Paris equation written as [1]:

$$\frac{da}{dN} = C(\Delta K)^m \tag{31}$$

where C and m are material constants. In the third stage, an accelerated crack growth occurs leading to unstable fracture when the stress intensity factor reaches the fracture toughness (K_{IC}) of the material [1].

In order to estimate the service life, both non-destructive testing (NDT) methods such as liquid penetrant testing (PT), magnetic particle testing (MT), radiographic testing (RT), ultrasonic testing (UT), and eddy current testing etc. and LEFM concepts are used. The initial crack, a_o (a_{NDT}), if present, is detected using NDT techniques in the first stage of the curve. Then, the LEFM concept is applied to estimate the critical crack length or final crack length (for fast catastrophic fracture), a_c, using equation (32).

$$a_c = \frac{Q}{\pi}\left(\frac{K_{IC}}{\sigma Y}\right)^2 \tag{32}$$

The Paris law of crack growth is then used to estimate the number of cycles needed for the crack to grow from a_o to the size a_c. Integrating equation (31) between the limits of initial crack size, a_o, and final crack size, a_c gives:

$$N = \int_{a_0}^{a_c} \frac{1}{C \cdot \Delta K^m} da \tag{33}$$

and after solving

$$N = \frac{1}{\frac{(m-2)}{2}\pi^{m/2} C Y^m (\Delta\sigma)^m a_0^{(m-2)/2}}\left[1 - \left(\frac{a_0}{a_c}\right)^{(m-2)/2}\right] \tag{34}$$

For the special case of $m = 2$, equation (34) can be simplified to:

$$N = \frac{1}{\pi C Y^2 (\Delta\sigma)^2} \ln\left(\frac{a_c}{a_0}\right) \tag{35}$$

Equation (35) is often used for service life estimation using LEFM concept.

3.3 Limitations of LEFM

LEFM is not valid when significant plastic deformation precedes failure because loading of a cracked body is usually accompanied by plastic deformation and other nonlinear effects near the crack tip. Moreover, the conditions of LEFM listed above in section 3.1 must be met. Situations where the extent of inelastic deformation is pronounced necessitate the use of nonlinear theories and will be dealt with in the next chapter.

Composites are one of the important classes of materials for which fracture mechanics is frequently not applicable because cracks tend to be spatially distributed throughout the material. In this situation, the usual fracture mechanics requirement of a single, well-defined crack is not met. Also, the requirements of homogeneous and isotropic materials are not met for most composite systems [5].

Chapter 4

Elastic Plastic Fracture Mechanics

Linear elastic stress analysis of sharp cracks predicts infinite stresses at the crack tip as the plastic zone size (r) approaches zero. This represents a state of stress singularity and implies that there is a crack with zero crack tip radius. However, real materials have an atomic structure, and the minimum finite tip radius is about the inter-atomic distance which limits the stresses at the crack tip [8]. More importantly, most engineering metallic materials are subjected to an irreversible plastic deformation when the yield strength is exceeded near the crack tip and so in reality there is a plastic zone surrounding the crack tip. Thus, linear elastic fracture mechanics is no longer useful for predicting the stress field equations when plastic zone size ($r >> a$) is large and therefore, the most attractive approach is the elastic plastic fracture mechanics (EPFM) to control the fracture process in this situation [1,3,8].

It is the objective of this chapter to study the elastic plastic fracture mechanics with particular emphasis on the size of the crack tip plasticity and its impact on stress distribution as well as a few models of the configuration or the shape of the crack tip plasticity. Brief overview of the crack tip opening displacement concept, the J-integral concept, life estimation based on EPFM and its limitations and dynamic fracture mechanics are included.

4.1 Crack Tip Plasticity and its Impact on Stress Distribution

The shape and size of the plastic zone is an important parameter in order to set an approximate limit for both LEFM and EPFM theories. Therefore, it is essential to have a thorough knowledge of the shape and size of the plastic zone. The stress field equations, listed in previous chapters, plasticity corrections are not necessary if small scale yielding occurs ($r << a$). In fact, an inelastic (plastic) deformation occurs when the yield strength (σ_{ys}) is exceeded near the crack tip and the crack behaves as if it is larger than the actual size. The internal tensile stress (σ_y) is limited to the ultimate tensile stress (σ_u) due to strain hardening that occurs at the plastic zone. Furthermore, the plastic zone size reaches a maximum magnitude when $\sigma_y = \sigma_u$ and, consequently; the effective crack size $a_e = a + r$ becomes the new actual size, which extends through the plastic zone due to initiation and coalescence of voids. This may repeat and continue until the actual crack reaches a critical value at the onset of crack propagation for fracture or separation [1,3].

The crack tip plastic zone and the stress distribution, both elastic and elastic plastic, are illustrated in Figure 10 [3]. It is assumed that the loading is in mode I and the crack growth occurs along its plane so that $\theta = 0$ and $y = 0$ along the crack line (x-axis). Solving equations (22) and (24), the stress along the crack line can be given by:

$$\sigma_y = \sigma\sqrt{\frac{a}{2r}} \tag{36}$$

In order to account for plasticity effects adjacent to the crack tip, setting $\sigma_y = \sigma_{ys}$ in equation (36) yields the plastic zone size as:

$$r = \frac{a}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2 \tag{37}$$

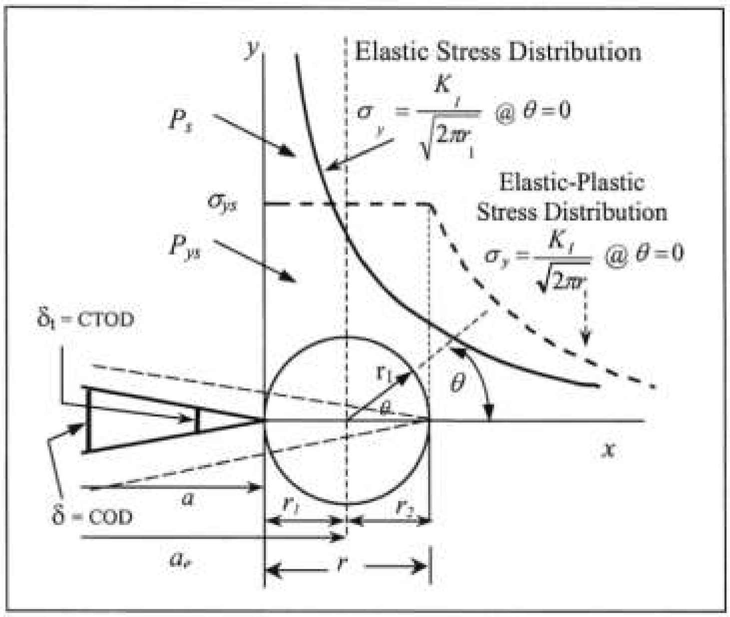

Figure 10: Crack tip plastic zone and stress distribution [3].

The elastic stress distribution in the vicinity of a crack tip exhibits an infinite stress as *r* tends to zero as shown in Figure 10. However, yielding due to plastic deformation leads to stress redistribution or stress relaxation at the plastic zone. Therefore, the local stresses are limited to the yield strength (σ_{ys}) of the material. As can be seen in Figure 10, this elastic stress distribution takes over from the yield stress at a distance *r* from the actual crack tip. Moreover, the stresses and strains both within and outside the plastic zone will be determined by *K* and the stress intensity approach can still be used. In addition, the stress intensity factors within and outside the boundary of the plastic zone are different in magnitude so that K_I (plastic) > K_I (elastic). In fact, K_I (plastic) must be defined in terms of plastic stresses and displacements in order to characterize the crack growth, and subsequent ductile fracture. Thus, the elastic plastic analysis of the stress field around the crack tip helps in determining the effective stress intensity factor (K_e) or the corrected stress intensity factor. Therefore, it is necessary to include the plastic zone size as a correction parameter that accounts for plasticity effects adjacent to the crack tip. Various investigators attempted to find the plastic zone size and the most widely known approaches such as Irwin and Dugdale models have been used to describe the plastic zone size. Irwin's and Dugdale's models are described in the following paragraphs.

The Plastic Zone Size According to Irwin: Irwin argued that the occurrence of plasticity makes the crack behave as if it is larger than its physical size i.e. the effect on the plastic zone is to artificially extend the crack by a distance r_I (see Figure 10) known as Irwin's plastic zone correction [1,3]. In order to account for the changes due to the artificial crack extension or virtual crack length, the crack length a can be replaced by a_e referred to as the effective crack length. Therefore, replacing $a_e = a + r$ in equation (25) provides the effective stress intensity factor as:

$$K_I = Y\sigma\sqrt{\pi(a+r)} = Y\sigma\sqrt{\pi a_e} \tag{38}$$

and substituting the value of r from equation (37) gives:

$$K_I = Y\sigma\sqrt{\pi a\left[1+\frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2\right]} \tag{39}$$

The Plastic Zone Size According to Dugdale: Dugdale's analysis assumes that all plastic deformation concentrates in the form of narrow strips in front of the crack, the so-called strip yield model [3,8]. The plastic zones that occur in the form of narrow strips extending a distance r each, and carrying the yield stress σ_{ys} as shown in Figure 11(a) and (b) which will also be further used to relate the crack tip opening displacement (δ_t) and the J-integral in subsequent sections.

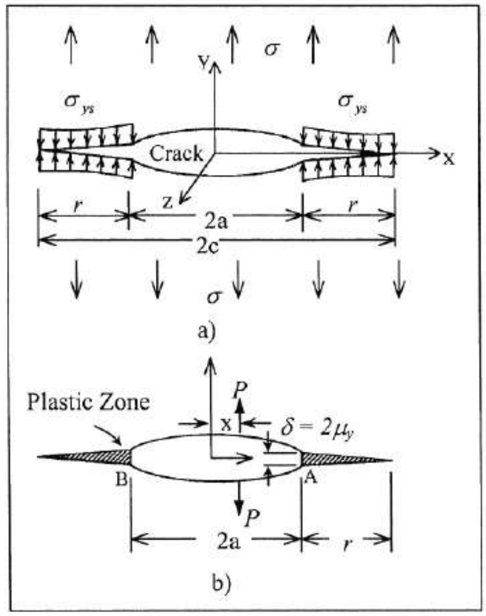

Figure 11: Dugdale plastic zone strip model (a) Dugdale crack (b) wedge crack [3].

The phenomenon of crack closure occurs due to internal stresses since they tend to close the crack in the region where $a < x < c$. Dugdale also argued that the effective crack length is longer than the physical length and therefore, the crack length a can be replaced by the virtual crack length (a+r). Therefore, according to Dugdale's model, the expression for r and K_I can be given by [3]:

$$r = \frac{a}{2}\left(\frac{\pi a}{2\sigma_{ys}}\right)^2 \tag{40}$$

$$K_I = Y\sigma\sqrt{\pi a\left[1 + \frac{1}{2}\left(\frac{\pi\sigma}{2\sigma_{ys}}\right)^2\right]} \tag{41}$$

Comparison of Irwin's equation (39) and Dugdale's equation (41) yields:

$$r[Irwin] = \frac{8}{\pi^2}[Dugdale] = 0.81r[Dugdale] \tag{42}$$

Based on Irwin's and Dugdale's approximations of plastic zone size, the normalized stress intensity factors vs. stress ratio are plotted in Figure 12 [3]. It is clear in the Figure 12 that the similarity exits between two curves when $0 < \sigma/\sigma_{ys} \leq 0.2$, however, curves significantly differ when the stress ratio (σ/σ_{ys}) tends to one. At large stress ratios, there is a significant difference in normalized stress intensity factors. Therefore, it is suggested that at large stress ratios, both Irwin's and Dugdale's approximation methods should be used very carefully [3].

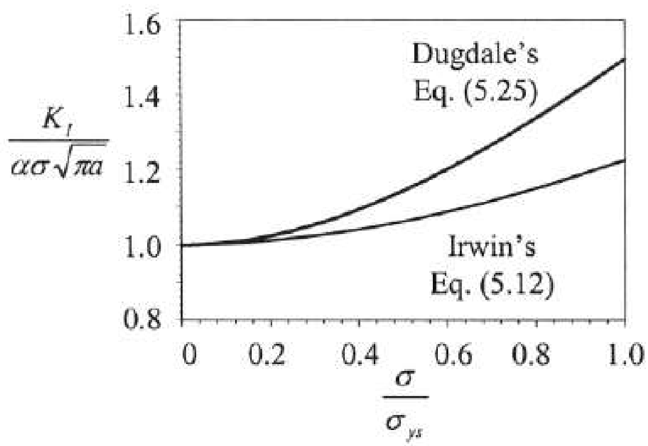

Figure 12: Normalized stress intensity factor vs. stress ratio [3].

Linear elastic fracture criterion can not be applied to crack growth at a large scale yielding situations i.e. when the plastic zone size is large (r >>a) and the physical sense of K_{IC} acting as the critical parameter is lost to a great extent. Therefore, nonlinear fracture criteria such as the crack tip opening displacement (δ_t-model) and the J-integral (J-model) are used to control fracture criteria in large scale yielding situations. These two models are briefly described in subsequent sections.

4.2 Crack Tip Opening Displacement Concept

The crack opening displacement concept is used as a fracture criterion for the study of crack initiation in situations where significant plastic deformation precedes fracture. It is believed that the stresses around the crack tip reach the critical value and therefore the fracture is controlled by the amount of plastic strain. The separation of the crack faces or crack opening displacement (COD = δ), especially very close to the crack tip, is due to the amount of crack tip plastic strain [1,3]. The plastic zone correction models (Irwin's and Dugdale's models) appear to be suitable for correcting the crack opening displacement. According to these models, shown in Figure 10 and 11, δ corresponds to the upper and lower relative displacement of the crack edges. The crack opening displacement tends to reach crack tip opening displacement ($\delta \rightarrow \delta_t$) when the measurements of the crack opening displacement are made close enough to the crack tip. The crack opening displacement (δ) is defined as twice the crack tip displacement in the y-direction (see Figure 11b) [3]. Hence,

$$\delta = 2\mu_y \tag{43}$$

According to crack configuration, the crack opening displacement can be defined as [3]:

$$\delta = \frac{4\sigma}{E}\sqrt{a^2 - x^2} \quad \text{(Uncorrected)} \tag{44}$$

Substituting $a = a + r$, equation (44) becomes:

$$\delta = \frac{4\sigma}{E}\sqrt{(a+r)^2 - x^2} \quad \text{(Corrected)} \tag{45}$$

If $x = a$, then $\delta = \delta_t$

$$\delta_t = \frac{4\sigma}{E}\sqrt{(a+r)^2 - x^2} \approx \frac{4\sigma}{E}\sqrt{2ar} \tag{46}$$

Inserting the value of *r* from equations (37) and (40) from Irwin's and Dugdale's models into equation (46), provides the crack tip opening displacement as:

$$\delta_t = \frac{4a\sigma^2}{E\sigma_{ys}} \text{ (Irwin)} \tag{47}$$

$$\delta_t = \frac{2\pi a\sigma^2}{E\sigma_{ys}} \text{ (Dugdale)} \tag{48}$$

These two equations can also be related as:

$$\delta_t^{Irwin} = \frac{2}{\pi}\delta_t^{Dugdale} \tag{49}$$

The crack tip opening displacement is a measure of the fracture toughness of solid materials that undergo ductile-to-brittle transition and exhibit elastic plastic or fully plastic behaviour. Crack extension begins when the crack tip opening displacement reaches some critical value. In other words, the fracture criterion can be written as [3]:

$$\delta_t = \delta_c \tag{50}$$

where δ_c is the critical value of the crack tip opening displacement which is a characteristic of the material at a given temperature, plate thickness, strain rate and environmental conditions. The magnitude of δ_c, a measure of fracture toughness, is used as a fracture criterion when K_{IC} requirements are not met [1,3].

4.3 The J-integral

The J-integral model treats the presence of large deformations and characterizes the state of plastic stresses and strains in tough materials. It is a measure of the fracture toughness at the onset of slow crack growth for elastic and elastic plastic metallic materials and therefore used as a failure criterion. In order to describe J-integral, Figure 13 illustrates a two-dimensional crack being surrounded by two arbitrary counter-clockwise contours Γ_1 and Γ_2 [3].

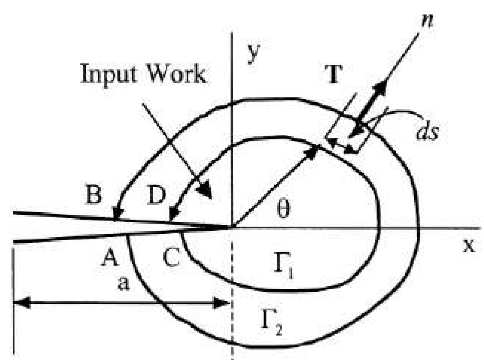

Figure 13: J-integral contours around the crack surfaces [3].

A contour integral or line integral known as the J-integral that encloses the crack front can be given by [1,3]:

$$J = \int_{r} \left(W dy - \frac{\vec{T}\partial\vec{\mu}}{\partial x} ds \right) \tag{51}$$

with J = Effective energy release rate (MPa.m or MN/m)

W = Elastic strain energy density or plastic loading work (J/m^3)

$\vec{\mu}$ = Displacement vector at ds

ds = Differential element along the contour

η = Outward unit normal to Γ

$\vec{T}\left(\frac{\partial\vec{\mu}}{\partial x}\right)ds$ = Input work

s = Arc length

$\vec{T}$ = Tension vector (traction forces) on the body bounded by Γ

Γ = Arbitrary counter-clockwise contour

In addition, the work input and the strain energy density are defined as [1,3]:

$$\vec{T} = \sigma_{ij}\eta \quad (i, j) = 1, 2, 3 \tag{52}$$

$$W = \int \sigma_{ij} d\varepsilon_{ij} \tag{53}$$

In case of pure tension loading, *W* can be defined as [3]:

$$W = \frac{\sigma_{yy}}{2E} \tag{54}$$

J-integral can also be defined as the energy release to the crack tip during crack growth. In other words, the J-integral along a contour around the crack is the change in potential energy (*dU*) for a virtual crack extension *da*. Thus, the J-integral can also be written as [3]:

$$J = -\frac{dU}{da} \tag{55}$$

In short, J-integral include following fundamental characteristics [1,3,8]:

1. It exhibits remarkable, independent path for linear or nonlinear elastic materials. The path independent nature of the integral allows the integration path to be taken close or far away from the crack tip
2. J-integral is zero for any closed paths i.e. J-integral vanishes along the closed contours Γ_1 and Γ_2
3. It is also a measure of the straining at the crack tip that accounts for significant plastic deformation at the onset of crack initiation

4. J integral is invariable in magnitude when the contour lies either inside or outside of the plastic zone

J-integral can also be considered as an attractive candidate for a fracture criterion when the requirements of K_{IC} as a fracture criterion are not met. Under opening mode loading, the fracture criterion can be expressed as [1,3]:

$$J = J_c \tag{56}$$

where J_c is a material property for a given thickness. Under plane strain conditions, the critical value of J, called J_{IC}, is related to the plane strain fracture toughness K_{IC} as [1,3]:

$$J_{IC} = \frac{1-\upsilon^2}{E} K_{IC}^2 \tag{57}$$

Since J is the available energy per unit crack extension at the crack tip, it becomes the controlling factor in characterizing the plastic behaviour of ductile materials containing stationary cracks especially in a large scale yielding case. But, in elastic plastic materials, crack blunting may occur at the crack tip and the fracture criterion used for determining the onset of crack instability depends on the amount of plasticity ahead of the crack tip. Although the J-integral is based on purely elastic (linear or nonlinear) analysis, J-integral fracture criterion can also be applied to tough materials [1,3].

Crack Tip Opening Displacement and J-integral Relationship: Dugdale's model, indicated in Figures 11, can be used to relate the crack tip opening displacement (δ_t) and the J-integral provided that the path of integration is arbitrarily chosen in the elastic regime and the contour curve Γ is taken around the plastic zone boundary [1,3]. Figure 14 shows an arbitrary contour, also shown in Figure 11(b), which is shrunk to a shape similar to the Dugdale's strip yield [3].

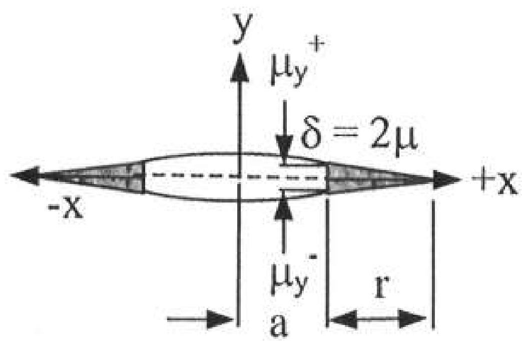

Figure 14: Dugdale's strip yield for COD measurements [3].

The limits of the J-integral under pure tension loading (mode I) becomes $x = a$ at the lower crack side before localized yielding occurs and $x = a + r$ at the upper crack side after yielding. Along the crack plane, dy = 0 and the traction force becomes $\vec{T} = \sigma_{yy} = \sigma_{ys}$. Thus, the J-integral can be given by [3]:

$$J_I = \int_r \vec{T} \frac{\partial \vec{\mu}}{\partial x} ds = -\int_a^{a+r} \sigma_{ys} \frac{\partial}{\partial x}\left(\mu_y^+ - \mu_y^-\right) dx \tag{58}$$

$$J_I = \int_a^{\delta_t} \sigma_{ys} d\left(\mu_y^+ - \mu_y^-\right) = \sigma_{ys}\delta_t \tag{59}$$

where μ_y^+ and μ_y^- are the upper and lower displacements.

4.4 Dynamic Fracture Mechanics

In quasi-static situations, the applied load varies quite slowly and the crack is assumed either to be stationary or to grow in a controlled stable manner. The concepts of fracture mechanics covered so far were devoted only to quasi-static situations in which the kinetic energy was relatively insignificant compared to other energy terms and was omitted. This section presents the basic concepts and the salient points of dynamic fracture mechanics in which the dynamically loaded stationary or growing cracks are considered. Moreover, in dynamic fracture mechanics, rapid motions are generated in the medium and inertia effects also become important. Broadly speaking, dynamic loads give rise to high stress levels near the cracks and the fracture takes place so rapidly that there is insufficient time for the yielding to develop. In addition, energy is released within a short time that leads to rapid crack propagation and promotes brittle fracture. Moreover, the mechanical properties of the material depend markedly on the time for which the applied loading is maintained in the solid. For these reasons, it is essential to include the kinetic energy term, K_e, in the Griffith's energy balance equation during dynamic loading. In order to derive an expression for the kinetic energy, velocity of the crack must be known. Several investigators attempted for a quantitative prediction of the speed and energy of a rapidly moving crack. The following key assumptions were made in deriving the kinetic energy and the velocity of a central crack in an infinite plate subjected to a uniform time-independent uniaxial stress perpendicular to the plane of the crack [1].

1. The stress and displacement fields for the dynamic problem are the same as those for the static problem, with the same crack length
2. The crack is travelling at a constant speed
3. The crack speed is small compared to the shear wave speed in the body

The kinetic energy term (K_e) and velocity (V) during dynamic loading can be defined as [1]:

$$K_e = \frac{1}{2} k^2 \rho V^2 a^2 \frac{\sigma^2}{E^2} \qquad (60)$$

$$V = 0.38 v_s \left(1 - \frac{a_0}{a}\right) \quad (61)$$

where ρ denotes the mass density, a_o is one-half of the crack length at $t = 0$, ν_s is the speed of the longitudinal waves in the material which is equal to the speed of sound and k is a constant.

The dynamic stress intensity factor can be given by the expression [1]:

$$K(t) = K(V)K(0) \quad (62)$$

where $K(0)$ can be approximated as the value of the stress intensity factor for a static crack of the same length as that for moving crack and $K(V)$ is a geometry independent function of the crack speed. The quantity $K(V)$ decreases monotonically with the crack speed and can be approximated by [1]:

$$K(V) = 1 - \frac{V}{C_R} \quad (63)$$

where C_R denotes the Rayleigh wave speed. From equation (63), the dynamic stress intensity factor becomes equal to zero when the crack speed V becomes equal to C_R.

Similar to the static case, a fracture criterion for dynamic crack propagation can also be established. When the strain energy release rate (G) becomes equal to a critical value, dynamic crack propagation occurs; this is equivalent to a critical stress intensity factor criterion. For small scale yielding, the concept of K-dominance for stationary cracks can be extended to dynamic cracks and the fracture criterion for a propagating crack can be given by [1]:

$$K(t) = K_{ID}(V, t) \quad (64)$$

where $K_{ID}(V, t)$ is assumed to be a material property that represents the resistance of the material to dynamic crack propagation. It can be determined experimentally and depends on the crack speed and environmental conditions. The dynamic stress intensity factor $K(t)$ is a function of the loading, crack length and geometrical configuration of the cracked body.

4.5 Life Estimation based on EPFM

LEFM principles are most frequently applied; however, elastic plastic fracture mechanics is used for life estimation when the conditions of LEFM are not met. EPFM can be applied under the following conditions [8]:

1. For very short cracks where the plastic zone size is no longer relatively small
2. In situations of high constraint e.g. cracks in thick sections.

Thus, EPFM is used to predict the life of the power generating and chemical processing components such as thick-walled vessels and pipes where most cracks occur in high pressure parts. EPFM are also used in large thick-sectioned welded structures of offshore industry [8].

4.6 Limitations of the EPFM

Similar to LEFM, EPFM also have limitations because a proper description of elastic plastic fracture behaviour is not possible by means of a straight forward single parameter concept. For elastic plastic fracture mechanics, two parameters namely crack tip opening displacement and J-integral have found a general acceptance, but non have received widespread recognition. Moreover, due to its complexity, EPFM is not as well developed as the LEFM concept [8].

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