Name: ______________________________ Student Number: ____________________

Tutorial Number: __________

MATHEMATICS 1NN3
Test #1

Solutions

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DAY CLASS
DURATION OF TEST: 1 hour
MCMASTER UNIVERSITY TERM TEST

Saturday, February 3, 2007

THIS TEST INCLUDES 7 PAGES AND 12 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING YOUR COPY OF THE TEST IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Instructions:

1. Only the Casio FX-991 calculator is allowed to be used on this test.
2. Put your name and student number at the top of each page.
3. In part A, PRINT the letter corresponding to the answer of your choice on **page 2**, in the box beside the corresponding question number below.
4. **A blank answer is an automatic zero for any question** in part A, even if the correct solution is circled on the question itself. Incorrect or multiple answers are also worth zero marks. No negative marks or part marks will be assigned.
5. In part B, provide complete solutions on this exam paper in the space provided below each question. Part marks are available.
6. Each question in part A is worth 1 mark, and in part B each question is worth 3 marks.
7. Rough work paper will be provided upon request. All rough work must be handed in with the test, but any solutions written on the rough paper will **NOT** be graded.
8. Good Luck!

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Part A

Question	Ans.	Question	Ans.
#1	f	#6	f
#2	c	#7	c
#3	b	#8	e
#4	f	#9	d
#5	e		

Total Part A: /9

Part B

Question	Grade
#10	/3
#11	/3
#12	/3

Total Part B: /9

Net Grade:

/18

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PART A

Enter the letter of the correct solution in the appropriate box on the first page.

No part marks will be given for this section

1. Which of the following is equivalent to:

$$\lim_{n\to\infty} \cos\left(\frac{n\pi}{2}\right)$$

a) $(-1)^n$ **b)** -1 **c)** 0 **d)** 1 **e)** ∞ **f)** DNE

2. Which of the following is equivalent to the expression:

$$\lim_{n\to\infty} \sin\left(n^2\pi\right)$$

a) $(-1)^n$ **b)** -1 **c)** 0 **d)** 1 **e)** ∞ **f)** DNE

3. Compute the value of the expression:

$$\sum_{n=1}^{\infty}\frac{2^{n-1}}{3^n} = \Sigma \frac{1}{3}\left(\frac{2}{3}\right)^{n-1} \Rightarrow \frac{\frac{1}{3}}{1-2/3}$$

a) 0 **b)** 1 **c)** $\frac{3}{2}$ **d)** 2 **e)** ∞ **f)** DNE

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4. The following expression converges to:

$$\sum_{n=1}^{\infty} n^n$$

a) 0 **b)** 1 **c)** 2 **d)** $n^n(\ln(n)+1)$ **e)** $\frac{n}{1-n}$ **f)** It does not converge

5. Evaluate the integral:

$$\int_0^1 \frac{1}{\sqrt{x}}dx = \int_0^1 x^{-1/2}dx = 2x^{1/2}\Big|_0^1 = 2$$

a) −1 **b)** 0 **c)** $\frac{1}{2}$ **d)** 1 **e)** 2 **f)** It is divergent

6. Evaluate the expression:

$$\int_{-\infty}^{\infty} x^3 dx$$

a) 0 **b)** $\frac{1}{2}$ **c)** $-\frac{1}{2x^2}$ **d)** $\frac{1}{b^2}$ **e)** ∞ **f)** It is divergent

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7. Evaluate the integral:

$$\int \frac{dx}{x^2-4}$$

$\int \frac{1}{(x-2)(x+2)}\,dx = \frac{A}{x+2} + \frac{B}{x-2}$

a) $\ln\left|\frac{x-2}{x+2}\right|+C$ b) $\ln\left|\frac{x+2}{x-2}\right|+C$ (c) $\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$ d) $\frac{1}{4}\ln\left|\frac{x+2}{x-2}\right|+C$

e) $\frac{1}{2x}\ln\left|x^2-4\right|+C$ f) $\ln\left|x^2-4\right|+C$

8. Which of the following is the appropriate form for the partial fraction decomposition of:

$$\frac{1}{x^2(x^2+x+1)}$$

a) $\frac{A}{x}+\frac{B}{x^2+x+1}$ b) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^2+x+1}$ c) $\frac{A}{x}+\frac{Bx+C}{x^2+x+1}$ d) $\frac{A}{x}+\frac{Bx}{x^2+x+1}$

(e) $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+x+1}$ f) $\frac{A}{x}+\frac{B}{x^2}+\frac{Cx}{x^2+x+1}$

9. Evaluate the integral:

$$\int_0^2 \sqrt{4-x^2}\,dx$$

a) $\sin(4)+2$ b) $\sin(2\vartheta)+2\vartheta+C$ c) $\frac{1}{2}\cos(2\vartheta)+2\vartheta+C$

(d) π e) $\frac{\pi}{4}$ f) $\pi-2$

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PART B

Provide complete solutions in the spaces below the questions.

10. Given the sequence:

$$a_n = \frac{1}{2 - a_{n-1}}, \qquad a_1 = 0$$

and given $a_n < 2$ for all n, show using induction that $\{a_n\}$ is monotonically increasing.

$a_1 = 0 \qquad a_2 = \frac{1}{2} > 0 = a_1$ } base case (1)

if $a_n > a_{n-1}$ } induction step (1)

$2 - a_n < 2 - a_{n-1}$

/3

$\Rightarrow \frac{1}{2-a_n} > \frac{1}{2-a_{n-1}}$ since both are the same sign.

(1) $\Rightarrow a_{n+1} > a_n$ QED by induction

11. Evaluate the integral:

$$\int x^3\sqrt{4+x^2}\,dx$$

(1) let $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\, d\theta$

$\sqrt{4+x^2} = 2\sec\theta \qquad x^3 = 8\tan^3\theta$

(1) $\Rightarrow \int 32 \tan^3\theta \sec^3\theta\, d\theta$

$= \int 32(\tan^2\theta \sec^2\theta)(\underbrace{\sec\theta\tan\theta\, d\theta}_{d\sec\theta})$

/3

$= \int 32[\sec^4\theta - \sec^2\theta]\, d(\sec\theta)$

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$= \frac{32}{5}\sec^5\theta - \frac{32}{3}\sec^3\theta + C$

(1) $= \frac{1}{5}(\sqrt{4+x^2})^5 + \frac{4}{3}(\sqrt{4+x^2})^3 + C$

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12. Is the following series convergent or divergent? If it is convergent, find the sum. If it is divergent, explain why.

$$\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$$

(1) Look at S_m

$$S_m = 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{m-1}} - \frac{1}{\sqrt{m}} + \frac{1}{\sqrt{m}} - \frac{1}{\sqrt{m+1}}$$

(1)

$$\Rightarrow S_m = 1 - \frac{1}{\sqrt{1+m}}$$

(1)

$$\lim_{m\to\infty} S_m = S = \boxed{1}$$

/3

THE END