Practice Midterm Solutions for MAT 2379 (Spring/Summer 2018)
Introduction to biostatistics

Question	Answer	Question	Answer
1	B	4	E
2	A	5	A
3	C	6	A

Multiple Choice Questions

[**1**] 1. Here are results from a group of 20 students chosen at random for a test in biostatistics:

85, 75, 85, 90, 50, 65, 70, 57, 85, 120,
56, 40, 85, 60, 70, 34, 79, 52, 138, 45.

The first and third quartile are 53 and 85, respectively. Identify all the outliers.

A) 120 and 138 are the only outliers
B) 138 is the only outlier
C) there are no outliers
D) 34 , 120 and 138 are the only outliers
E) 34, 40, 120 and 138 are the only outliers

Answer: B
Solution: We will compute the fences:

$$\text{lower fence} = q_1 - 1.5\text{IQR} = 53 - 1.5(85 - 53) = 5$$

and

$$\text{upper fence} = q_3 + 1.5\text{IQR} = 85 + 1.5(85 - 53) = 133.$$

The only value outside of the fences is 138. So 138 is the only outlier.

[**1**] 2. According to the CDC, 22% of the adults in the United States smoke. Suppose we sample 10 americans. What is the probability that our sample will contain two or fewer smokers?

A) 0.617 B) 0.318 C) 0.533 D) 0.393 E) 0.528

Answer: A

Solution: Let X be the number of smokers in a sample of $n = 10$ americans. X has a binomial distribution with $n = 10$ and $p = 0.22$. We want

$$\begin{aligned} P(X \le 2) &= P(X=0) + P(X=1) + P(X=2) \\ &= \binom{10}{0} p^0(1-p)^{10} + \binom{10}{1} p^1(1-p)^9 + \binom{10}{2} p^2(1-p)^8 \\ &= (0.22)^0(0.78)^{10} + 10(0.22)^1(0.78)^9 + 45(0.22)^2(0.78)^8 \\ &= 0.6169 = 61.7\%. \end{aligned}$$

[**1**] 3. An Athlete suspected of having used steroids is given two tests that operate independently of each other if steroids have been used. Test A has a probability of 0.9 of being positive if steroids have been used. Test B has probability of 0.8 of being positive if steroids have been used. What is the probability that *neither* test is positive if steroids have been used.

A) 0.72 B) 0.38 C) 0.02 D) 0.06 E) 0.32

Answer: C

Solution: Let us assume that steroids have been used. Let A be the event that Test A is positive and let B the event that Test B is positive. Since A and B are independent, then $P(A \cap B) = P(A)P(B) = (0.9)(0.8) = 0.72$. The probability that *neither* test is positive if steroids have been used is:

$$\begin{aligned} P(A' \cap B') &= 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - (0.9 + 0.8 - 0.72) = 0.02. \end{aligned}$$

[1] 4. A simple urine test was developed for a particular disease. It was determined that it has a specificity of 93% and a sensitivity of 91%. Assuming the the prevalence of the disease is 2.5%, compute the positive predictive value of the test.

A) 0.75 B) 0.925 C) 0.015 D) 0.785 E) 0.25

ANSWER=E

Solution: Let $T+$ be the event that the test is positive and $U+$ be the event of truly having the disease. We have

$$P(T+|U+) = 0.91, \quad P(T-|U-) = 0.93, \quad \text{and } P(U+) = 0.025.$$

By the total probability rule, the probability that an individual from this population would test positive for this disease is

$$\begin{aligned} P(T+) &= P(T+|U+)P(U+) + P(T+|U-)P(U-) \\ &= (0.91)(0.025) + (1-0.93)(1-0.025) = 0.091. \end{aligned}$$

The positive predictive value of the test is

$$P(U+|T+) = \frac{P(U+\cap T+)}{P(T+)} = \frac{P(T+|U+)P(U+)}{P(T+)} = \frac{(0.91)(0.025)}{0.091} = 0.25 = 25\%.$$

[1] 5. Consider an experiment where we study the ability of snails to hold firmly onto a smooth substrate. We would like to know if the species of snail is independent of the ability. The snail is attached to a smooth rock and we expose it to water and record if it has the ability to resist the current. Suppose that for such an experiment that the probability that the snail belongs to a particular species A and it is resistant is 0.288. The probability that a snail resisted is 0.48 and suppose that species A represents 60% of the snails under study.

Let A="species A" and B="resistant". Compute $P(B|A)$ and determine if A and B are independent.

A) $P(B|A) = 0.48$; the two events are independent
B) $P(B|A) = 0.48$; the two events are not independent

C) $P(B|A) = 0.6$; the two events are independent
D) $P(B|A) = 0.6$; the two events are not independent
E) $P(B|A) = 0.288$; the two events are independent
F) $P(B|A) = 0.288$; the two events are not independent

Answer is A.
solution: We want $P(B|A) = P(A \cap B)/P(A) = 0.288/0.6 = 0.48$. Since $P(B|A) = 0.48 = P(B)$, then A and B are independent.

[**1**] 6. Police report that 97.12% of drivers stopped on suspicion of drunk driving are given a breath test, 12.96% a blood test, and 10.08% both tests. Consider the next driver stopped on suspicion of drunk driving. What is the probability that they are given a breath test but they are not given a blood test?

A) 0.8704 B) 0.0000 C) 0.1296 D) 0.0288 E) 0.0544

Answer is D.
solution: Let A be the event that they are given a breath test and let B be the event that they are given a blood test. We have

$$P(A) = 0.9712, \quad P(B) = 0.1296, \quad \text{and} \quad P(A \cap B) = 0.1008.$$

We want $P(A \cap B') = P(A) - P(A \cap B) = 0.9712 - 0.1008 = 0.8704.$

Short Answer Questions

1. The weights of a certain population of young adult females are approximately normally distributed with a mean of 132 pounds and a standard deviation of 15.

[1] a) Find the probability that a subject selected at random from this population will weigh more than 155 pounds.

[1] b) Find the probability that a subject selected at random from this population will weigh between 105 pounds and 145 pounds.

[1] c) Find a value c, such that $P(X > c) = 0.8$, where X is the weight of a randomly chosen individual from this population.

Solution: Let X be the weight of a randomly chosen female from this population, then $X \sim N(\mathrm{E}[X] = 132, \mathrm{Var}[X] = 15^2)$.

a) We want

$$\begin{aligned} P(X > 155) &= 1 - P(X \leq 155) = 1 - P\left(Z < \frac{155 - 132}{15}\right) \\ &= 1 - \Phi(1.53) = 1 - 0.9370 = 0.063. \end{aligned}$$

b) We want

$$\begin{aligned} P(105 < X < 145) &= P\left(\frac{105 - 132}{15} < Z < \frac{145 - 132}{15}\right) \\ &= \Phi(0.87) - \Phi(-1.80) = 0.8078 - 0.0359 = 0.7719. \end{aligned}$$

c) We want c such that $P(X > c) = 0.8$. Thus, we want c such that

$$0.2 = P(X \leq c) = P\left(Z < \frac{c - 132}{15}\right) = \Phi\left(\frac{c - 132}{15}\right).$$

From the table, we have $\Phi(-0.84) = 0.2005$ and $\Phi(-0.85) = 0.1977$. So we can take

$$-0.845 = \frac{c - 132}{15}.$$

Thus, $c = (-0.845)(15) + 132 = 119.325$.

2. Most female black bears have their first mating between the ages 3 and 5 years old. From a large sample of female black bears, we construct the following distribution for X, the size of the litter:

x	1	2	3	4	5
$f(x) = P(X = x)$	0.105	0.512	0.300	0.051	0.032

[2] (a) Compute $P(X > 3)$ and $P(\{X \leq 1\} \cup \{X \geq 4\})$

[1] (b) Compute the expected size of a litter.

[1] (c) Compute the standard deviation of the size of a litter.

[1] (d) Compute the following probability

$$P\left(-1 \leq \frac{X-\mu}{\sigma} \leq 1\right), \quad \text{where } \mu = E[X] \text{ and } \sigma = \sqrt{\text{Var}[X]}.$$

solution:

(a) $P(X > 3) = P(X = 4) + P(X = 5) = 0.083$ and

$$\begin{aligned} P(\{X \leq 1\} \cup \{X \geq 4\}) &= P(X \leq 1) + P(X \geq 4) \\ &= P(X = 1) + P(X = 4) + P(X = 5) = 0.188 \end{aligned}$$

(b) The expected size of a litter is

$$E[X] = 1(0.105) + 2(0.512) + \cdots + 5(0.032) = 2.393.$$

(c) The standard deviation of the size of a litter is

$$\begin{aligned} \sigma_X &= \sqrt{\text{Var}(X)} = \sqrt{1^2(0.105) + 2^2(0.512) + \cdots + 5^2(0.032) - 2.393^2} \\ &= 0.8617. \end{aligned}$$

(d)

$$\begin{aligned} P\left(-1 \leq \frac{X-\mu}{\sigma} \leq 1\right) &= P(\mu - \sigma \leq X \leq \mu + \sigma) \\ &= P(2.393 - 0.8617 \leq X \leq 2.393 + 0.8617) \\ &= P(1.5313 \leq X \leq 3.2547) \\ &= P(X = 2) + P(X = 3) \\ &= 0.512 + 0.3 = 0.812. \end{aligned}$$