For the multiple choice questions 1-28 you do not need to show your work.
Enter your answers on the scantron sheet using a pencil.

Problem 1. (*2 marks*) The exponential function $f(x) = b^x$ $(b > 0)$, whose graph is passing through the point $(2, 9/4)$ is

(A) $f(x) = (2/3)^x$
(B) $f(x) = (3/2)^x$
(C) $f(x) = 2^x$
(D) $f(x) = 3^x$
(E) none of the above

Problem 2. (*2 marks*) If $\ln(\ln x + 1) = 1$ then

(A) $x = 0$
(B) $x = 1$
(C) $x = e^{e-1}$
(D) $x = e^{e+1}$
(E) none of the above

Problem 3. (*2 marks*) If $a = \log_3 \sqrt{27}$ and $b = \ln(0.01) + \ln \dfrac{100}{e}$ then

(A) $a > 0 > b$
(B) $a > b > 0$
(C) $0 > a > b$
(D) $b > a > 0$
(E) none of the above

Problem 4. (*2 marks*) The values of x in the interval $[0, 2\pi]$ that satisfy the equation

$$\frac{\tan x}{\sqrt{3}} - 1 = 0$$

are

(A) $\pi/3$, $4\pi/3$
(B) $\pi/3$, $2\pi/3$
(C) $\pi/6$, $5\pi/6$, $7\pi/6$
(D) $\pi/6$, $7\pi/6$
(E) none of the above

Problem 5. (*2 marks*) Which of the following equalities (if any) is true ?

(A) $\left(\sin\frac{\pi}{7}\right)^2 + \left(\cos\frac{\pi}{7}\right)^2 = \frac{1}{7}$

(B) $2\left(\cos\frac{\pi}{7}\right)^2 = 2 + \cos\frac{2\pi}{7}$

(C) $\tan\frac{\pi}{7} = \frac{\sqrt{5}}{2}$

(D) $\left(\sin\frac{\pi}{7} + \cos\frac{\pi}{7}\right)^2 = 1 + \sin\frac{2\pi}{7}$

(E) none of the above are true

Problem 6. (*2 marks*) The domain of the function $f(x) = e^{\arccos x}$ is

(A) $[-1, 1]$
(B) $(-\infty, \infty)$
(C) $[0, \pi]$
(D) $[1, e^{\pi}]$
(E) none of the above

Problem 7. (*2 marks*) If $f(x) = \dfrac{x+1}{x-2}$ then $f^{-1}(x) =$

(A) $\dfrac{x-2}{x+1}$
(B) $\dfrac{x+2}{x-1}$
(C) $\dfrac{2x+1}{x-1}$
(D) $\dfrac{2x+2}{1-x}$
(E) none of the above

Problem 8. (*2 marks*) $\sin\Big(\tan^{-1}\Big(\dfrac{3}{4}\Big)\Big) =$

(A) 3/5
(B) $-3/5$
(C) 4/5
(D) $-4/5$
(E) none of the above

Problem 9. (*2 marks*)

$$\lim_{x\to 1} \frac{|3x-3|}{x-1}$$

(A) is equal to 3
(B) is equal to -3
(C) is equal to 0
(D) does not exist
(E) none of the above

Problem 10. (*2 marks*) $\lim_{x\to 2^+} \arctan(\ln(x-2))$

(A) is equal to $-\pi/2$
(B) is equal to $\pi/2$
(C) is equal to 0
(D) does not exist
(E) none of the above

Problem 11. (*2 marks*)

$$\lim_{x\to 0}\Big(\frac{1}{x} - \frac{1}{x^2+x}\Big)$$

(A) is equal to -1
(B) is equal to 1
(C) is equal to 0
(D) does not exist
(E) none of the above

Problem 12. (*2 marks*) $\lim_{x\to\infty} e^{-x}\sin(x^3+2x)$

(A) is equal to $-e$
(B) is equal to e
(C) is equal to 0
(D) does not exist
(E) none of the above

Problem 13. (*2 marks*)

$$\lim_{x\to-\infty} \frac{\sqrt{4x^{10}+x^4+1}}{x^5+1}$$

(A) is equal to -2
(B) is equal to 2
(C) is equal to 0
(D) does not exist
(E) none of the above

Problem 14. (*2 marks*) The graph of the function

$$f(x) = \frac{5x^2+1}{x^2-3x+2}$$

(A) has one horizontal asymptote and no vertical asymptotes
(B) has no horizontal asymptotes and two vertical asymptotes
(C) has two horizontal asymptotes and two vertical asymptotes
(D) has one horizontal asymptote and two vertical asymptotes
(E) none of the above

Problem 15. (*2 marks*) If $f(x)$ and $g(x)$ are continuous functions such that $f(0) = 6$ and

$$\lim_{x \to 0}(2f(x)g(x) - f(x)^2) = -36$$

then $g(0) =$

(A) -1
(B) 0
(C) 1
(D) 2
(E) none of the above

Problem 16. (*2 marks*) If $f(x)$ is continuous on $[3, 5]$, $f(3) = -1$, $f(5) = 2$, which of the following statements is true ?

(A) there is $c \in (3, 5)$ such that $f(c) = 0$
(B) there is $c \in (3, 5)$ such that $f'(c) = 0$
(C) there is $c \in (3, 5)$ such that $f(c) = 4$
(D) $f(x)$ is increasing on the interval $[3, 5]$
(E) none of the above are true

Problem 17. (*2 marks*) For what value of the constant a is the function

$$f(x) = \begin{cases} 2\cos x & \text{if } x \leq 0 \\ a + \ln(x+1) & \text{if } x > 0 \end{cases}$$

continuous on $(-\infty, \infty)$?

(A) for $a = 0$
(B) for $a = 1$
(C) for $a = 2$
(D) for any value of a
(E) none of the above

Problem 18. (*2 marks*) If $f(x) = g(x)e^x$, $g(0) = 2$, $g'(0) = 3$, $g''(0) = \pi$ then $f''(0) =$
(A) π
(B) $2 + \pi$
(C) $5 + \pi$
(D) $8 + \pi$
(E) none of the above

Problem 19. (*2 marks*) For which pair $f(x)$, a does the equality $f'(a) = \lim_{h \to 0} \dfrac{(4+h)^3 - 64}{h}$ hold ?
(A) $f(x) = x^3$, $a = 4$
(B) $f(x) = x^3$, $a = 0$
(C) $f(x) = (x+4)^3$, $a = 4$
(D) $f(x) = (x+4)^3 - 64$, $a = 4$
(E) none of the above

Problem 20. (*2 marks*) The slope of the tangent line to the graph of

$$f(x) = \frac{\cos x}{x^2 + 1}$$

at $x = \pi$ is

(A) 0

(B) $\dfrac{2\pi}{(\pi^2+1)^2}$

(C) $-\dfrac{2\pi}{(\pi^2+1)^2}$

(D) $-\dfrac{1}{\pi^2+1}$

(E) none of the above

Problem 21. (*2 marks*) An equation of the normal line to the graph of $f(x) = \dfrac{1}{3}x\sqrt{x}$ at $x = 16$ is

(A) $y = \dfrac{1}{2}x + \dfrac{40}{3}$

(B) $y = \dfrac{1}{2}x + \dfrac{16}{3}$

(C) $y = -\dfrac{1}{2}x + \dfrac{88}{3}$

(D) $y = -\dfrac{1}{2}x + \dfrac{80}{3}$

(E) none of the above

Problem 22. (*2 marks*) The values of x for which the tangent line to graph of

$$f(x) = x^3 - 15x + 1$$

is horizontal, are

(A) $x = 0$
(B) $x = 0$ and $x = 15$
(C) $x = -\sqrt{5}$ and $x = \sqrt{5}$
(D) $x = 5$
(E) none of the above

Problem 23. (*2 marks*)

$$\lim_{x \to 0} \frac{\sin(3x)}{\pi x}$$

(A) is equal to $1/(3\pi)$
(B) is equal to $3/\pi$
(C) is equal to 0
(D) does not exist
(E) none of the above

Problem 24. (*2 marks*) Which of the following functions (if any) is <u>not</u> differentiable at $x = 1$?

(A) $f(x) = \dfrac{x^2 + \sqrt{x} + 1}{x - 3}$
(B) $g(x) = |x - 2|$
(C) $h(x) = 2|x - 1|$
(D) $s(x) = 1$
(E) none of the above

Problem 25. (*2 marks*)

$$\Big(\tan(\sin(2^x))\Big)' =$$

(A) $\sec^2(\sin(2^x))$

(B) $\sec^2 x \cos x \; 2^x \ln 2$

(C) $\sec^2(\sin(2^x)) \cos(2^x) 2^x \ln 2$

(D) $\sec^2(\cos(2^x \ln 2))$

(E) none of the above

Problem 26. (*2 marks*) If

$$ye^x + x^2 + 2y^3 = 0$$

then $\dfrac{dy}{dx} =$

(A) $\dfrac{-x^2 - 2y^3}{e^x}$

(B) $\dfrac{-2x - 6y^2}{e^x}$

(C) $\dfrac{-2x - ye^x - 6y^2}{e^x}$

(D) $\dfrac{-ye^x - 2x}{e^x + 6y^2}$

(E) none of the above

Problem 27. (*2 marks*) An equation of the tangent line to the curve

$$\sin(x+y) = 2x - 2y$$

at $(-\pi, -\pi)$ is

(A) $y = x$

(B) $y = \frac{1}{3}x - \frac{2\pi}{3}$

(C) $y = -\pi$

(D) $y = \frac{1}{2}x - \frac{\pi}{2}$

(E) none of the above

Problem 28. (*2 marks*)

$$\left(\sin^{-1}(x + \frac{1}{4})\right)'\Big|_{x=0} =$$

(A) $\frac{4}{\sqrt{15}}$

(B) $-\frac{4}{\sqrt{15}}$

(C) $\frac{1}{\sqrt{15}}$

(D) $-\frac{1}{\sqrt{15}}$

(E) none of the above

SHOW YOUR WORK AND PROVIDE CLEAR EXPLANATION.
Unjustified answers may receive little or no credit.

Problem 29. (*8 marks*) Let $f(x)$ be the function given by

$$f(x) = \begin{cases} x + 1 - \dfrac{\pi}{4} & \text{if } x < -1 \\ \arctan x & \text{if } -1 \leq x \leq 1 \\ 2e^{2x} & \text{if } x > 1 \end{cases}$$

(a) State the value of the indicated limit, if it exists, in the space provided. If a limit does not exist, write DNE.

(*i*) $\lim\limits_{x \to -1^-} f(x)$

(*ii*) $\lim\limits_{x \to -1^+} f(x)$

(*iii*) $\lim\limits_{x \to -1} f(x)$

(*iv*) $\lim\limits_{x \to 1^-} f(x)$

(*v*) $\lim\limits_{x \to 1^+} f(x)$

(*vi*) $\lim\limits_{x \to 1} f(x)$

(b) Is f continuous at -1 ? (Answer *yes* or *no*). Answer:

(c) Is f continuous at 1 ? (Answer *yes* or *no*). Answer:

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Problem 30. (*6 marks*) A table of values for f, g, f', and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

(a) If $h(x) = f(g(x))$, find $h'(1)$.

(b) If $H(x) = g(f(x))$, find $H'(1)$.