Université d'Ottawa
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University of Ottawa
Faculty of Engineering
Department of
Mechanical Engineering

MCG4308: Mechanical Vibration Analysis
Midterm Exam

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Policy

The present exam is closed book and closed notes. Illegible work and loose sheets will not be graded. All electronic devices, with the exception of non programmable calculators, must be turned off during the test.

Problem 1

Consider the schematics of a translational oscillator in Fig. 1, where x_1 and x_2 are absolute displacements with respect to the wall.

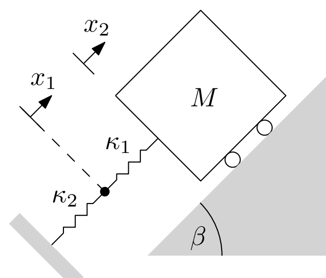

Fig. 1: Translational oscillator.

1. Use the energy method to obtain the natural frequency of the system. Stiffness κ_1 and κ_2 should appear explicitly.
2. Use the energy method to derive the mathematical model in terms of a scalar ordinary differential equation.

Problem 2

The solid rectangular bar in Fig. 2 pivots around point O. Point G is the centre of mass, and the total mass is m.

1. Derive the mathematical model in terms of a scalar ordinary differential equation for θ, and linearize around $\theta = 0$.
2. Determine a set of coefficients of the linearized ODE such that the solution around $\theta = 0$ is asymptotically stable.

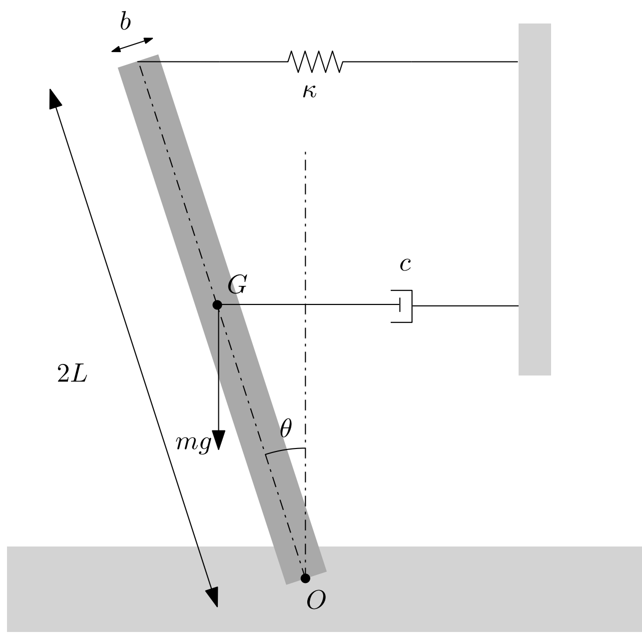

Fig. 2: Solid rectangular bar rotating around a fixed point.

Problem 3

The plot in Fig. 3 is the nondimensional amplitude $Xk/F_0 = X\omega_n^2/f_0$ of the steady-state response of a linear harmonic oscillator to an harmonic force $F_0 \cos \omega t$.

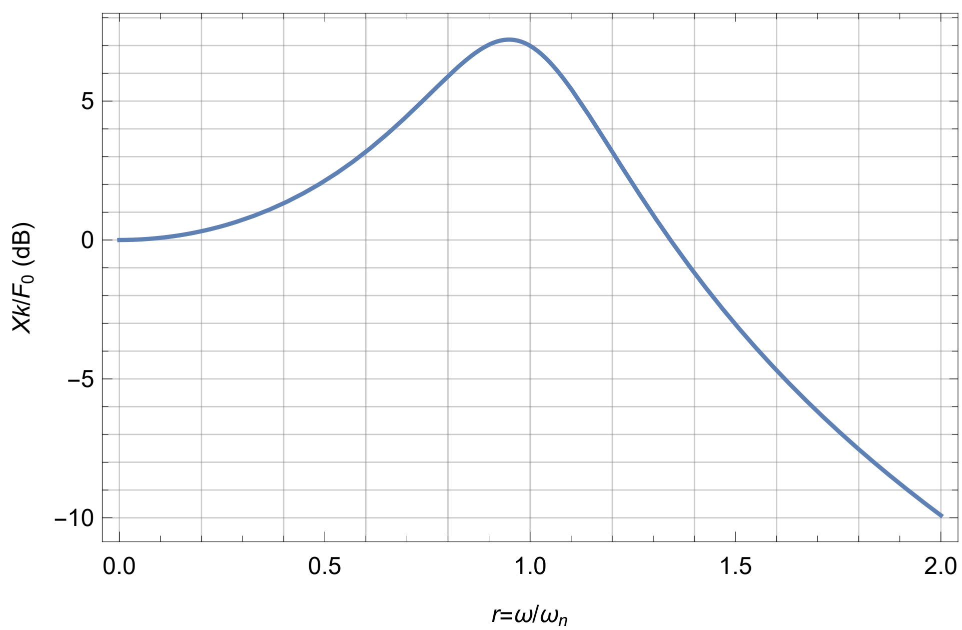

Fig. 3: Nondimensional amplitude plot of the steady state response of an harmonically excited oscillator.

1. Use the plot to estimate the damping ratio of the system.

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Formula sheet

- Definition of damping ratio and damped natural frequency:

$$\zeta = \frac{c}{2\omega_n M}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \tag{1}$$

  where c is the damping coefficient and M is the mass.

- Underdamped simple harmonic oscillator: relation between amplitude and phase and initial conditions

$$A = \frac{\sqrt{(\omega_d x_0)^2 + (v_0 + \zeta\omega_n x_0)^2}}{\omega_d}, \quad \phi = \tan^{-1}\frac{\omega_d x_0}{v_0 + \zeta\omega_n x_0} \tag{2}$$

- Polar moment of inertia of a disk of radius R and mass M, with respect to the centre G:

$$J_G = \frac{mR^2}{2} \tag{3}$$

- Polar moment of inertia of a rectangular bar of sides b and h and mass M, with respect to the centre of mass G:

$$J_G = \frac{M}{12}\left(b^2 + h^2\right) \tag{4}$$

- Parallel axis theorem: the polar moment of inertia with respect to an axis passing by point O at distance d from the centre of mass G is

$$J_0 = J_G + Md^2 \tag{5}$$

  where M is the total mass.

- Equivalent stiffness of flexible structures: will be provided in the text if necessary.

- Steady-state response of an harmonically excited underdamped system (cosinusoidal harmonic excitation)

$$x_p(t) = \frac{f_0}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}\cos\left(\omega t - \tan^{-1}\frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2}\right) \tag{6}$$

  The nondimensional amplitude is obtained from the expression above by collecting ω_n^4 in the denominator, and by defining the frequency ratio $r = \omega/\omega_n$.

- Harmonically excite underdamped system: peak frequency and peak magnitude

$$\omega_{\text{peak}} = \omega_n \sqrt{1 - 2\zeta^2}, \qquad \left(\frac{X\omega_n^2}{f_0}\right)_{\text{peak}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \tag{7}$$

- Base excitation: displacement transmissivity

$$\frac{X}{Y} = \sqrt{\frac{1 + (2\zeta r)^2}{\left(1 - r^2\right)^2 + (2\zeta r)^2}} \tag{8}$$

- Base excitation: force transmissivity

$$\frac{F_T}{kY} = r^2 \frac{X}{Y} \tag{9}$$