Université d'Ottawa
Faculté de genie

École d'ingénierie et de technologie de l'information

University of Ottawa
Faculty of Engineering

School of Information Technology and Engineering

uOttawa

L'Université canadienne
Canada's university

MCG 3307
Control Systems

MIDTERM EXAMINATION

Length of Examination: 80 minutes
Professor: Ali Karime

March 1, 2017

Family Name: ____

Other Names: ____

Student Number: ___________

Signature ___________________________

Closed-book exam. Calculators are allowed.

If you do not understand a question, clearly state an assumption and proceed.

At the end of the exam, when time is up:

- Stop working and turn your exam upside down.
- Remain silent.
- Do not move or speak until <u>all</u> exams have been picked up, and a TA or the Professor gives the go-ahead to leave.

QUESTION 1 (15 points)

Part a

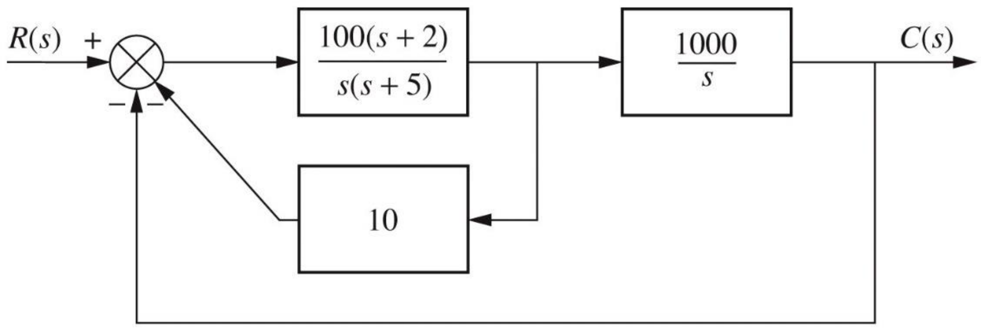
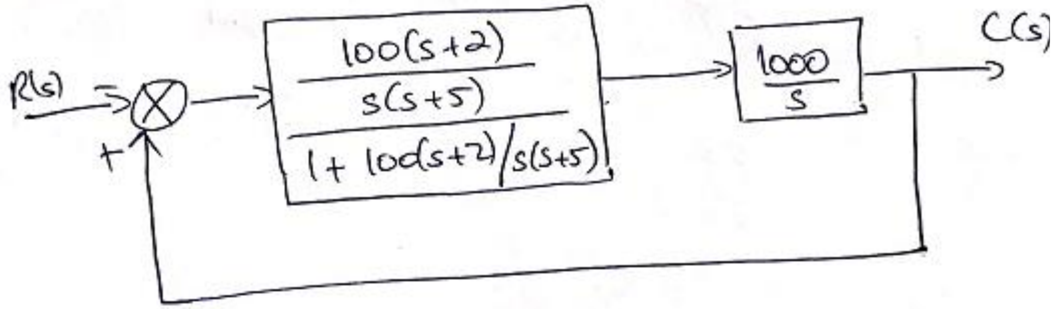
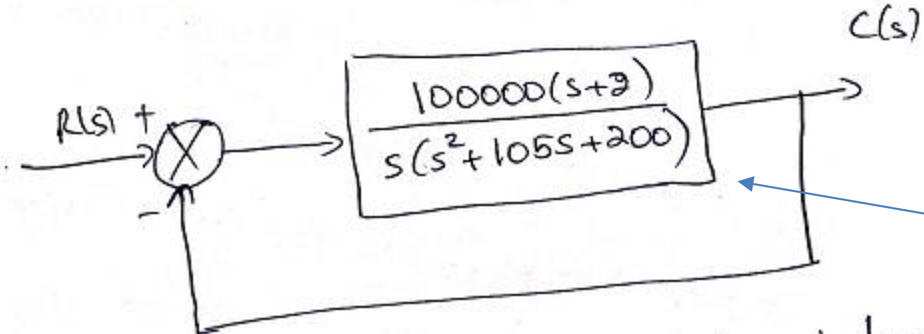


Figure 1

Determine the type of the system in Figure 1. **[7]**





4 points for finding the inner loop transfer function

we can clearly see that the system is type 1

3 points for specifying the type

Part b

Consider the system shown in Figure 2.

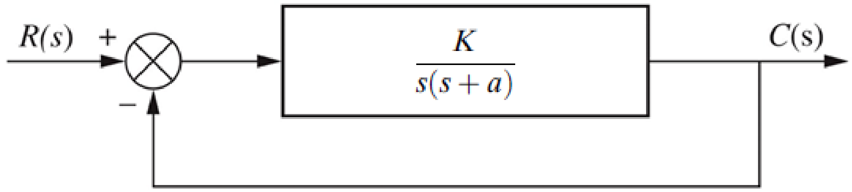


Figure 2

Find the value of K and a to yield a 1% error in the steady state and a 10% overshoot. [**8**]

First let's find the closed-loop T.F

$$T.F = \frac{\frac{K}{s(s+a)}}{1+\frac{K}{s(s+a)}} = \frac{K}{s^2+as+K}$$

2 points for finding the closed loop transfer function

$$e_{ss} = 1\% \Rightarrow \frac{1}{K_V} = 0.01 \Rightarrow K_V = 100 = \frac{K}{a}$$

$$\Rightarrow \boxed{K = 100a}$$

2 points for finding relationship between K and a

$a = 2\zeta\omega_n$ or $\omega_n^2 = K \Rightarrow \omega_n = \sqrt{K}$

$\% OS = 10\% \Rightarrow e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.1 \Rightarrow$

$\frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = -2.3$ and $\zeta = 0.591$

1 point for finding ζ

1 point for finding relationship W_n and K

$a = 2\zeta\omega_n = 2(0.591)\sqrt{K} = 1.182\sqrt{1000a}$

$\Rightarrow a^2 = 1.397(1000a) \Rightarrow \boxed{a = 139.7}$

and $\boxed{K = 13970}$

2 points for finding relationship the answer for a and K

QUESTION 2 (20 points)

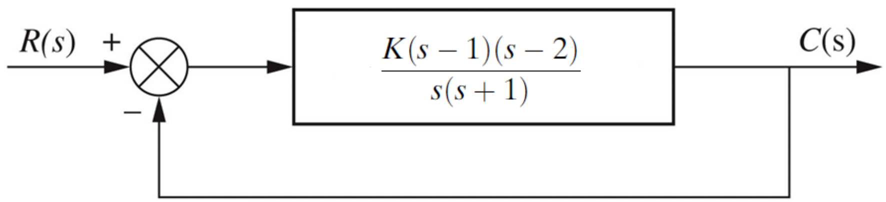


Figure 3

For the unity feedback system of figure 3, find the following:

(a) The breakaway and break-in points [**6**]

First Method

The breakaway and breakin points satisfy the equation

$$\sum_{1}^{m} \frac{1}{z_i+\sigma} = \sum_{1}^{n} \frac{1}{p_i+\sigma} \Rightarrow \frac{1}{\sigma-1}+\frac{1}{\sigma-2} = \frac{1}{\sigma}+\frac{1}{\sigma+1}$$

6 points for finding sigmas

$$\frac{1}{\sigma-1}+\frac{1}{\sigma-2}-\frac{1}{\sigma}-\frac{1}{\sigma+1}=0 \quad \Rightarrow 4\sigma^2-4\sigma-2=0$$

2 points for specifying the break-in and breakaway points

$$\Rightarrow \quad \sigma_{1,2} = \frac{4 \pm \sqrt{(4)^2-4(4)(-2)}}{8}$$

$$\Rightarrow \sigma_1 = -0.37 \text{ and } \sigma_2 = 1.37$$

breakaway point is at $\sigma_1 = -0.37$

break-in point is at $\sigma_2 = 1.37$

Second method

$$K\,G(s)\,H(s) = -1 \Rightarrow \frac{K\,(s-1)(s-2)}{s(s+1)} = -1 \Rightarrow$$

If student used this method, 2 points for finding K

$$K = \frac{-s(s+1)}{(s-1)(s-2)} = \frac{-s^2-s}{s^2-3s+2}$$

Let σ the breakaway or breakin point $\Rightarrow$

4 points for finding sigmas

$$K = \frac{-\sigma^2-\sigma}{\sigma^2-3\sigma+2}$$, to find the points

$\frac{dK}{d\sigma} = 0$, we find $\sigma_1 = -0.37$ and $\sigma_2 = 1.37$

2 points for specifying the break-in and breakaway points

For this part, combine for marks of part b and c

(b) The $j\omega$-axis crossing **[5]**

$$CLTF = \frac{\frac{K(s-1)(s-2)}{s(s+1)}}{1+\frac{K(s-1)(s-2)}{s(s+1)}} = \frac{K(s-1)(s-2)}{s^2(K+1)+s(1-3K)+2K}$$

1 points for finding CLTF

Routh table

s^2	$K+1$	$2K$
s	$1-3K$	0
s^0	$2K$	0

3 points for Routh table

for $K>0$, the only line that can generate a line of 0 is s

$1-3K=0 \Rightarrow K=\frac{1}{3}$

2 points for finding K and 1 point for properly writing the stability interval shown in part c

the auxiliary polynomial is $P(s) = (K+1)s^2 + 2K = 0$

for $K=\frac{1}{3}$

$P(s) = \left(\frac{1}{3}+1\right)s^2 + 2\left(\frac{1}{3}\right) = 0 \Rightarrow \frac{4}{3}s^2 + \frac{2}{3} = 0$

$\Rightarrow s = \pm j\sqrt{\frac{1}{2}} \Rightarrow s = \pm j0.71$

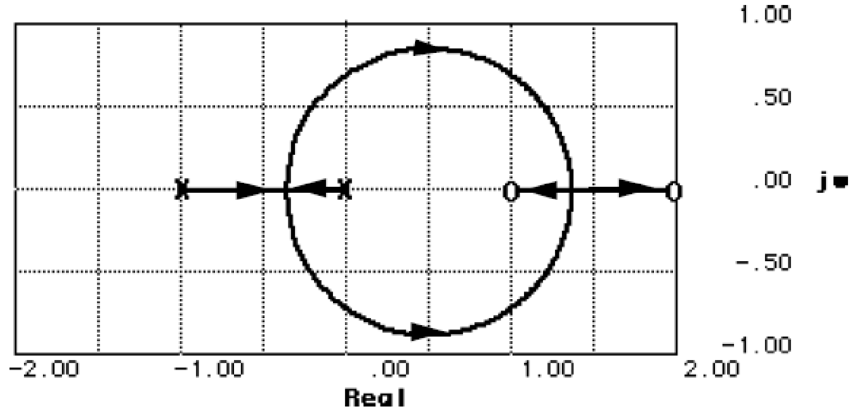
3 points for finding the crossing points s

(c) The gain K at $j\omega$-axis crossing. What is the interval of K for which the system is stable? [**5**]

We have already found that $K = \frac{1}{3}$ at the $j\omega$ axis crossing. the system is stable

for $0 < K \leq \frac{1}{3}$

(d) Sketch the root locus for the system of Figure 3. You must show all the points of interest on the sketch. [**4**]



1.5 points for properly showing the jw crossing and breakaway and break-in points on the root locus and 2.5 points for the root locus sketch. If the students showed a rectangular shape instead of a circular shape for the root locus it is ok as long as the root locus is symmetrical around the real axis

QUESTION 3 (15 points)

Part a

Consider the Routh table shown below. Notice that the s^5 row was originally all zeros. Tell how many roots are in the right half-plane, left half-plane and on the $j\omega$-axis. Briefly explain your answer. [**5**]

s^7	1	2	-1	-2
s^6	1	2	-1	-2
s^5	$\cancel{0}$ 3	$\cancel{0}$ 4	$\cancel{0}$ -1	0
s^4	1	-1	-3	0
s^3	7	8	0	0
s^2	-15	-21	0	0
s^1	-9	0	0	0
s^0	-21	0	0	0

Between s^6 and s^0, there is 1 sign change, therefore there is one RHP , 1 LHP (due to symmetry rule), and 4 poles on the *jw*-axis. On the other hand, between s^7 and s^6 there is no sign change, therefore there exists one LHP.

In conclusion: **1 RHP, 2 LHP, and 4 *jw* poles**

The student must explain the reasoning before giving his/her answer, if the answer is given without proper explanation then the maximum point given is 2

Part b

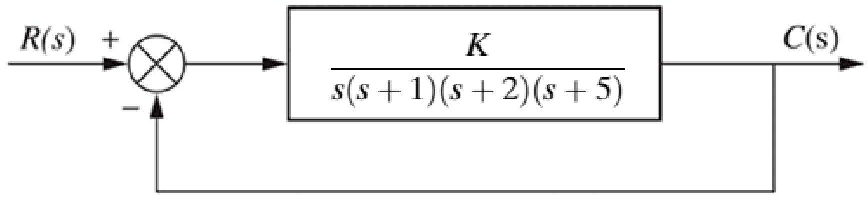


Figure 4

Find the value of K and the frequency that will make the system in Figure 4 marginally stable. [**10**]

The CLTF is

$$T(s) = \frac{K}{s^4 + 8s^3 + 17s^2 + 10s + K}$$

1 points for finding the CLTP

s^4	1	17	K
s^3	8	10	0
s^2	$\frac{126}{8}$	K	0
s^1	$-\frac{32}{63}K + 10$	0	0
s^0	K	0	0

3 points for finding the Routh table

the system is marginally stable if there exists two poles on the $j\omega$ axis.

the only line that can produce a line of zeros is s^1, therefore $-\frac{32}{63}K + 10 = 0$

$\Rightarrow 32K = 630 \Rightarrow K = 19.69$

1 point for stating when the system is stable

1 point identifying the line that can produce a line of zeros

1 point for find the gain K

the auxilary polynomial

$P(s) = \frac{126}{8}s^2 + K = 0 \Rightarrow \frac{126}{8}s^2 + 19.69 = 0$

$\Rightarrow s^2 = -1.25 \Rightarrow s = \pm j1.118$

$\Rightarrow$ the system is marginally stable for $K = 19.69$ and $w_n = 1.118$

2 points for finding s

1 point for finding the frequency