

Math 267 Midterm I

Feb 13, 2009

Duration: 50 minutes

Name: _____ Student Number: _____

This exam should have 9 pages, including this cover sheet. No textbooks, calculators, or other aids are allowed. One page of notes (two-sided) is allowed. There are 4 questions in this exam, each worth 10 points.

Problem 1 (10 points)

Let $f(t)$ be periodic with period π , and

$$f(t) = \begin{cases} 0 & \text{if } -\frac{\pi}{2} \leq t < 0 \\ t & \text{if } 0 \leq t < \frac{\pi}{2}. \end{cases}$$

- (a) Sketch the graph of $f(t)$ (show three periods).
- (b) Find the complex Fourier series of $f(t)$.

Problem 2 (10 points)

Let $g(t) = \sin(3t) - \cos(8t)$.

- (a) What is the fundamental period of $g(t)$?
- (b) Find $\hat{g}(k)$ (that is, the complex Fourier coefficients of $g(t)$).

Problem 3 (10 points)

Consider an elastic string of length $L = 5$ the ends of which are fixed. Let $u(x, t)$ denote the vertical displacement. In this case, along with the appropriate boundary conditions, $u(x, t)$ satisfies

$$u_{tt} = 4u_{xx}, \quad 0 < x < 5, \quad t > 0$$

and initial conditions

$$u(x, 0) = \sin\left(\frac{4\pi}{5}x\right), \quad 0 \leq x \leq 5,$$

and

$$u_t(x, 0) = \sin\left(\frac{\pi}{5}x\right) - \sin\left(\frac{7\pi}{5}x\right), \quad 0 \leq x \leq 5.$$

(a) Specify the boundary conditions that $u(x, t)$ satisfies.

(b) Find $u(x, t)$.

Problem 4 (10 points)

Let $f(t)$ be periodic with period 2π , and suppose the complex Fourier series coefficients of $f(t)$ are

$$\widehat{f}(k) = 2^{-|k|}, \quad \text{for all integers } k.$$

- (a) Is $f(t)$ real-valued? Odd or even? Justify your answers.
- (b) Find the real (trigonometric) Fourier series of $f(t)$?
- (c) Calculate $\frac{1}{2\pi} \int_{-\pi}^{\pi} |f(t)|^2 dt$.

