

**RYERSON UNIVERSITY**  
**DEPARTMENT OF MATHEMATICS**

**CMTH110 — DISCRETE MATHEMATICS I**  
**MIDTERM TEST**

March 13, 2019

**INSTRUCTIONS**

1. Duration: 1.5 hours
2. Exam is closed book. No calculators are allowed.
3. You are allowed **one** 8.5" × 11" formula sheet (two-sided). The information on the sheet must be hand-written, with your own hand-writing.
4. Marks (out of) are shown in brackets.
  - (a) All the answers on the test must be accompanied by a full explanation. Marks will not be given solely on a short answer. Show all your work.
  - (b) Write your solutions in the space provided. If you need more space, use the back of the page. Indicate this fact on the original page, making sure that your solution cannot be confused with any rough work which may be there.
5. Do not separate the sheets.
6. Make sure that your test paper is complete; there are 9 questions on 6 pages.
7. Exam is printed single-sided on white paper.
8. Have your student card available on your desk.

Last Name (Print): \_\_\_\_\_ First Name (Print): \_\_\_\_\_

Student I.D. \_\_\_\_\_

Signature \_\_\_\_\_

Grade \_\_\_\_\_ /40

[3 marks] (1) Construct the truth table for  $\sim p \rightarrow (\sim q \vee r)$ . Is this statement a tautology?

$p$	$q$	$r$	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

[4 marks] (2) Show that

$$p \rightarrow (q \wedge r) \equiv (\sim q \vee \sim r) \rightarrow \sim p$$

For your proof use logical equivalences. Do not use the truth table.

[5 marks] (3) Use the truth-table to determine whether the following argument is valid:

$$\begin{array}{l} (p \wedge q) \rightarrow \sim r \\ p \vee \sim q \\ \sim p \rightarrow q \\ \hline \therefore r \end{array}$$

$p$	$q$	$r$	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

[4 marks] **(4)** Prove that the following statement is true

$$\exists y \in \mathbb{R} \text{ such that } \forall x \in \mathbb{Z}, x^2 - 3 > y.$$

[4 marks] **(5)** Prove that the following statement is false

$$\exists y \in \mathbb{R} \text{ such that } \forall x \in \mathbb{R}, x \leq y.$$

(Hint: Negate the statement first and then show that the negation is true.)

[3+3=6 marks] **(6)** Prove or disprove. Clearly state your answer: true/false.

(a)  $(2k - 4) \cdot (3k + 6)$  is divisible by 5 for every integer  $k$ .

(b) For all integers  $a$  and  $b$ , if  $a$  and  $b$  are odd, then  $a - 3b$  is even.

[4 marks] **(7)** Prove by contraposition that if  $x^2$  is even, then  $x$  is even.

[5 marks] **(8)** Convert 65 (in decimal notation) to binary notation, find two's complement of this number (relative to a fixed bit length 8), and convert the result back to decimal notation.

[5 marks] **(9)** Find the last digit of  $2^{10000}$ .  
(Hint: Use modular arithmetic.)