

# ENGR301- Revision 1

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# Critical Path Method (CPM)

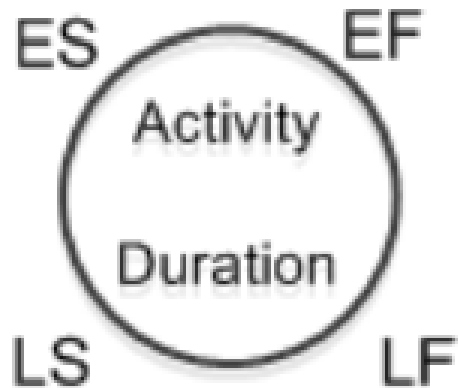
- Technique for scheduling project activities.
- Helps you find:
  - How long it will take to complete the project.
  - Which activities are **critical**
- Simple Example:

# Definitions:

- Predecessor = Preceding activity
- Successor = Succeeding activity
- ESD= Early Start Date
- EFD= Early Finish Date
- LSD= Late Start Date, the latest date the activity can start without delaying the project.
- LFD = Late Finish Date, the latest date the activity can finish without delaying the project.
- Total Float (TF): How long can the activity be delayed without delaying the whole project.
  - $TF = LFD - EFD$  or  $TF = LSD - ESD$

# Representations

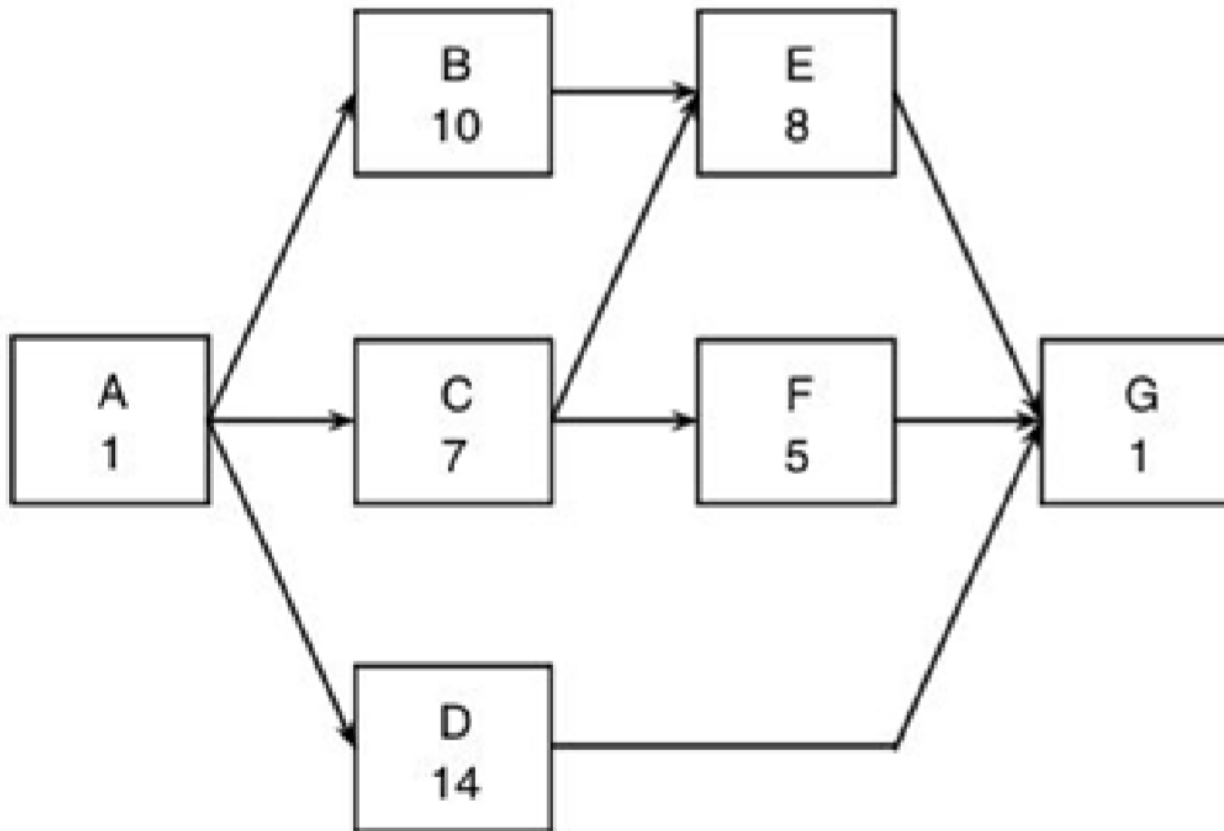
Activity:



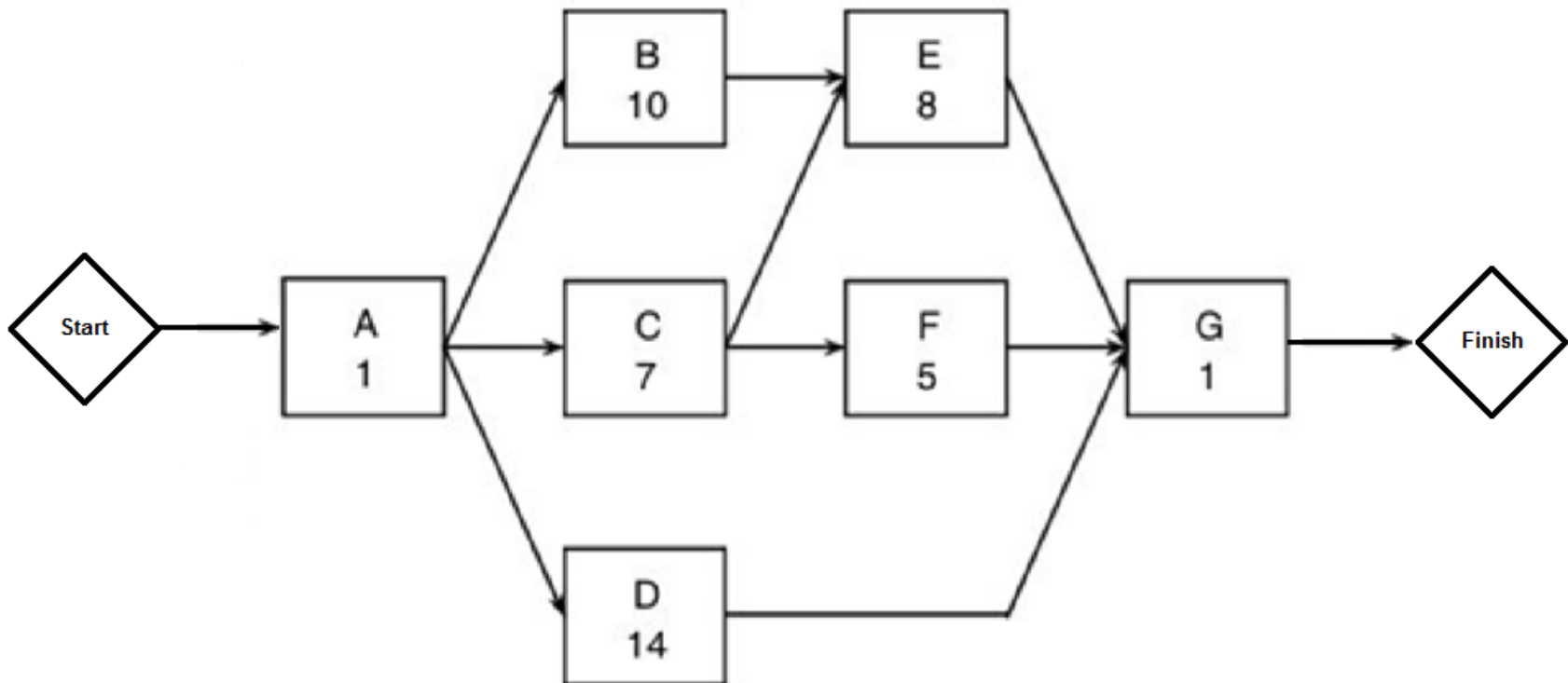
Relationship:



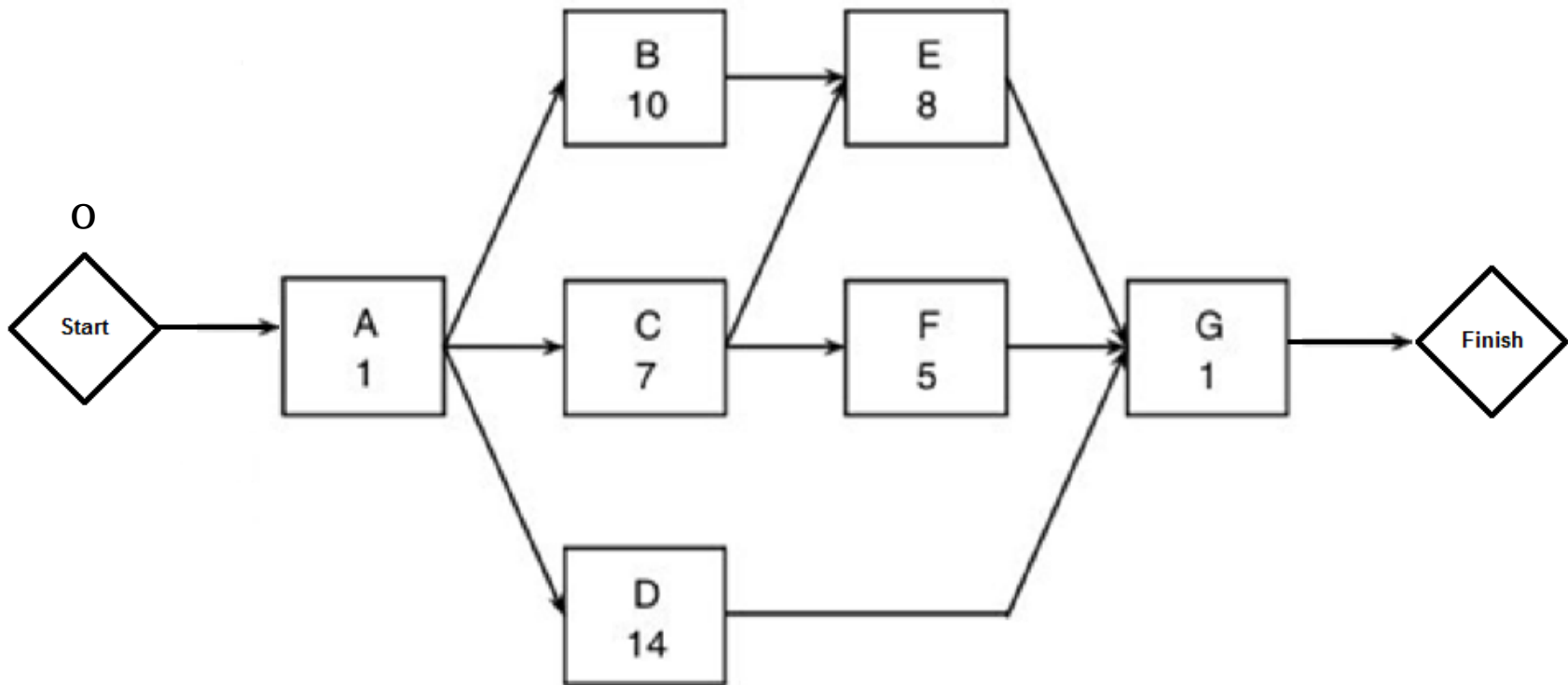
# CPM Example: Forward pass



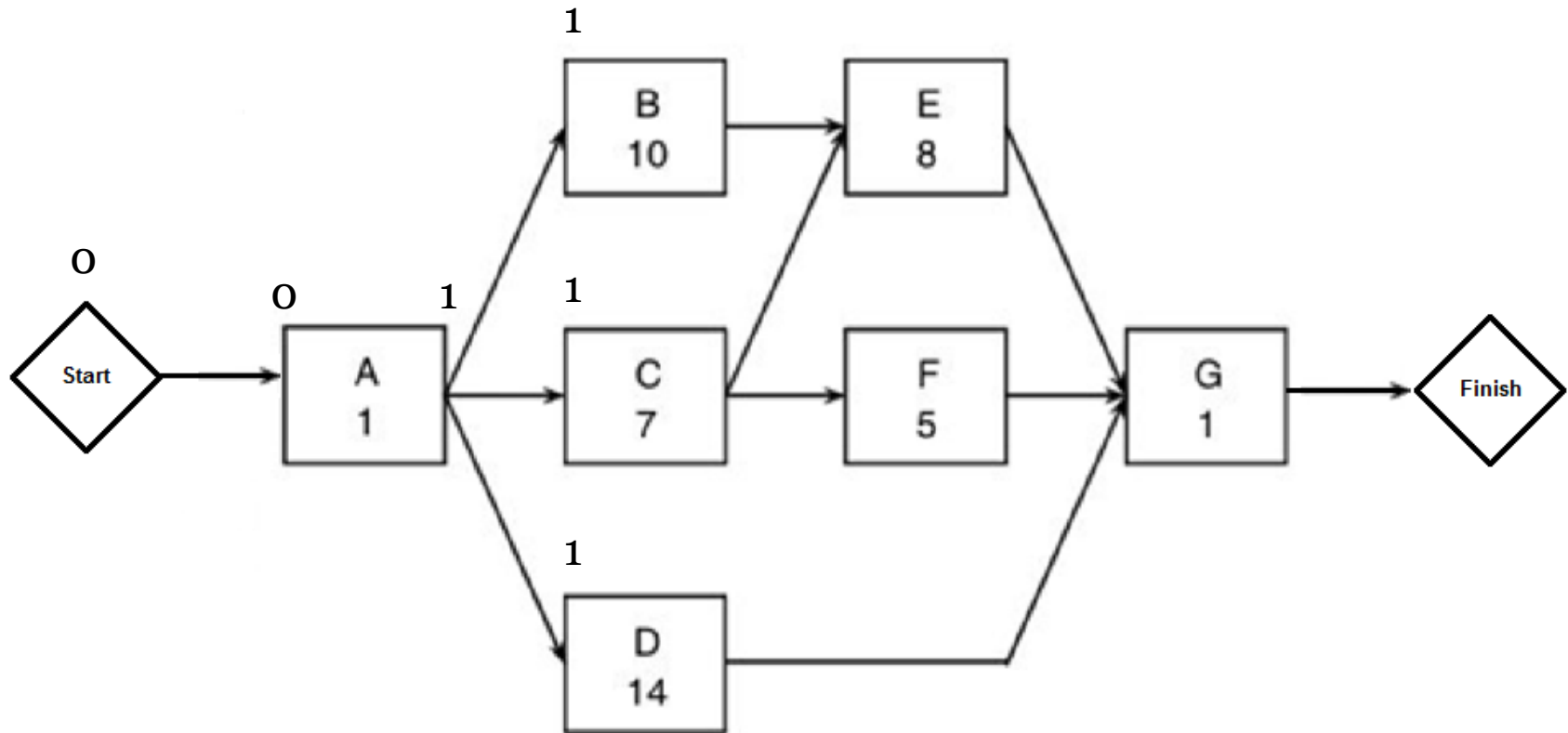
# CPM Example: Forward pass



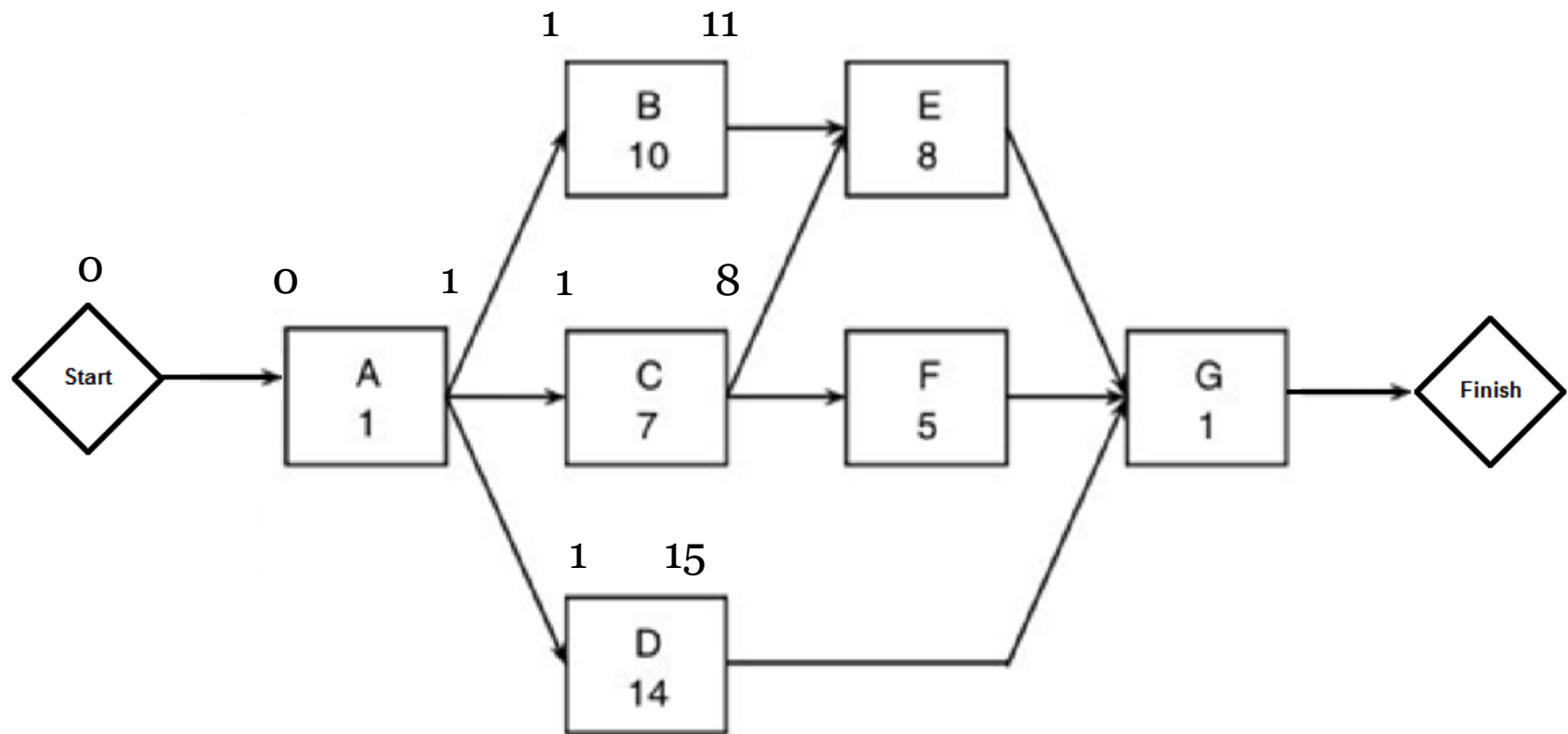
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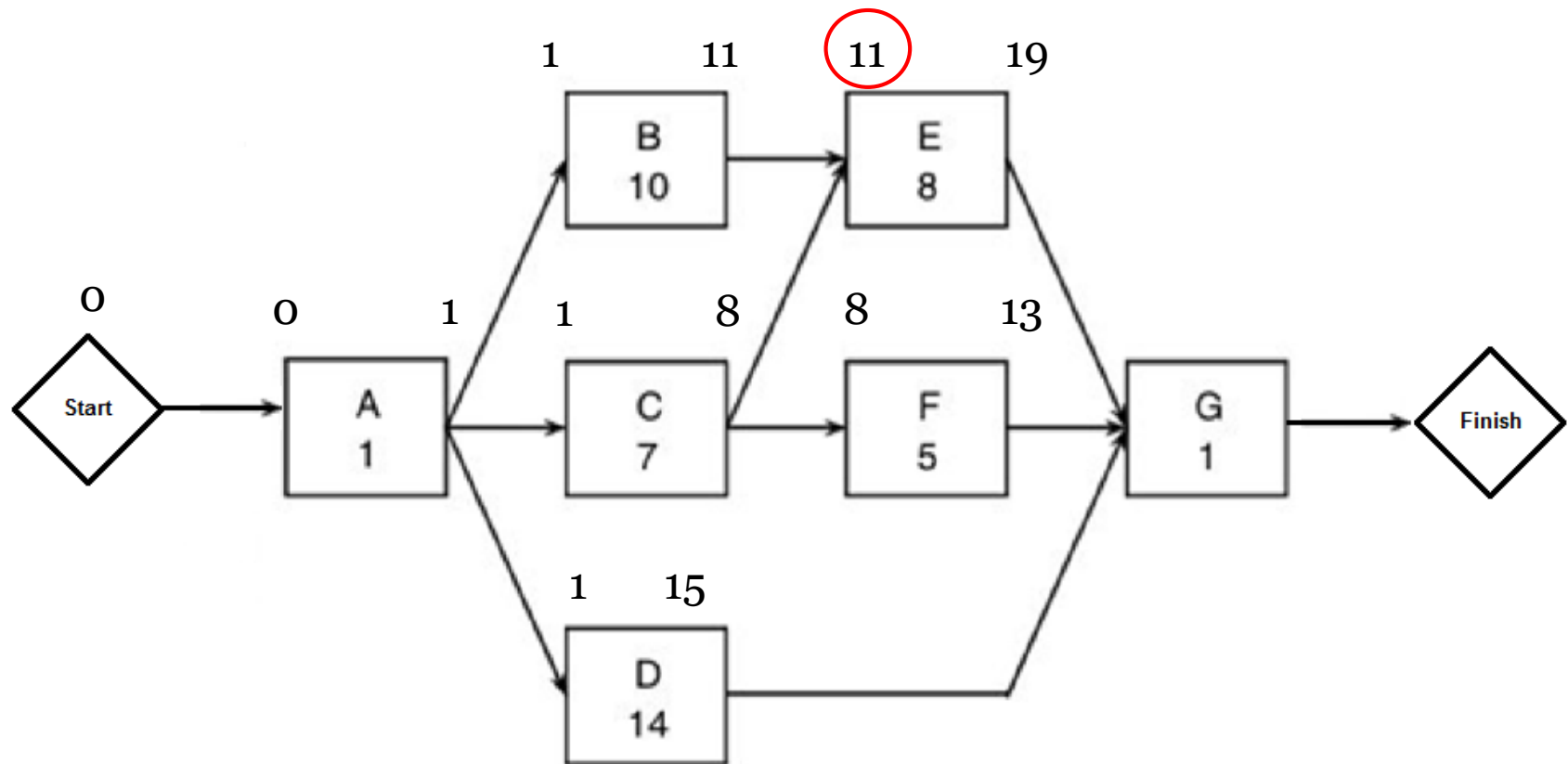
# CPM Example: Forward pass



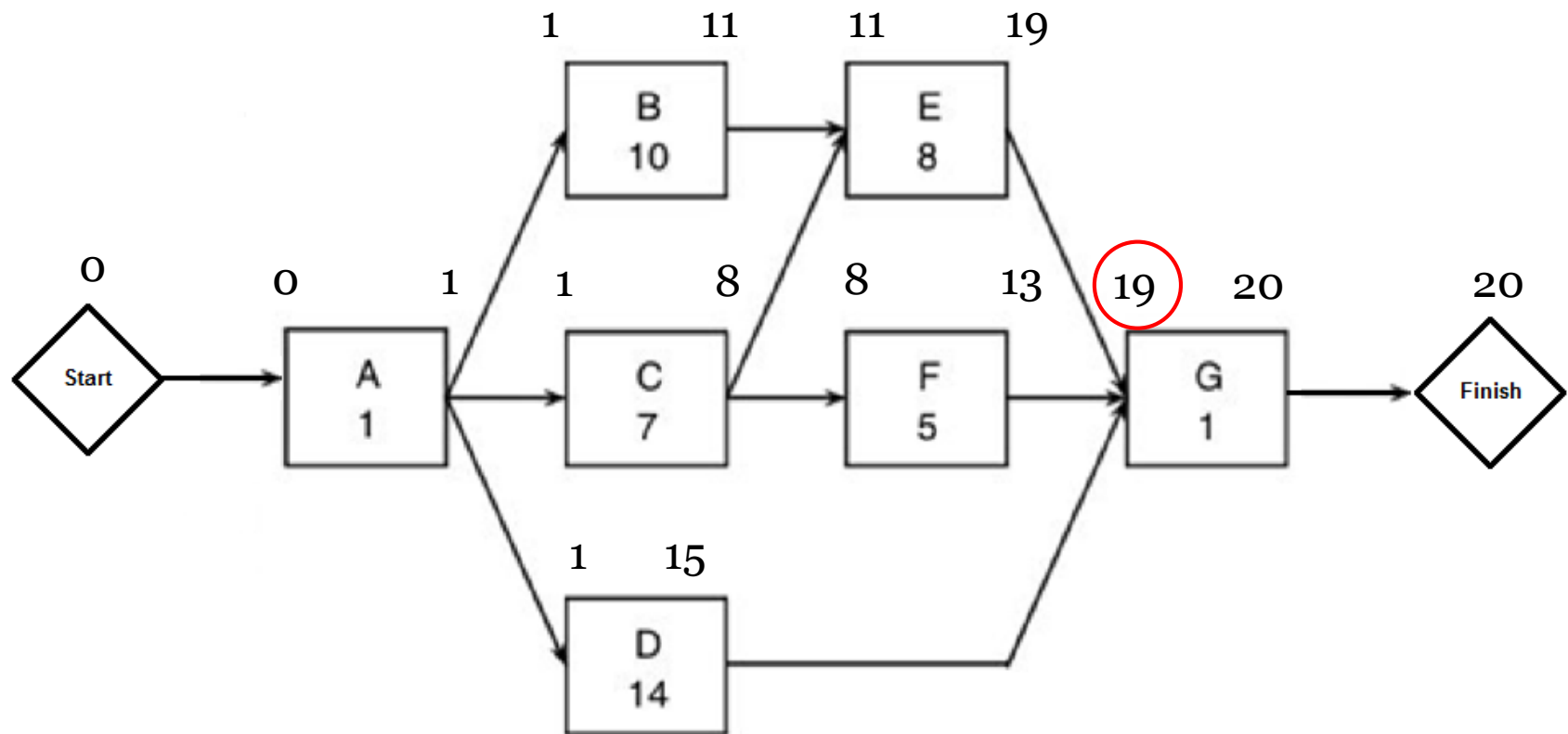
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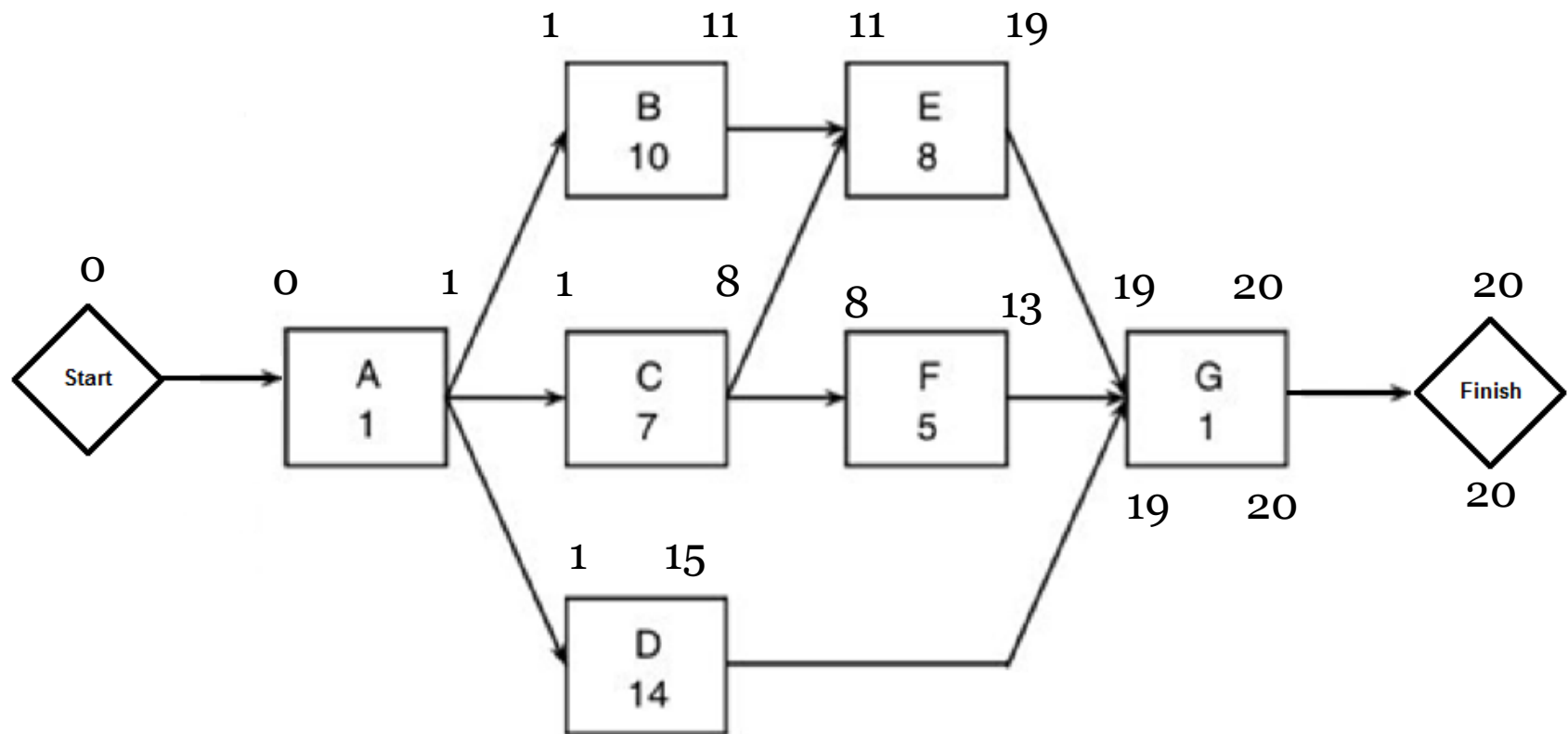
# CPM Example: Forward pass



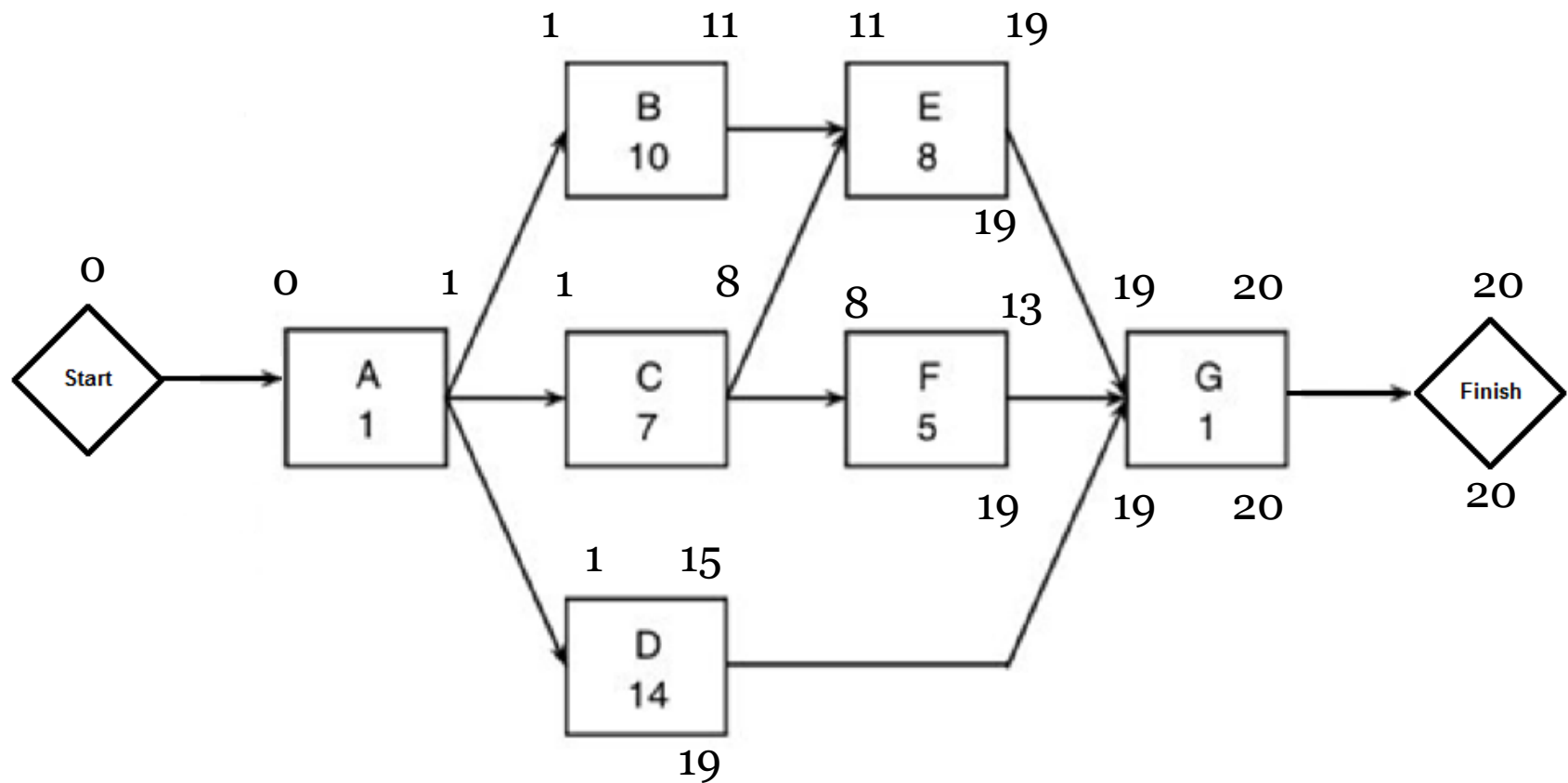
# CPM Example: Forward pass



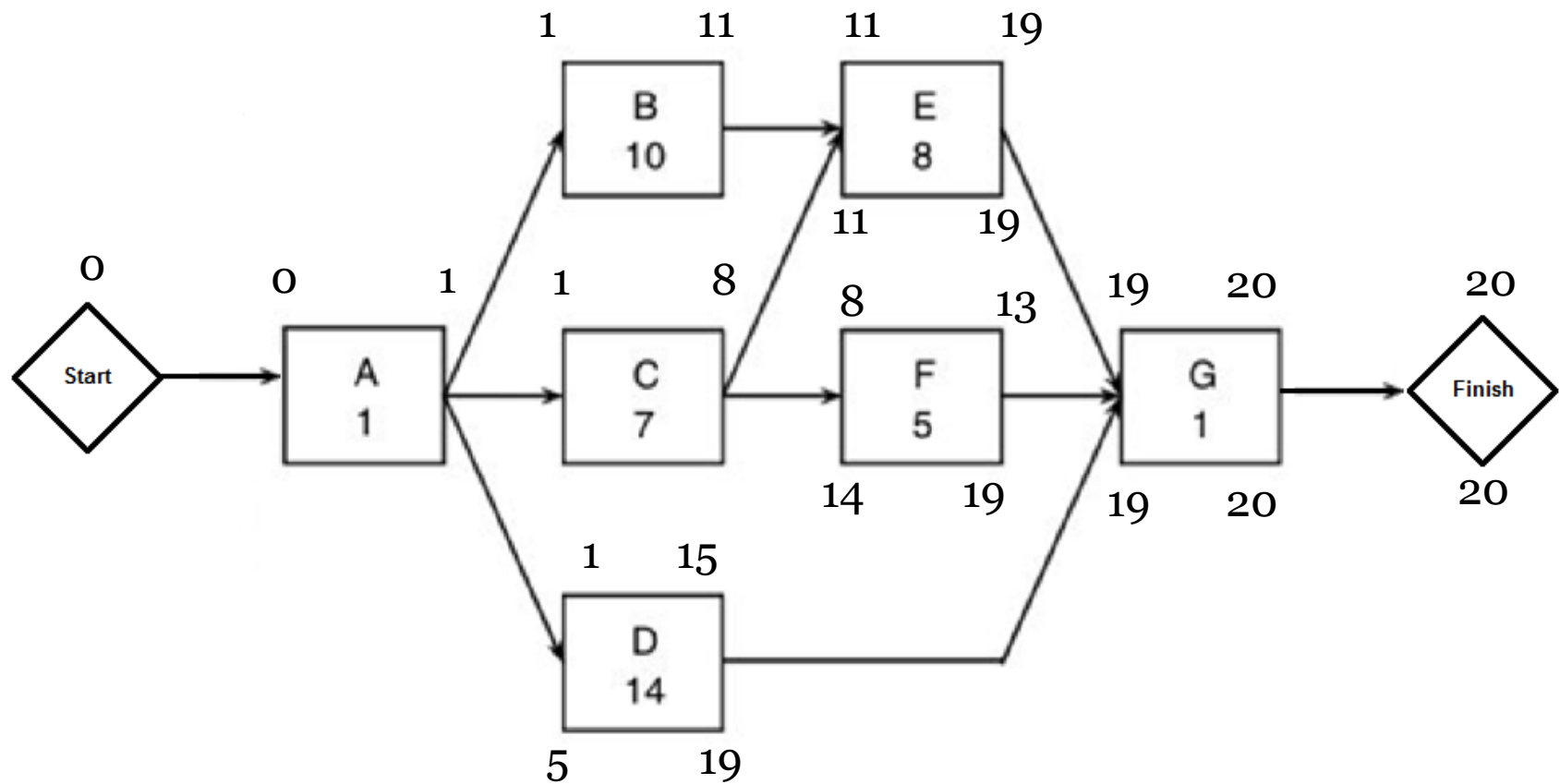
# CPM Example: Backward pass



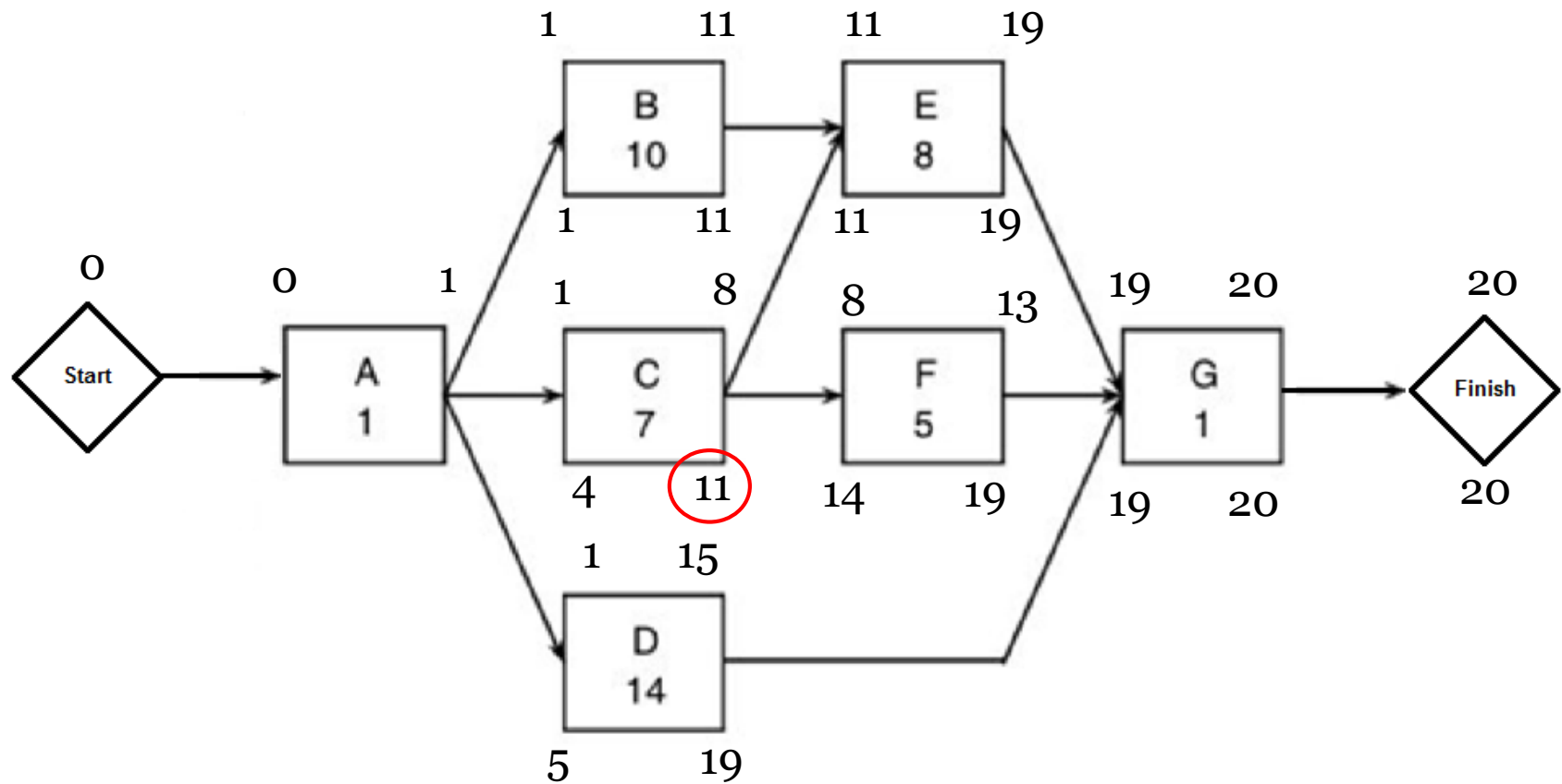
# CPM Example: Backward pass



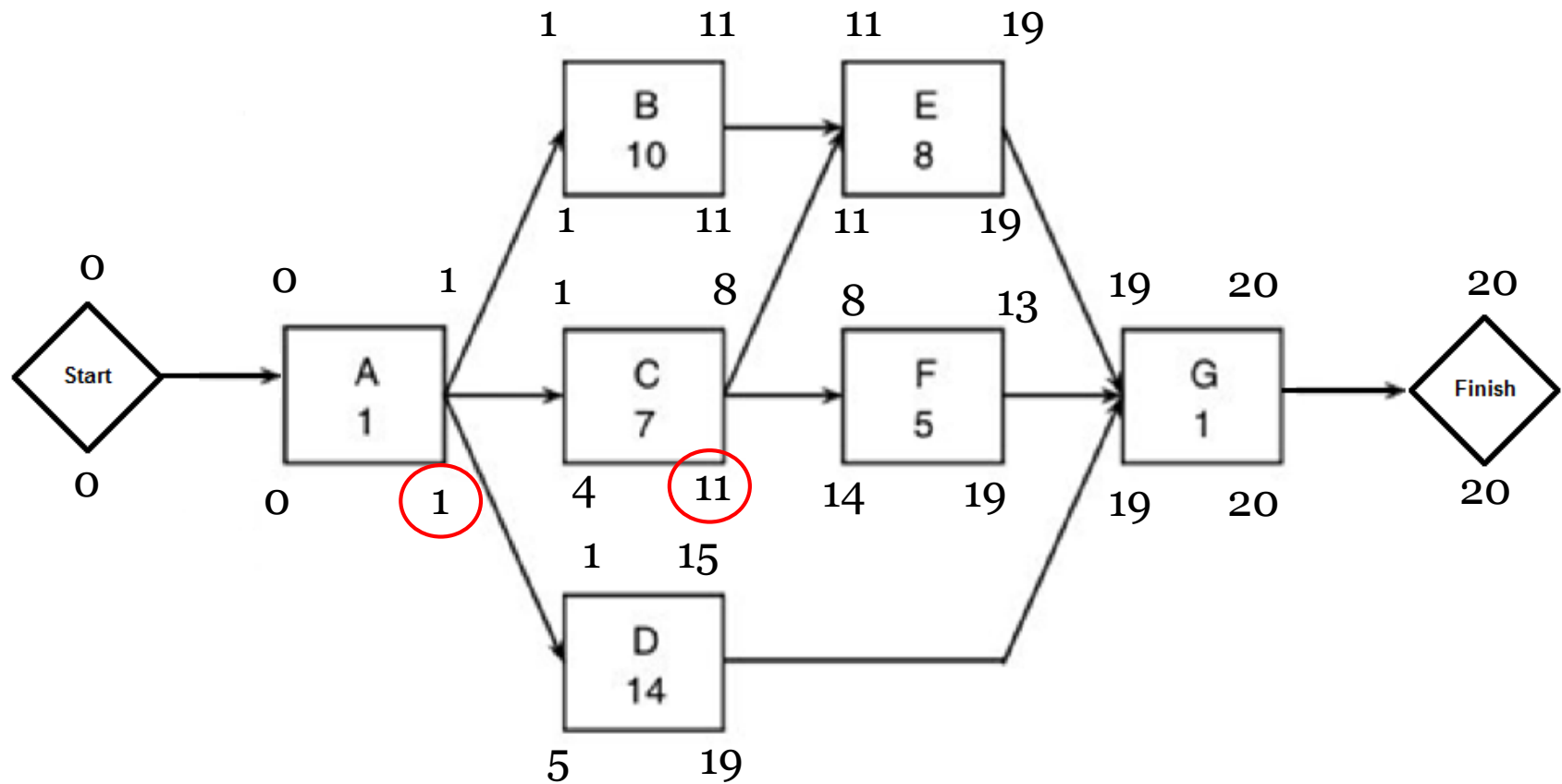
# CPM Example: Backward pass



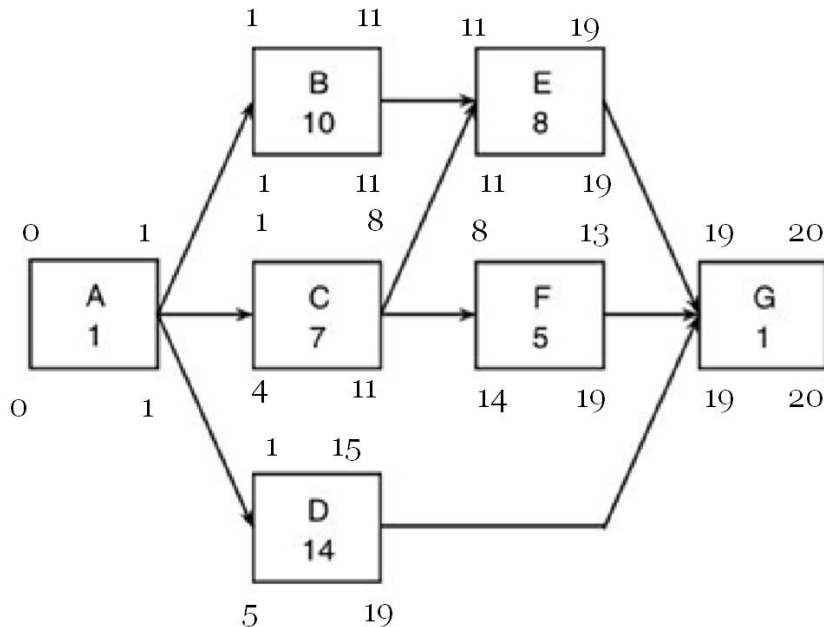
# CPM Example: Backward pass



# CPM Example: Backward pass



# Float

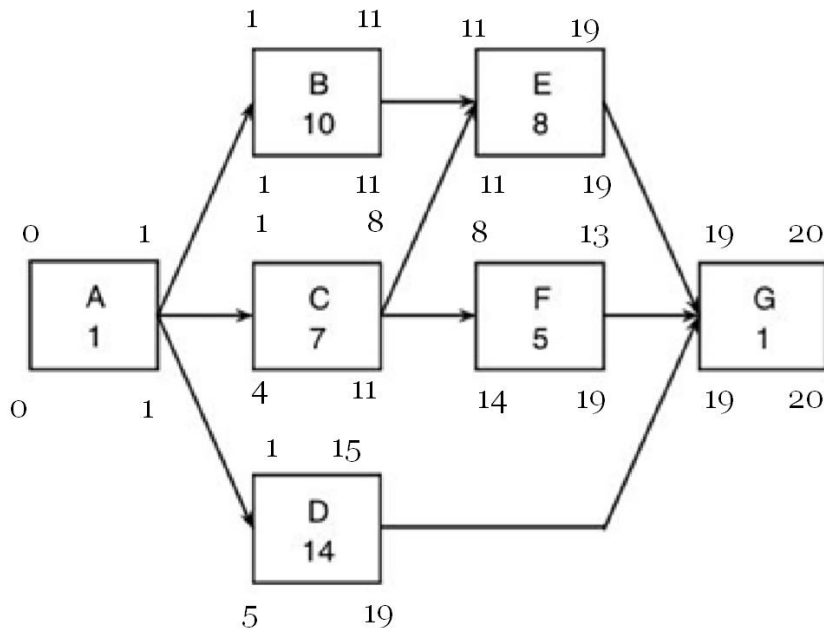


Remember, the float for any given activity is either the LSD-ESD or LFD-EFD. (Same answer)

Activity	Equation	Float
A	1-1	0
B	11-11	0
C	11-8	3
D	19-15	4
E	19-19	0
F	19-13	6
G	20-20	0

The float values correspond to the number of days an activity can be delayed without delaying the overall completion date of the project.

# Critical Path



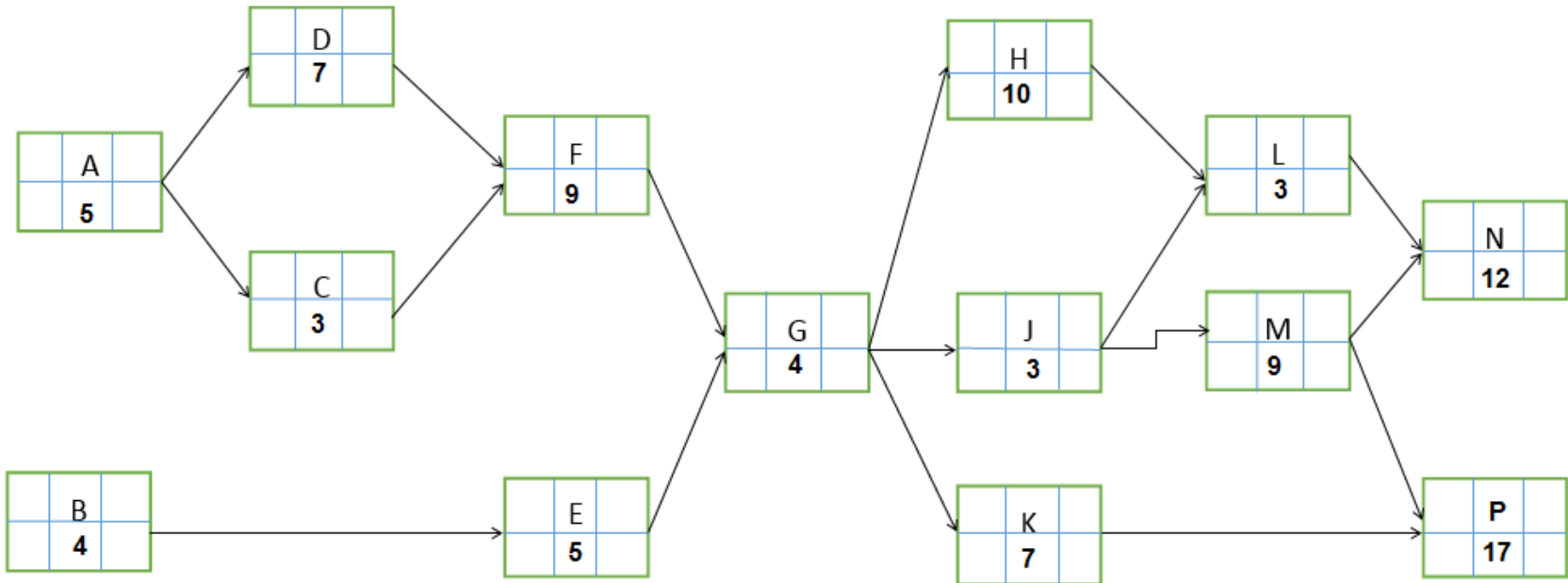
Activity	Equation	Float
A	1-1	0
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C	11-8	3
D	19-15	4
E	19-19	0
F	19-13	6
G	20-20	0

Activities on the critical path have float = 0

CP: the longest path to complete the project

A-B-E-G is the critical path.

# Midterm Example



# Youtube Video Review of CPM

- <http://www.youtube.com/watch?v=SF53ZZsP4ik>

# Cash flow analysis

**Table 2.3** Summary of Discrete Compounding Interest Factors

To Find	Given	Factor	Symbol	Name
$P$	$F$	$(1 + i)^{-n}$	$(P F,i,n)$	Single payment, present worth factor
$F$	$P$	$(1 + i)^n$	$(F P,i,n)$	Single payment, compound amount factor
$P$	$A$	$\frac{(1 + i)^n - 1}{i(1 + i)^n}$	$(P A,i,n)$	Uniform series, present worth factor
$A$	$P$	$\frac{i(1 + i)^n}{(1 + i)^n - 1}$	$(A P,i,n)$	Uniform series, capital recovery factor
$F$	$A$	$\frac{(1 + i)^n - 1}{i}$	$(F A,i,n)$	Uniform series, compound amount factor
$A$	$F$	$\frac{i}{(1 + i)^n - 1}$	$(A F,i,n)$	Uniform series, sinking fund factor
$P$	$G$	$\frac{[1 - (1 + ni)(1 + i)^{-n}]}{i^2}$	$(P G,i,n)$	Gradient series, present worth factor
$A$	$G$	$\frac{(1 + i)^n - (1 + ni)}{i[(1 + i)^n - 1]}$	$(A G,i,n)$	Gradient series, uniform series factor
$P$	$A_1, j$	$\frac{1 - (1 + j)^n(1 + i)^{-n}}{i - j}$ for $i \neq j$	$(P A_1,i,j,n)$	Geometric series, present worth factor
$F$	$A_1, j$	$\frac{(1 + i)^n - (1 + j)^n}{i - j}$ for $i \neq j$	$(F A_1,i,j,n)$	Geometric series, future worth factor

# Compound interest example

- Assume that you deposit \$1,000 in an account earning 5% per year compounded yearly for 3 years. What is the value of the account after 3 years and what is the accumulated interest at the end of the 3<sup>rd</sup> year?

5%

## End-of-Period Compound Interest Factors

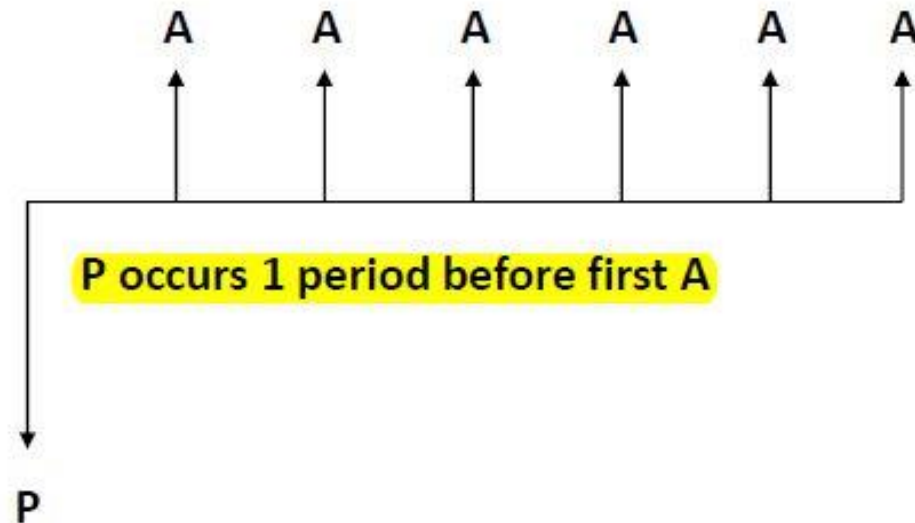
5%

N	Single Payment		Uniform Payment Series				Arithmetic Gradient		N
	Compound Amount Factor F/P	Present Worth Factor P/F	Capital Recovery Factor A/P	Present Worth Factor P/A	Sinking Fund Factor A/F	Compound Amount Factor F/A	Present Worth Factor P/G	Uniform Payment Factor A/G	
1	1.050	.9524	1.0500	.952	1.0000	1.000	0	0	1
2	1.103	.9070	.5378	1.859	.4878	2.050	.907	.488	2
3	1.158	.8638	.3672	2.723	.3172	3.153	2.635	.967	3
4	1.216	.8227	.2820	3.546	.2320	4.310	5.103	1.439	4
5	1.276	.7835	.2310	4.329	.1810	5.526	8.237	1.903	5
6	1.340	.7462	.1970	5.076	.1470	6.802	11.968	2.358	6
7	1.407	.7107	.1728	5.786	.1228	8.142	16.232	2.805	7
8	1.477	.6768	.1547	6.463	.1047	9.549	20.970	3.245	8
9	1.551	.6446	.1407	7.108	.0907	11.027	26.127	3.676	9
10	1.629	.6139	.1295	7.722	.0795	12.578	31.652	4.099	10
11	1.710	.5847	.1204	8.306	.0704	14.207	37.499	4.514	11
12	1.796	.5568	.1128	8.863	.0628	15.917	43.624	4.922	12
13	1.886	.5303	.1065	9.394	.0565	17.713	49.988	5.322	13
14	1.980	.5051	.1010	9.899	.0510	19.599	56.554	5.713	14
15	2.079	.4810	.0963	10.380	.0463	21.579	63.288	6.097	15
16	2.183	.4581	.0923	10.838	.0423	23.657	70.160	6.474	16
17	2.292	.4363	.0887	11.274	.0387	25.840	77.140	6.842	17
18	2.407	.4155	.0855	11.690	.0355	28.132	84.204	7.203	18
19	2.527	.3957	.0827	12.085	.0327	30.539	91.328	7.557	19
20	2.652	.3770	.0802	12.460	.0302	33.066	98.600	7.902	20

# Answer:

- $F = P(F/P, i\%, n)$
- $F = 1000(F/P, 5\%, 3)$
- $F = 1000(1.158)$
- $F = \$1,158$
- Accumulated interest =  $1,158 - 1,000 = \$158$

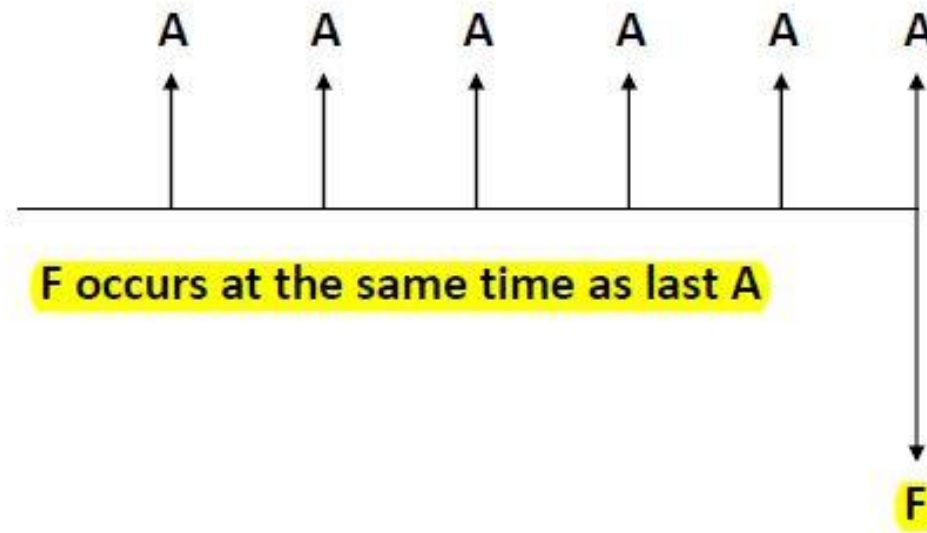
# Uniform payment series



$(P|A,i,n)$  Uniform series, present worth factor

$(A|P,i,n)$  Uniform series, capital recovery factor

# Uniform payment series



$(F|A,i,n)$  Uniform series, compound amount factor

$(A|F,i,n)$  Uniform series, sinking fund factor

# Arithmetic Gradient Series (G)

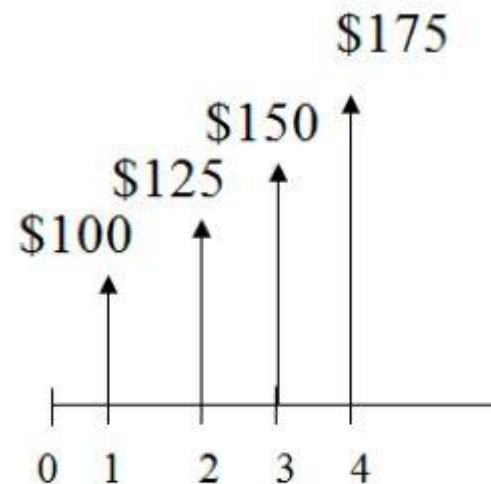
- Cash flows that increase or decrease by a constant amount are considered arithmetic gradient cash flows.
- The amount of increase (or decrease ) is called the gradient.

$(P|G,i,n)$  Gradient series, present worth factor

$(A|G,i,n)$  Gradient series, uniform series factor

***Converts the G to A !!***

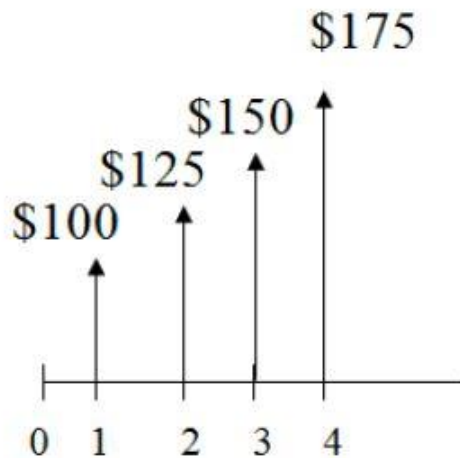
# Increasing arithmetic gradient



$$\mathbf{G = \$25}$$

$$\text{Base} = \$100$$

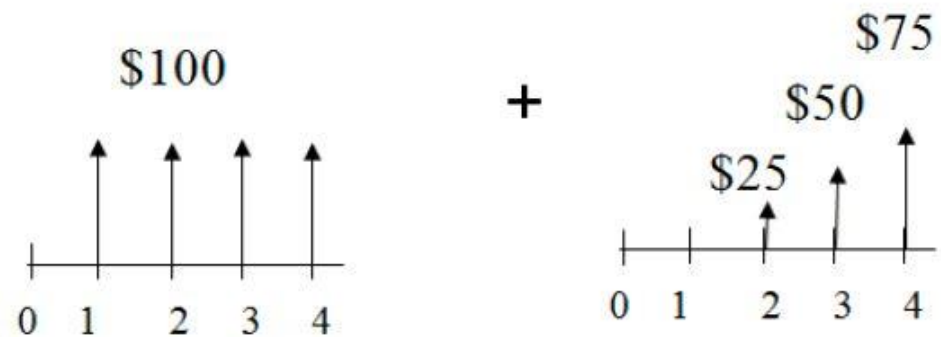
# Increasing arithmetic gradient



**G = \$25**

Base = \$100

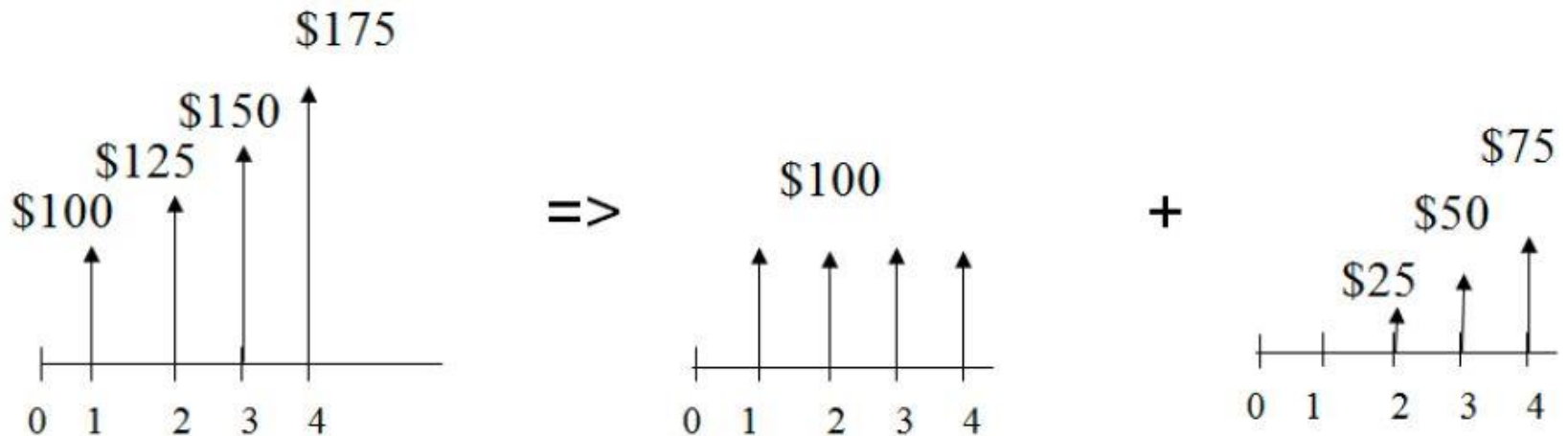
$\Rightarrow$



Note: the gradient series by convention starts in year 2.

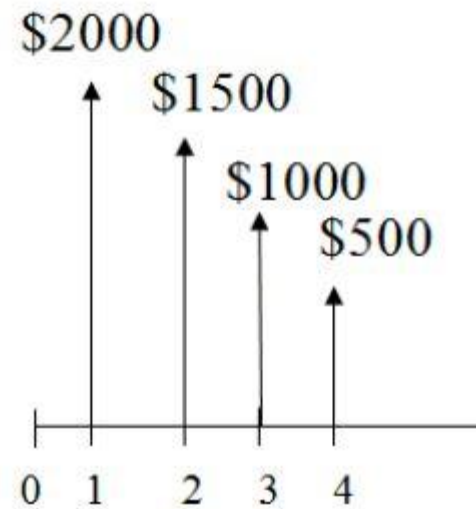
# Increasing arithmetic gradient

- To get the P:



$$P = \$100(P/A, i, 4) + \$25(P/G, i, 4)$$

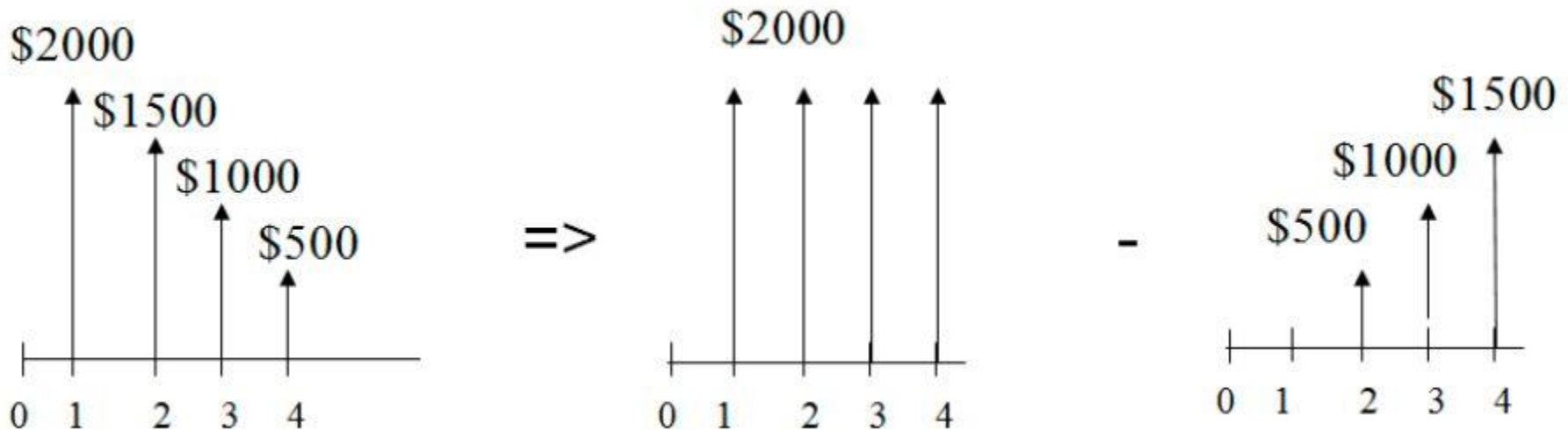
# Decreasing arithmetic gradient



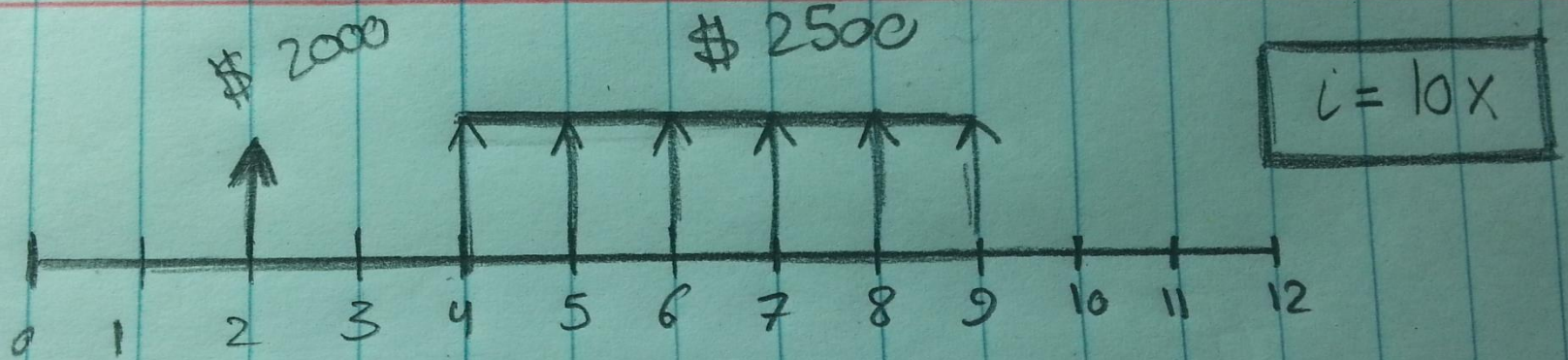
$$\mathbf{G = -\$500}$$

$$\text{Base} = \$2000$$

# Decreasing arithmetic gradient



# Example 1



\* present worth ?

\* future worth ?

# Example 2

