MAT 1348 B

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Proposition :- A declarative sentence (that states a fact or an argument) that is either T or F ( but not both )

Ottawa is the capital of Canada

$1+1=3$

$2x+6=8$ neither T nor F

What time is it ?

It is now snowing ~~out~~ here

propositional variables p, q, r, s . . . with true or false value T/F.

Negation

$\neg p$ : It is not the case that p

p: square of 3 is 9.
q:- Rubik's cube has 6 sides.
what are $\neg p$ , $\neg q$ ?

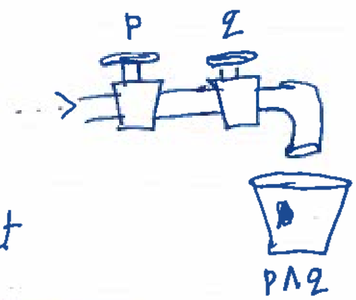
Truth Table

| p | $\neg p$ |
|---|---|
| T | F |
| F | T |

Compound Propositions

Logical operators / Connectives

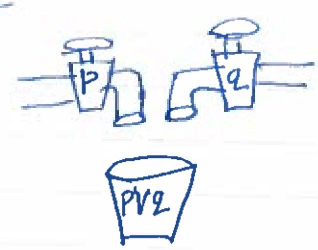
AND / conjunction




example John is hardworking & John is smart

OR / Disjuction

| p | q | $p \vee q$ |
|---|---|---|
| F | F | F |
| F | T | T |
| T | F | T |
| T | T | T |




example students who have taken mathematics or computer science can enroll in this class .

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exclusive OR .

Dinner menu includes soup or salad .

p : ~~soup you can take~~ Dinner includes soup
q : ~~you can take~~ Dinner includes salad

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| F | F | F |
| F | T | T |
| T | F | T |
| T | T | F |

Logic & Bit Operation Switch between T/F & 1/0

exclusive NOR

| $p$ | $q$ | $p \odot q$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$\neg(p \oplus q)$

(NOR, NAND)

XOR in real life ?

evaluate the following :

$p \land T \equiv$

$p \land F \equiv$

$p \land \neg p \equiv$

$p \lor T \equiv$

$p \lor F \equiv$

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p : You have the flu.

q : You miss the final exam

r : You pass the course

ex 1 :– $(p \land q) \lor (\neg q \land r)$ ?

ex 2 :– neither you miss the final exam nor you have flu.

$\Rightarrow$ it is not the case that you missed the final

and

it is not the case that you have flu.

$$\neg q \land \neg p$$

| p | q | $\neg p$ | $\neg q$ | $\neg p \land \neg q$ |
|---|---|---|---|---|
| F | F | T | T | T |
| F | T | T | F | F |
| T | F | F | T | F |
| T | T | F | F | F |

It is also

$\neg (p \lor q)$

NOR

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## Conditional statement (if, then)

If p then q $\qquad\qquad$ p implies q

If p, q

p only if q

p is sufficient for q

(a sufficient condition for q is p)

a necessary condition for p is q.

q if p $\qquad$ q when p $\qquad$ q whenever p

q is necessary for p

q follows from p

| p | q | $p \to q$ | |
|---|---|---|---|
| F | F | T | OK |
| F | T | T | OK |
| T | F | F | not OK |
| T | T | T | OK |

$p \to q$ is false iff p is T & q is F

Hypothesis $\longrightarrow$ conclusion

E1 $\quad$ p:- You give me a million dollar. $\qquad$ $p \to q$

q:- I will be your best friend.

E2 $\quad$ If you get 100% on the final, you will get an A+

Q1:— r: 8 is an odd number $\qquad$ what is the truth value of $r \to s$?

s: 9 is composite (not prime)

Soln:— Since hypothesis is false, the condition $r \to s$ is true.

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E3. If $\underbrace{\text{Maria has blue eyes}}_{p}$ then $\underbrace{2+3=5}_{q}$

q is T

$p \to q$ is True
T/F

If Maria has blue eyes then $2+1=5$

This is true if p is True (even though q is false)

$p \to q$

Converse $q \to p$
contrapositive $\neg q \to \neg p$
Inverse $\neg p \to \neg q$

| $p$ | $q$ | $\neg p$ | $\neg q$ | $p \to q$ | $q \to p$ | $\neg q \to \neg p$ | $\neg p \to \neg q$ |
|---|---|---|---|---|---|---|---|
| F | F | T | T | T | T | T | T |
| F | T | T | F | T | F | T | F |
| T | F | F | T | F | T | F | T |
| T | T | F | F | T | T | T | T |

$$(p \to q) \equiv (\neg q \to \neg p)$$

Biconditional $p \leftrightarrow q$

p is necessary and sufficient for q
p iff q

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| F | F | T |
| F | T | F |
| T | F | F |
| T | T | T |

You can take the flight iff you buy a ticket