

Phil 377 Assignment 2

Due: July 26, 2019 at 1 pm

General Instructions: For each of the following exercises, each sub-question is worth 1 point. If a question is worth more than 1 point, that will be indicated. A small user manual for typesetting proofs will also be provided as an additional resource. **Note: spelling and grammar matter!**

1. For each item below, provide a Fitch-style proof of the claim.

Proofs from single premises

- (a) $P \rightarrow (Q \rightarrow R) \vdash (P \rightarrow Q) \rightarrow (P \rightarrow R)$
- (b) $(E \wedge L) \rightarrow M \vdash (\neg M \wedge L) \rightarrow \neg E$
- (c) $(\neg D \rightarrow \neg B) \wedge (D \rightarrow (B \rightarrow C)) \vdash \neg B \vee (D \wedge C)$

Equivalences

- (d) $A \wedge (B \vee C) \dashv\vdash (A \wedge B) \vee (A \wedge C)$
- (e) $\neg(A \wedge B) \dashv\vdash A \rightarrow \neg B$

Theorems

- (f) $\vdash (P \vee P) \rightarrow P$
- (g) $\vdash (A \rightarrow B) \vee (B \rightarrow A)$

Proofs from multiple premises

- (h) $(L \rightarrow M), (M \rightarrow \neg L) \vdash \neg L$
- (i) $(E \wedge \neg L) \rightarrow M, \neg(M \vee L) \vdash \neg E$

Inconsistency

- (j) $A \vee (\neg B \rightarrow C), \neg(A \vee C), \neg B \vdash \perp$

2. Consider the propositional language with only \rightarrow and \perp as connectives. Is that language truth functionally complete? Explain your answer. (**2 points**)
3. Consider the following truth function ($\mathcal{A} \downarrow \mathcal{B}$) given by the characteristic truth table:

A	B	$A \downarrow B$
T	T	F
T	F	T
F	T	T
F	F	F

Note that the syntax of this connective is that of a binary connective so for example: $C \downarrow (B \downarrow A)$ would be a formula.

- (a) Represent this function in our language using (\vee, \wedge, \neg) by the standard method, i.e., the method discussed in lecture 5.
- (b) Offer a more economical way to represent this function in our propositional language, i.e., you may use any of the connectives ($\neg, \vee, \wedge, \rightarrow, \leftrightarrow, \perp$), but the goal is to represent the function with fewer symbols than the standard method.
- (c) Consider a propositional language consisting of atoms and \downarrow as the only connective. Would that language be truth functionally complete? Explain your answer.

For the next questions, if you are writing the assignment by hand, please print out these pages and copy your final answers directly on to those pages and either scan them or attach them to your assignment. Highlighting columns: if you are doing it by hand, just use a highlighter, if you are doing it in L^AT_EX, replace the appropriate column specification in the tabular environment with a ‘y’. For example:

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\begin{tabular}{|cccccccccc|}
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becomes

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\begin{tabular}{|cccccycccccy|}
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if the relevant columns to highlight were 9 and 14 (starting from the first column).

4. Complete a full truth table showing that $(E \wedge L) \rightarrow M \vDash (\neg M \wedge L) \rightarrow \neg E$. Indicate which column(s) show this by highlighting it(them) in yellow. (2 points)

E	L	M	$(E \wedge L) \rightarrow M \vDash (\neg M \wedge L) \rightarrow \neg E$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

5. Complete a full truth table showing that $\{A \vee (\neg B \rightarrow C), \neg(A \vee C), \neg B\}$ is unsatisfiable. Indicate which column(s) show this by highlighting it(them) in yellow. (2 points)

A	B	C	$A \vee (\neg B \rightarrow C), \neg(A \vee C), \neg B$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

6. Complete a full truth table showing that $(A \rightarrow B) \vee (B \rightarrow C)$ is a tautology. Indicate which column(s) show this by highlighting it(them) in yellow. (2 points)

A	B	C	$(A \rightarrow B) \vee (B \rightarrow C)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	