CARLETON UNIVERSITY

LABORATORY REPORT

COURSE No.	EXPERIMENT No.
PYS1007A	Exp. # 2

Measurement of Density

Purpose:

The purpose of this laboratory experiment is to measure and analyse the density of a metal cylinder using two different methods and to statistically compare them the results of the two methods.

Theory:

The density (ρ) can be defined as the mass per unit volume of a substance. (ManualPHY1007 p.65, 2019) There is different method to determine the density of a specific substance depending on its prosperities. In the present study, the density of a metal cylinder is assessed using two different methods. First, using the direct measurement method and, second, using Archimedes principle. Finally, the values obtained using the two methods will be statistically compared to determine of they are consistent using the T-test.

In the first section of this experiment, the density of the cylinder is determined using the direct measuring method. In this method, the volume of the cylinder is found by calculation the surface area of the base of the cylinder multiplied by the length of the cylinder. The mass of the cylinder will be determined using a conventional digital scale.

Equation 1.1

$$\rho = \frac{M}{V},$$

where V is the volume of the object and the M is the mass.

(ManualPHY1007 p.65, 2019)

The following equation is used to determine the volume of a cylinder:

Equation 1.2

$$V = \left(\pi \frac{d^2}{4}\right) X\ l$$

Where V is the volume of the cylinder, d is the diameter of the cylinder, and l is the length of the cylinder.

(ManualPHY1007 p.65, 2019)

The following equation is used to determine the density:

Equation 1.3

$$\rho = \frac{4M}{\pi l d^2}$$

Where ρ is the density, the M is the mass, the l is the length and the d is the diameter.

(ManualPHY1007 p.65, 2019)

The standard devotion of the measurement will be used to determine the sample errors. The standard deviation will be calculated using the logger pro desktop application. To determine the experimental error of the calculated density the following equation can be used:

Equation 1.4

$$\sigma_\rho = \rho\sqrt{\left(\frac{\sigma_M}{M}\right)^2 + \left(\frac{\sigma_l}{l}\right)^2 + 2^2\left(\frac{\sigma_d}{d}\right)^2}$$

Where σ is the standard deviation, σX is the standard deviation of X, ρ is the density, the M is the mass, the l is the length and the d is the diameter.

(ManualPHY1007 p.65, 2019)

In the second section of this paper, the density of the same cylinder is found using Archimedes' Principle. Archimedes principle puts in relation the volume of the rater displaced by and object when it is submerged in it and the volume of the object. The principle also explains that an object immersed in a fluid experiences a force pointing in the upwards direction (trying to lift the object). This force is equal in magnitude to the weight of the displaced water. The following formula represent this relation:

Formula 2.1

$$F = \rho_f V_f g$$

where F is the force exerted upward on the immersed object, g is the acceleration due to gravity, ρ_f the density of the fluid and V_f the volume of fluid displaced.

(ManualPHY1007 p.67, 2019)

Therefore, if the object placed in water has a smaller weigh then the weight if the displaced water, that object will float. This the object is heavier than the displaced water, then this object will sink.

From This information, it can be concluded that the wright of the object in the water is equal to the weight of the object in the air minus the force of the water that is pushing to lift the object submerged. The relationship between these variables can be expressed in the following formula:

Formula 2.2:

$$W_{water} = W_{air} - \rho_W V_o g$$

Where the volume of the object (V_o), the weight of the object in water (W_{water}), ρ_W is the density of water and g is the gravitational acceleration.

(ManualPHY1007 p.67, 2019)

Using these data, the density of the object can be calculated using the following formula:

Formula 2.3:

$$\rho_o = \rho_w \times \left(\frac{W_{air}}{W_{air} - W_{water}}\right)$$

Where ρ_W is the density of water, ρ_o is the density of the object, $W_{water} is\ the\ weight\ in\ the\ water\ and\ W_{air}\ is\ the\ weight\ in\ the\ air.$

The error related to the density calculate by the formula 2.3 can be calculated using the following formula:

Formula 2.4:

$$\sigma_{\rho_o} = \rho_w \sqrt{\left(\frac{W_{water}\sigma_{air}}{(W_{air} - W_{water})^2}\right)^2 + \left(\frac{W_{air}\sigma_{water}}{(W_{air} - W_{water})^2}\right)^2}$$

Where ρ_W is the density of water, ρ_o is the density of the object, $W_{water} is\ the\ weight\ in\ the\ water\ and\ W_{air}\ is\ the\ weight\ in\ the\ air.$

(ManualPHY1007 p.68, 2019)

Finally, the average and the standard deviation of the mean was calculated using logger pro. to compare the density value obtained using the direct measurement and using Archimedes' principle, a statistical T-test was performed:

Formula 3.1

$$\frac{|P_{dm} - p_{arc}|}{\sqrt{\sigma_{P_{dm}}^2 + \sigma_{p_{arc}}^2}} \leqslant 2$$

Where P_{dm} is the density using the direct measurement method, p_{arc} is the density using Archimedes' principle, σ_X is the standard error related to X.

If the test is less than 2 then the weighted mean and error should be calculated as

Formula 3.2

$$p_{weighted} = \frac{P_{dm}\sigma_2^2 + p_{arc}\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$$

Formula 3.3

$$\sigma_{weighted} = \frac{\sigma_1 \sigma_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

where $\sigma_1 is\ \sigma_{P_{dm}}$, $\sigma_2 is\ \sigma_{p_{arc}}$, $p_{weighted}$ is the weighted mean and $\sigma_{weighted}$ is it's related error.

If the test is greater than 2 then the arithmetic mean and error should be calculated as

Formula 3.4

$$p_{av} = \frac{(P_{dm} + p_{arc})}{2}$$

Formula 3.5

$$\sigma = \frac{|P_{dm} - p_{arc}| + (\sigma_{P_{dm}} + \sigma_{p_{arc}})}{2}$$

Where p_{av} is the arithmetic mean, P_{dm} is the density using the direct measurement method, p_{arc} is the density using Archimedes' principle, σ_X is the standard error related to X.

Note: while using the micrometer, the initial reading and the final reading are used. Thus, the final reading is the difference between both values and the standard deviation can be calculated using the following formula:

Formula 4.1

$$\sigma_{d_{corr}} = \sqrt{\sigma_{d_{meas}}^2 + \sigma_{d_0}^2}.$$

Where $\sigma_{d_{corr}}$ is the error on the final reading, $\sigma_{d\ meas}$ is th standard deviation of the final reading and σ_{do} is the standard deviation of the initail reading

Apparatus:

- The Vernier Caliper (0.001 cm)
- The Micrometer (+/- 0.0005 cm)
- The Scale (+/- 0.02 g)
- LoggerPro file
- metallic cylinder
- beaker
- cradle

Figure 1: Vernier caliper.

Figure 2: micrometer

Procedure

1.1.1. Procedure (Part I-Direct measurement method)

Mass measurement:

A digital balance was used to measure the mass of the metallic cylinder. The measurement was done in quadruplicate and the data was logged in logger pro. The average and the standard deviation of the mean was calculated using logger pro.

Length and diameter measurement (Vernier Caliper)

The length of the cylinder was measured using the Vernier caliper. The measurement was done in quadruplicate and the data was logged in logger pro. The average and the standard deviation of the mean was calculated using logger pro.

The diameter of the cylinder was measured using the micrometer. The measurement was done in quadruplicate and the data was logged in logger pro. The average and the standard deviation of the mean was calculated using logger pro. To calculate the corrected average diameter, the diffrence between the initial reading and the final reading is calculated.

Calculate the error on the average diameter (σ_d), formula 4.1 is used.

Data analysis:

The average, the sample standard deviation, the standard deviation of the mean is obtained using logger pro.

the volume of the cylinder is found by calculation the surface area of the base of the cylinder multiplied by the length of the cylinder. The volume of the cylinder is calculated using equation 1.2. The density of the cylinder is calculated using equation 1.3.

The standard deviation will be calculated using the logger pro desktop application. To determine the experimental error of the calculated density equation 1.4 is used.

Part II-Archimedes principle:

The same cylinder as in part 1 is used. Its weight in air, W_{air}, was determined using the digital scale. The measurement was done in quadruplicate and the data was logged in logger pro. The average and the standard deviation of the mean was calculated using logger pro.

A beaker is filled with water and the cylinder is completely submerged in the water while being attached to the scale. The measurement was done in quadruplicate and the data was logged in logger pro. The average and the standard deviation of the mean was calculated using logger pro.

the density of the object is calculated using the formula 2.3.

The error related to the density calculate is calculated using the following formula 2.4:

Analysis

The average and the standard deviation of the mean was calculated using logger pro. to compare the density value obtained using the direct measurement and using Archimedes' principle, a statistical T-test was using formula 3.1

If the test is less than 2 then the weighted mean and error should be calculated as using formula 3.2 and formula 3.3 (respectively)

If the test is greater than 2 then the arithmetic mean and error should be calculated as using formula 3.4 and formula 3.5 (respectively)

Observations:

Table 1: The mass (in grams) of the cylinder (N=4). The average and Standard deviation of the mean is calculated by logger pro

Trial number	Mass (grams) Error (+/- 0.02 g)
1	14.19
2	14.19
3	14.19
4	14.20
Average	14.1925
S. Dev.	0.0050

Table 2: Length (cm) of the cylinder (N=4). The average and Standard deviation of the mean is calculated by logger pro

Trial number	Length (cm) Error (+/- 0.001 cm)
1	1.868
2	1.868
3	1.868
4	1.868
Average	1.868
S. Dev.	0.000000

Table 3: Diameter (cm) of the cylinder and the Zero of the micro-meter (N=4). The average and Standard deviation of the mean is calculated by logger pro

Trial number	Zero of the micrometer (cm) Error (+/- 0.0005 cm)	Measured diameter (cm) Error (+/- 0.0005 cm)
1	-0.003	1.907
2	-0.003	1.906

3	-0.004	1.907
4	-0.003	1.905
Average	-0.0033	1.90625
S. Dev.	0.0005	0.00096

Calculations and analysis for part I (the direct method):

To calculate the diameter of the cylinder, the difference between D0 and Diameter measured:

Diameter corrected = Diameter measure – D0

D corrected (cm) = 1.90065 – (- 0.0033)

D corrected = 1.90395 cm

To determine the error related to the calculated diameter, formula 4.1 can be used:

$$\sigma_{d_{corr}} = \sqrt{\sigma_{d_{meas}}^2 + \sigma_{d_0}^2}$$

$$\sigma_{d_{corr}} = \sqrt{0.0005^2 + 0.00095_{d_0}^2}$$

$$\sigma_{d_{corr}} = 0.0011\ cm$$

to determine the density, equation 1.3 is used

$$\rho = \frac{4M}{\pi l d^2}$$

$$\rho = \frac{4\ (14.193)}{\pi(1.868)(1.90359)^2}$$

$$\rho = 2.6696\ g/cm^3$$

To determine the experimental error of the calculated density the following equation can be used:

Equation 1.4

$$\sigma_\rho = \rho\sqrt{\left(\frac{\sigma_M}{M}\right)^2 + \left(\frac{\sigma_l}{l}\right)^2 + 2^2\left(\frac{\sigma_d}{d}\right)^2}$$

$$\sigma_\rho = 2.6696\sqrt{\left(\frac{0.005}{14.193}\right)^2 + \left(\frac{0.00}{1.868}\right)^2 + 2^2\left(\frac{0.0011}{1.904}\right)^2}$$

$$\sigma_\rho = 0.0032\ \ g/cm^3$$

Thus, the density using the direct measurement method is 2.6696 +/- $0.0032\ \ g/cm^3$

Observation for section II (Archimedes Law)

Trial number	Weight in air (g) Error (+/- 0.001 g)	Weight in water (g) Error (+/- 0.001 g)
1	14.22	8.95
2	14.23	8.95
3	14.23	8.95
4	14.23	8.95
Average	14.2275	8.95
S. Dev.	0.005000	0.000000

Calculation for section II:

Using these data, the density of the object can be calculated using the following formula:

Formula 2.3:

$$\rho_o = \rho_w \times \left(\frac{W_{air}}{W_{air} - W_{water}}\right)$$

$$\rho_o = 1 \times \left(\frac{14.2275}{14.2275 - 8.95}\right)$$

$$\rho_o = 2.6959 \text{ g/cm3}$$

The error related to the density calculate by the formula 2.3 can be calculated using the following formula:

Formula 2.4:

$$\sigma_{\rho_o} = \rho_w \sqrt{\left(\frac{W_{water}\sigma_{air}}{(W_{air} - W_{water})^2}\right)^2 + \left(\frac{W_{air}\sigma_{water}}{(W_{air} - W_{water})^2}\right)^2}$$

$$\sigma_{\rho_o} = 1 \sqrt{\left(\frac{8.95X0.005}{(14.2275 - 8.95)^2}\right)^2 + \left(\frac{14.2275X0.00}{(14.2275 - 8.95)^2}\right)^2}$$

$$\sigma_{\rho_o} = 0.0016 \text{ g/cm3}$$

Thus, the density using Archimedes principle is 2.6959 +/- 0.0016 g/cm3.

Comparison test calculation:

To compare the density value obtained using the direct measurement and using Archimedes' principle, a statistical T-test was performed:

Formula 3.1

$$\text{Test} = \frac{|\rho_{dm} - \rho_{arc}|}{\sqrt{\sigma^2_{\rho_{dm}} + \sigma^2_{\rho_{arc}}}}$$

$$\text{Test} = \frac{|2.6696 - 2.6959|}{\sqrt{0.0032^{\ 2} + 0.0016^{\ 2}}}$$

$$\text{Test} = 7.35$$

The value of the t-test is greater than 2 then the arithmetic mean and error should be calculated as

Formula 3.4

$$\rho_{av} = \frac{(\rho_{dm} + \rho_{arc})}{2}$$

$$\rho_{av} = \frac{(2.6696 \ \ + 2.6959)}{2}$$

$$\rho_{av} = 2.683 \ \ g/cm3$$

Formula 3.5

$$\sigma = \frac{|\rho_{dm} - \rho_{arc}| + (\sigma_{\rho_{dm}} + \sigma_{p_{arc}})}{2}$$

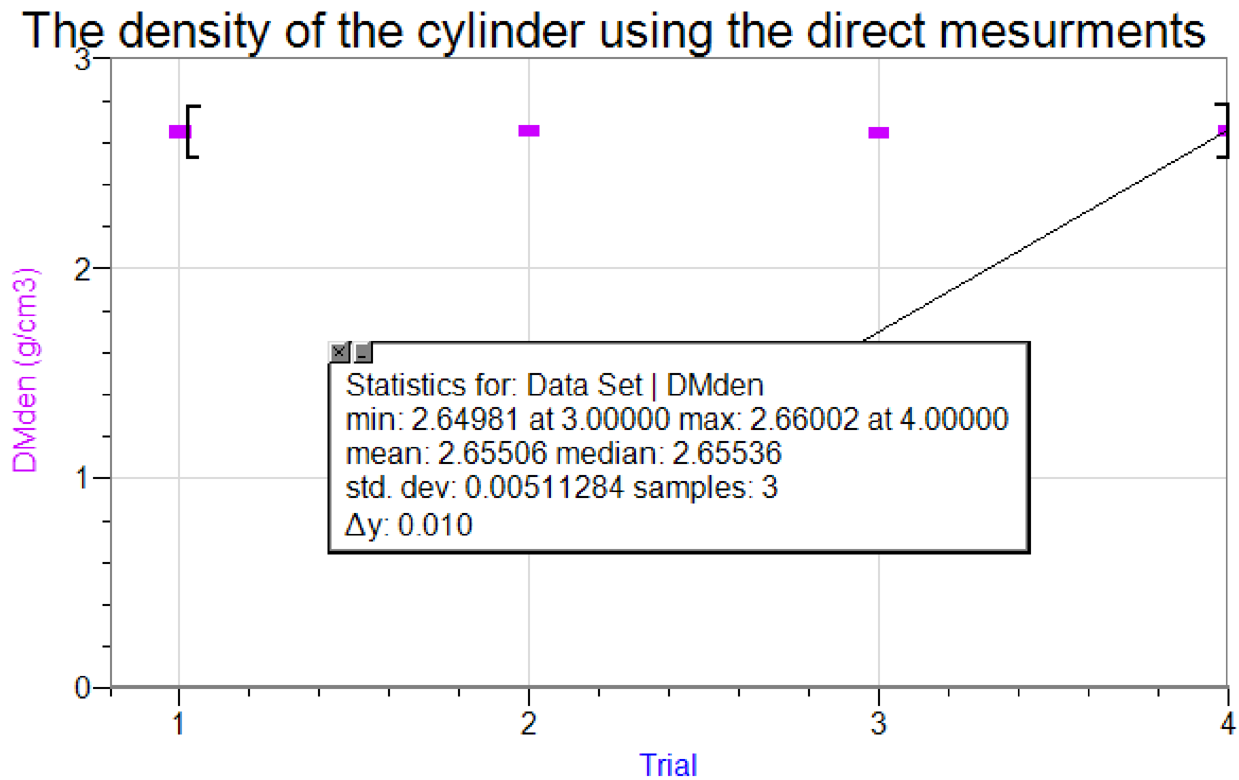
$$\sigma = \frac{|2.6696 - 2.6959| + (0.0032 \ + 0.0016\ )}{2}$$

$$\sigma = 0.015\ g/cm3$$

The arithmetic mean of the measured values for the density using the direct measurement method and Archimedes principle is 2.683 +/- 0.015 g/cm^3.

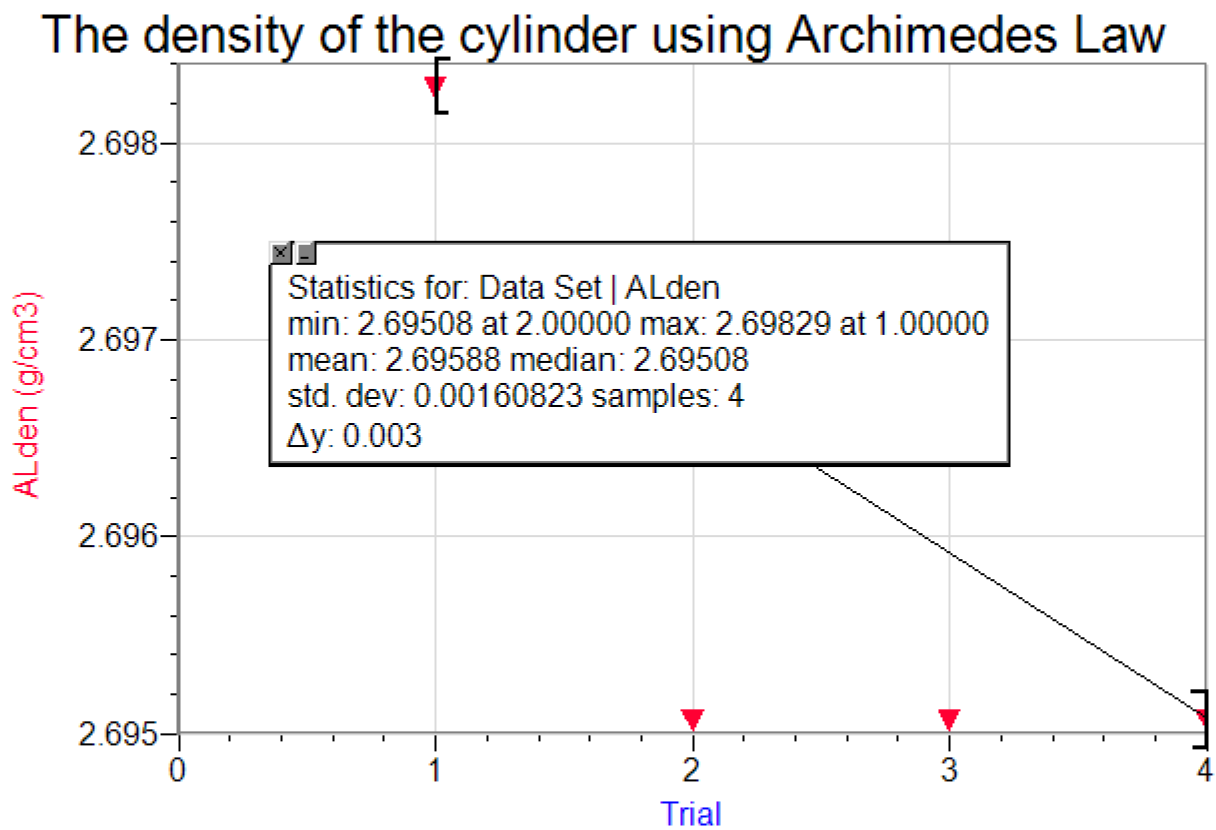
Graphs:

Figure 1: The density of the cylinder using the direct measurement method. The X axes is the density in g/cm3, and the Y axes is the trial number.



This histogram was generated using the application logger Pro.

Figure 2: The density of the cylinder using Archimedes principle. The X axes is the density in g/cm3, and the Y axes is the trial number.



This histogram was generated using the application logger Pro.

Results:

This experiment determined the following data:

The measured value for the density using the direct measurement method is 2.6696 +/- 0.0032 g/cm^3.

The measured value for the density using Archimedes principle is 2.6959 +/- 0.0016 g/cm3.

Comparing the average, the T-test has a value greater than 2, thus the two values are statistically significantly different (non consistent).

The arithmetic mean of the measured values for the density using the direct measurement method and Archimedes principle is 2.683 +/- 0.015 g/cm^3.

Discussion:

The purpose of the present experiment is to measure the density of the same cylinder using two different methods and comparing the result. Comparing the experimental value to the theoretical the accepted value $\left(\rho_{Al} = 2.70\ \frac{g}{cm^3}\right)$, it can be concluded that the measured value is close enough. This difference can be explained by the sources of errors explained further. If a consistency test at 2σ is conducted by 60 students in the lab that do the same experiment, 5% (3 students) will get a result that is inconsistent just because of random error. This is due to the nature of the random distribution and statistical concepts. Archimedes is the more precise and accurate compare to the direct measurement method since the whole in the middle of the cylinder is accounted for because using this method all irregularities in the shape is accounted for. The mass measurement has the biggest standard deviation, to improve this source of error, more precise scale can be used. In terms of the T-test, the two measurements of the density are not consistent. In fact, the two values obtained are very close two, but the error obtained is very small, thus the result's interval don't overlap and thus the results are inconsistent. In theory, they should be consistent since the same value is measured using two different methods. if there would be microscopic bubbles attached to the submerged apparatus for the second part of the experiment, this error source would cause and under estimation the value of the density since it will try to exert an upward force on the cylinder. If a fraction of a milliliter is displaced by such bubble, it would not result in a big error since the fraction of a milliliter is very small compared to the volume of the cylinder. if the density of the water was not corrected, there would not be a big error since the density of the water is 1.

Bonus: Contribution to the error on density due to the string and the hole through the cylinder:

The whole in the middle of the cylinder would cause an under estimation of the density using the direct method measurement, but it would not affect the density obtained using Archimedes principle. For the string, its mass is very small compared to the mass of the cylinder this it would not have a very significant error.