uOttawa

Civil Engineering Department

October 2nd, 2019

CVG2132 – FUNDAMENTALS OF ENVIRONMENTAL ENGINEERING

Homework #3 Professor: Chris Kinsley

Due Date: Oct. 11th, 2019 (3:30pm) – Dropbox « CVG 2132 », Mezzanine A (0.5) CBY

1) Compute the following –
   a) Derive the expressions for concentration exiting a CSTR and a PFR for a zero-order reaction. Which reactor design is more efficient in terms of volume and hydraulic retention times? Why? State assumptions.
   b) A chemical element degrades according to first order reaction. Its half-life is 25 days, determine rate constant, k. How long does it take for the element to degrade by 80%? Does $t_{1/2}$ depend on initial concentration?

Solution:

a)

For CSTR –

Assumptions

- steady state,
- **A→ B**,
- Influent and effluent are dilute.

From Lecture 6, slide 9, for a single reactant to single product irreversible reaction rate equation can be given by -

$$r = -\frac{dC}{dt} = kC^0 \quad \text{or} \quad \frac{dC}{dt} = -k$$

Performing mass balance for the constituent of interest in the CSTR

$$Accumulation = Input - Output + Generation$$

Since its in steady state, accumulation = 0

$$\left(\frac{dC}{dt}\right)_{accum} V = QC_0 - QC_t + \left(\frac{dC}{dt}\right) V$$

$$QC_O = QC_t + kV$$

$$Q(C_O - C_t) = kV$$

$$\frac{V}{Q} = \tau = \frac{C_O - C_t}{k}$$

$$\boldsymbol{C_t = C_O - k\tau}$$

For PFR –

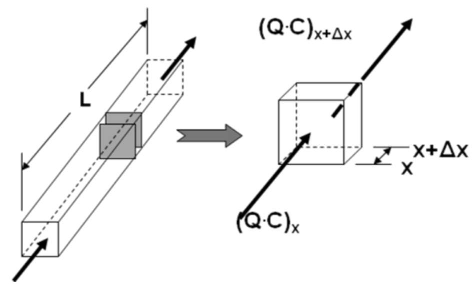
Assumptions

- Steady state,
- **A→ B**,
- Influent and effluent are dilute ($\rho_x = \rho_{x+\Delta x} = \rho_w$)

Performing a mass balance for the constituent of interest on the differential volume element of the PFR

$$Accumulation = Input - Output + Generation$$

Since its in steady state, accumulation = 0



$0 = Q_xC_x – Q_{x+\Delta x}C_{x+\Delta x} + rV$

$(Q_{x+\Delta x}C_{x+\Delta x} - Q_xC_x)/V = r$

Since the influent and effluent are dilute, $Q_{x+\Delta x} = Q_x = Q$

$C_{x+\Delta x} - C_x = \Delta C$

From diagram: $\Delta V = \Delta X * A$ where A is the cross-sectional area.

$$\frac{Q\Delta C}{A\Delta X} = r$$

From Lecture 6, slide 9, for a single reactant to single product irreversible reaction rate equation can be given by -

$$r = -\frac{dC}{dt} = kC^0 \quad \text{or} \quad \frac{dC}{dt} = -k$$

Performing mass balance for the constituent of interest in the PFR

$$Accumulation = Input - Output + Generation$$

Since its in steady state, accumulation = 0

Also,

$$\frac{Q\Delta C}{A\Delta X} = -k$$

$$\frac{Q}{A}\int_{C_O}^{C_t} dC = -K\int_0^L dX$$

$$Q(C_t - C_O) = -k(L)A$$

$$C_O - C_t = k\tau$$

$$\boldsymbol{C_t = C_O - k\tau}$$

As the design equations are the same for a zero-order removal, both of the reactors will perform with the same efficiency in terms of volume and hydraulic retention time.

b) According to lecture 6, slide 37,

first-order: $C = C_0 e^{-kt} \rightarrow 0.5C_0 = C_0 e^{-kt_{1/2}} \rightarrow \ln 0.5 = -kt_{1/2} \rightarrow t_{1/2} = -\frac{\ln 0.5}{k} = \frac{0.693}{k}$

$$t_{1/2} = 25\ days = \frac{0.693}{k}$$

$$k = 0.02772\ d^{-1}$$

If 80% of the element degrades, 20% of the element still remains.

$$\frac{C}{C_O} = e^{-kt}$$

$$\frac{C}{C_O} = 0.2 = e^{-kt}$$

$$\ln\left(\frac{C}{C_O}\right) = -kt$$

$$\ln(0.2) = -0.02772\ d^{-1} \times t$$

$$\boldsymbol{t = 58\ days}$$

2) Phosphorous is considered as one of the limiting factors of eutrophication in lakes and rivers. The goal of a pilot plant study is to treat phosphorous from influent wastewater. The process involves a CSTR that removes phosphorus according to second order kinetics with a k value of 5.2 $\frac{L}{mg.d}$. The CSTR has a flow rate of 520 L/d, a volume of 90 L and an initial concentration of 3.5 mg P/L.
   a) Derive the HRT equation for the second order CSTR
   b) If the effluent P concentration to prevent eutrophication is 0.03 mg P/L, do the current parameters meet this goal?
   c) If the flow rate were to be decreased by 30% and volume of CSTR increased to 130L, what is the new effluent P concentration?

Solution –

Assumptions

1. As there is no indication of change with time, steady state conditions apply
2. Unless stated otherwise. we can assume dilute streams.

Knowns and Unknowns

Known: Q = 520 L/d, V = 90 L, C_0 = 3.5 mg P/L, k = 5.2 $\frac{L}{mg.d}$

Unknown: HRT, C and New C

Determining the HRT equation for the second order CSTR

Performing a constituent mass balance around the system

$$Accumulation = Input - Output - Reaction$$

Accumulation = 0 due to steady state

$$0 = QC_0 - QC - rV_{CSTR}$$

$$0 = QC_0 - QC - kC^2V_{CSTR}$$

Divide by Q

$$0 = C_0 - C - kC^2\tau$$

$$\tau = \frac{(C_0 - C)}{C^2k}$$

$$\tau = \frac{\left(\frac{C_0}{C} - 1\right)}{kC}$$

Rearrange to form -

$$0 = kC^2\tau + C - C_O$$

a) We know that,

$$\tau = \frac{V}{Q} = \frac{90L}{520\ L/d} = \mathbf{0.173\ day}$$

b) Substitute the value of HRT obtained in the derived equation above,

$$0 = kC^2\tau + C - C_O$$

$$0 = \left(5.2\frac{L}{mg.d} \times C^2 \times 0.173\ day\right) + C - 3.5\frac{mg\ P}{L}$$

$$0 = 0.8996C^2 + C - 3.5$$

On solving the above quadratic equation, we get,

C = 1.49$\frac{mg\ P}{L}$

This effluent does not meet the requirements to prevent eutrophication.

c) Flowrate reduced by 30%, new volume of CSTR is 130L

$$New\ Q = 520\frac{L}{d} - \left(0.3 \times 520\frac{L}{d}\right) = 364\frac{L}{d}$$

$$New\ \tau = \frac{V}{Q} = \frac{130L}{364\ L/d} = 0.357day$$

$$0 = kC^2\tau + C - C_O$$

$$0 = \left(5.2\frac{L}{mg.d} \times C^2 \times 0.357\ day\right) + C - 3.5\frac{mg\ P}{L}$$

We get, new effluent concentration, **C = 1.129**$\boldsymbol{\frac{mg\ P}{L}}$

3) A CSTR reactor is used to treat an industrial waste using a reaction which destroys the waste according to first order kinetics, where k = 0.226 day^{-1}. The reactor volume is 450m^3, volumetric flow rate of the single inlet and exit is 55 m^3/day, and the inlet waste concentration is 100 mg/L.
   a) Derive the design equation.
   b) What is the outlet concentration?
   c) Determine the volume required for a PFR to obtain the same degree of pollutant reduction as the CSTR. Assume the flow rate and k are unchanged.
   d) Which system is more efficient (% removal / m^3 of volume)?

Solution-

Assumptions –

1. As there is no indication of change with time, steady state conditions apply
2. Unless stated otherwise. we can assume dilute streams.

a) Derivation of design equation

The simplified first-order removal reaction:

$$r = -\frac{dC}{dt} = kC^1 \qquad \text{or} \qquad \frac{dC}{dt} = -kC$$

Performing a mass balance for the constituent of interest on the CSTR reactor –

$$Accumulation = Input - Output - Reaction$$

$$\left(\frac{dC}{dt}\right)_{\text{accum}} V = QC_0 - QC_t + \left(\frac{dC}{dt}\right)V$$

Steady state = No accumulation

$$0 = QC_0 - QC_t - kC_tV$$

$$kC_tV = Q(C_0 - C_t)$$

$$\text{HRT} = \frac{V}{Q} = \frac{(C_0 - C_t)}{kC_t} = \frac{(C_0/C_t - 1)}{k}$$

b) Determine the HRT and C

$$HRT = \frac{V}{Q} = \frac{450\ m^3}{55m^3/d} = 8.18d$$

$$HRT \times k = \frac{C_O}{C} - 1$$

$$C = \frac{C_O}{(HRT \times k) + 1} = \frac{100\ mg/L}{(8.18d \times 0.226d^{-1}) + 1} = \mathbf{35.1\ mg/L}$$

c) The CSTR achieved a pollutant decrease of $\frac{C}{C_O} = \frac{35.1\ mg/L}{100\ mg/L} = 0.351$

Derive equation for PFR

$$Accumulation = Input - Output + Generation$$

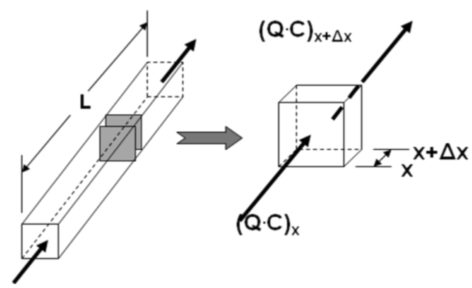
Since its in steady state, accumulation = 0

$0 = Q_xC_x – Q_{x+\Delta x}C_{x+\Delta x} + rV$

$(Q_{x+\Delta x}C_{x+\Delta x} - Q_xC_x)/V = r$

Since the influent and effluent are dilute, $Q_{x+\Delta x} = Q_x = Q$

$C_{x+\Delta x} - C_x = \Delta C$



From diagram: ΔV = ΔX*A where A is the cross-sectional area.

$$\frac{Q\Delta C}{A\Delta X} = r$$

The simplified first-order removal reaction:

$$r = -\frac{dC}{dt} = kC^1 \quad \text{or} \quad \frac{dC}{dt} = -kC$$

$$\frac{Q\Delta C}{A\Delta X} = -kC$$

$$\frac{Q}{A}\int_{C_O}^{C_t} \frac{dC}{C} = -k\int_0^L dX$$

$$\ln\left(\frac{C_t}{C_O}\right) = \frac{-kAL}{Q}$$

$$\ln\left(\frac{C_t}{C_O}\right) = \frac{-kV}{Q}$$

$$\ln(0.351) = \frac{-0.226d^{-1} \times V}{55m^3/d}$$

$$-1.046 = -0.004 \times V$$

$$\boldsymbol{V = 261.75m^3}$$

d) Volumetric efficiency of CSTR

$$Efficiency = \frac{C_O - C}{Volume\ of\ CSTR} = \frac{(100 - 35.1) \times \frac{mg}{L}}{450m^3} = 0.144\frac{\frac{mg}{L}}{m^3}$$

Volumetric efficiency of PFR

$$Efficiency = \frac{C_O - C}{Volume\ of\ PFR} = \frac{(100 - 35.1) \times \frac{mg}{L}}{261.75m^3} = 0.248\frac{\frac{mg}{L}}{m^3}$$

Note - As expected, PFR volume is smaller than the CSTR used initially. Hence, the same trend follows for volumetric flowrate.

4) The Ottawa wastewater treatment facility is investigating the potential of ammonia treatment at their facility. The aeration basin of the Ottawa facility is a baffled tank that can be modelled as an ideal PFR. The wastewater facility's influent flow rate is 634 000 m^3/d with an assumed influent ammonia concentration of 25 mg NH_3-N/L. The baffled tank has a volume of 23 000 m^3 and an ammonia removal reaction kinetics constant of $8.7\frac{L}{mgN.d}$. Note that the ammonia removal reaction is dependent on both the ammonia concentration and dissolved oxygen concentration, $r = -kC_{NH3\text{-}N}C_{O2}$. The Ottawa wastewater treatment facility is considering providing a constant dissolved oxygen concentration of 4 mg O_2/L or 10 mg O_2/L in the aeration basin to attain an effluent ammonia target of 1.25 mg NH_3-N/L.
   a) Derive the HRT equation for the baffled tank (state all assumptions and show all work).
   b) Determine which of the oxygen concentrations (4 mg O_2/L or 10 mg O_2/L) are required to meet the effluent ammonia requirement. As all options must be explored, please calculate both options.

a) Assumptions

1. As there is no indication of change with respect to time, we can assume steady state applies
2. Unless stated otherwise, assume dilute streams. Hence, the density of all streams is equivalent to the density of water i.e $\rho_W = \rho = 1000\ kg/m^3$

Based on the units of the k rate constant, we can assume that it is a pseudo first order reaction. Derive the pseudo-first order PFR equation

The problem statement describes that the concentration of oxygen therefore we can combine the C_{O2} and k_2 terms to get a first order k' term. Therefore $r = -k'C_{NH3\text{-}N} = -k_2C_{NH3\text{-}N}C_{O2}$

Accumulation = In – Out ± Generation

Accumulation = 0, due to steady state

As there is only a single stream entering and leaving the reactor and steady state conditions apply.

$Q_{in} = Q_{eff} = Q$

$0 = QC_0 - QC_{eff} + rV$

As this is a PFR set $C_0 = C_x$ and C_{eff} as $C_{x+\Delta x}$

$$0 = QC_x - QC_{x+\Delta x} + rV$$

$$r = \frac{Q(C_{x+\Delta x} - C_x)}{V}$$

$$V = \Delta V = A\Delta X$$

$$C_{x+\Delta x} - C_x = \Delta C$$

$$r = \frac{(Q\Delta C)}{(A\Delta X)}$$

$$r = -kC$$

$$-kC = \frac{(Q\Delta C)}{(A\Delta X)}$$

$$-\frac{A\Delta X}{Q} = \frac{\Delta C}{kC}$$

Integrate the length of the reactor

$$-\frac{A}{Q}\int_0^L dX = \frac{1}{k}\int_{C0}^{C}\frac{1}{C}dC$$

$$-\tau = \frac{(lnC - lnC_0)}{k}$$

$$\tau = \frac{(lnC_0 - lnC)}{k}$$

Given Information

$Q_{in} = 634\ 000\ m^3/d$

$V_{PFR} = 23\ 000\ m^3$

$K_2 = 8.7\ \frac{L}{mg.d} = 8.7\frac{m^3}{g\times d}$

$DO_1 = 4\ g\ O_2/m^3$

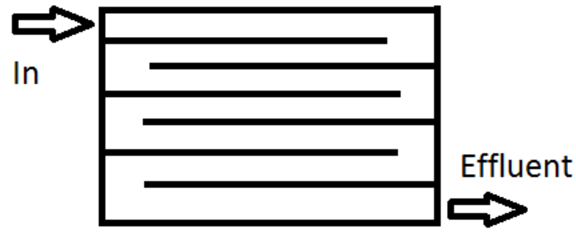
$DO_2 = 10\ g\ O_2/m^3$

$C_{in} = 25\ mg\ NH_3\text{-}N/L = 25\ g\ N/m^3$

Unknowns: C_{eff} at 4 g O_2/L and C_{eff} at 10 g O_2/L

The system diagram can be shown as below

Determine the effluent concentration with a DO of 4 g O_2/m^3



Using the equation previously defined, we get

T = ($\ln C_0 - \ln C$)/k

Where $C_0 = C_{in}$, $C = C_{eff}$, $\tau = V_{PFR}/Q$ and $k = k' = k_2 C_{O2}$

For C_{O2} = 4 g O_2/m^3; $k' = 8.7\frac{m^3}{g\times d} \times 4\frac{gO_2}{m^3} = 34.8\frac{1}{d}$

Rearranging the equation,

$$C_{eff} = C_{in} \times e^{-\tau k'}$$

$$\tau = \frac{Volume\ of\ PFR}{Q} = \frac{23000m^3}{634000\ m^3/d} = 0.036\ day$$

$$C_{eff} = \left(25\ \frac{g}{m^3}\right) \times e^{-\left(0.036day\times 34.8\frac{1}{d}\right)} = 7.14\frac{g}{m^3}$$

Determine the effluent concentration with a DO of 10 g O_2/m^3

Using the equation previously defined, we get

τ = ($\ln C_0 - \ln C$)/k

Where $C_0 = C_{in}$, $C = C_{eff}$, $\tau = V_{PFR}/Q$ and $k = k' = k_2 C_{O2}$

For C_{O2} = 10 g O_2/m^3; $k' = 8.7\ \frac{m^3}{g\times d} \times 10\frac{gO_2}{m^3} = 80\frac{1}{d}$

Rearranging the equation,

$$C_{eff} = C_{in} \times e^{-\tau k'}$$

$$\tau = \frac{Volume\ of\ PFR}{Q} = \frac{23000m^3}{634000\ m^3/d} = 0.036\ day$$

$$C_{eff} = \left(25\ \frac{g}{m^3}\right) \times e^{-\left(0.036day\times 80\frac{L}{d}\right)} = 1.09\frac{g}{m^3}$$

Therefore, a dissolved oxygen concentration of 10 g O_2/m^3 is required to attain a final effluent concentration lower than 1.25 mg NH_4^+-N/L.