MATH 2007A
Test 2 Solutions

October 18, 2018

[Marks]

[6] 1. Evaluate $\int \tan^2(x)\sec^6(x)\,dx$.

Solution:

$u = \tan(x) \;\Rightarrow$

$\int \tan^2(x)\sec^6(x)\,dx = \int \tan^2(x)[\tan^2(x)+1]^2\sec^2(x)\,dx = \int u^2(u^2+1)^2\,du$

$= \int u^2(u^4+2u^2+1)\,du = \int u^6+2u^4+u^2\,du = \frac{1}{7}u^7+\frac{2}{5}u^5+\frac{1}{3}u^3+C$

$= \frac{1}{7}\tan^7(x)+\frac{2}{5}\tan^5(x)+\frac{1}{3}\tan^3(x)+C.$

[6] 2. Evaluate $\int \frac{1}{(9-x^2)^{3/2}}\,dx$.

Solution:

$x = 3\sin(t) \;\Rightarrow$

$\int \frac{1}{(9-x^2)^{3/2}}\,dx = \int \frac{1}{[9\cos^2(t)]^{3/2}}3\cos(t)\,dt = \frac{1}{9}\int \sec^2(t)\,dt = \frac{1}{9}\tan(t)+C.$

$\sin(t) = \frac{x}{3} \;\Rightarrow\; \tan(t) = \frac{x}{\sqrt{9-x^2}} \;\Rightarrow\; \int \frac{1}{(9-x^2)^{3/2}}\,dx = \frac{1}{9}\frac{x}{\sqrt{9-x^2}}+C.$

[6] 3. Evaluate $\int_0^2 \frac{1}{\sqrt{4+x^2}}\,dx$.

Solution:

$x = 2\tan(t) \;\Rightarrow\; t = 0$ when $x = 0$ and $t = \frac{\pi}{4}$ when $x = 2$. Hence,

$\int_0^2 \frac{1}{\sqrt{4+x^2}}\,dx = \int_0^{\frac{\pi}{4}} \frac{1}{\sqrt{4\sec^2(t)}}2\sec^2(t)\,dt = \int_0^{\frac{\pi}{4}} \sec(t)\,dt$

$= \ln|\sec(t)+\tan(t)|\Big|_0^{\frac{\pi}{4}} = \ln(\sqrt{2}+1).$

[6] 4. Evaluate $\int \frac{7-x}{x^2+x-2}\,dx$.

Solution:

$\frac{7-x}{x^2+x-2} = \frac{7-x}{(x-1)(x+2)} = \frac{A}{x-1}+\frac{B}{x+2} = \frac{A(x+2)+B(x-1)}{(x-1)(x+2)} \;\Rightarrow$

$A(x+2)+B(x-1) = 7-x$, $x = -2 \;\Rightarrow\; -3B = 9 \;\Rightarrow\; B = -3$, and $x = 1 \;\Rightarrow$ $3A = 6 \;\Rightarrow\; A = 2$. Hence,

$\int \frac{7-x}{x^2+x-2}\,dx = \int \frac{2}{x-1} - \frac{3}{x+2}\,dx = 2\ln|x-1| - 3\ln|x+2| + C.$

[6] 5. Write down the partial fraction decomposition of $\frac{x^3-2x^2+1}{x^3(2x-3)(x^2+3)^3(3x^2-x+2)}$.

Do not solve for the constants.

Solution:

$$\frac{x^3 - 2x^2 + 1}{x^3(2x-3)(x^2+3)^3(3x^2-x+2)}$$
$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{2x-3} + \frac{Ex+F}{x^2+3} + \frac{Gx+H}{(x^2+3)^2} + \frac{Ix+J}{(x^2+3)^3} + \frac{Kx+L}{3x^2-x+2}.$$