**1.**

Determine whether the sequence $a_n = \frac{\ln n}{\sqrt{n+1}}$ converges or diverges; if it converges, find its limit L

A. Convergent, L = 0.5 B. Convergent, L = 0 C. Convergent, L = $\infty$ D. Divergent
E. Convergent, L = -0.5 F. Divergent, L = 0 G. none of the above

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln n}{\sqrt{n+1}} = \lim_{x\to\infty} \frac{\ln x}{\sqrt{x+1}}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2}\cdot\frac{1}{\sqrt{x+1}}} = \lim_{x\to\infty} \frac{2\sqrt{x+1}}{x}$$

$$= \lim_{x\to\infty} 2\sqrt{\frac{x+1}{x^2}} = \lim_{x\to\infty} 2\sqrt{\frac{1}{x}+\frac{1}{x^2}} = 0$$

**2.** Write a formula for the n-th entry of the sequence $a_1 = -1, a_2 = \frac{3}{4}, a_3 = -\frac{9}{16}, a_4 = \frac{27}{64}, \cdots$ and the sum of $s = \sum_{n=1}^{\infty} a_n$.

A. $a_n = (-1)^{n-1}\frac{3^n}{4^n}, s = -\frac{4}{7}$ B. $a_n = (-1)^n\frac{3^n}{4^{n-1}}, s = -\frac{12}{7}$ C. $a_n = (-1)^n\frac{3^{n-1}}{4^{n-1}}, s = -\frac{7}{4}$
D. $a_n = (-1)^n\frac{3^n}{4^n}, s = -\frac{3}{7}$ E. $a_n = (-1)^n\frac{3^n}{4^n}, s = -\frac{7}{4}$ F. $a_n = (-1)^n\frac{3^{n-1}}{4^{n-1}}, s = -\frac{4}{7}$
G. none of the above

$$a_n = (-1)^n \cdot \left(\frac{3}{4}\right)^{n-1}$$ , it is geometric with $r = \frac{-3}{4}$

$$S = \frac{-1}{1+\frac{3}{4}} = -\frac{4}{7}$$

**3.** Which of the following series is absolute convergent? I) $\sum_{n=1}^{\infty} \frac{n+4}{n^3+1}$ II) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ III) $\sum_{n=1}^{\infty} \frac{\cos n}{n!}$

A. I  B. I and II  C. I and III  D. ~~All of them~~  E. II and III  F. III  G. none of the above

I) $\sum_{n=1}^{\infty} \frac{n+4}{n^3+1} < \sum_{n=1}^{\infty} \frac{n+4}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2} + \sum_{n=1}^{\infty} \frac{4}{n^3}$ . abs. Convergent

II) $\left|\frac{(-1)^n}{n}\right| = \frac{1}{n}$ , decreases to zero ---- $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Convergent.

but. $\sum_{n=1}^{\infty} \frac{1}{n}$ --- divergent.

only conditional.

(III) $\sum_{n=1}^{\infty} \frac{|\cos n|}{n!} = \sum_{n=1}^{\infty} \frac{1}{n!}$

Ratio test: $\frac{1}{(n+1)!} \cdot n! = \frac{1}{n+1} \to 0 < 1.$

--- abs. Convergent.

**4.** What **best** describes the series $\sum_{n=1}^{\infty} \frac{(-1)^n e^n \sin n}{n!}$

A. Conditional convergent  B. Absolute convergent  C. Alternating absolute convergent

D. absolute divergent  E. Alternating divergent  F. Alternating conditionally convergent

G. none of the above

Not alternating since "sin n".

$\sum_{n=1}^{\infty} \frac{e^n |\sin n|}{n!} < \sum_{n=1}^{\infty} \frac{e^n}{n!}$

Ratio test: $\frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^n} = \frac{e}{n+1} \xrightarrow{n\to\infty} 0 < 1$

so. convergent. (absolutely)

**5.** How many terms do you need to estimate $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+4}$ so that the error is within 0.005?

A. 10 B. 11 C. 12 D. 13 E. 14 F. 15 G. none of the above

$$|R_n| \leq \frac{1}{(n+1)^2+4} \leq 0.005$$

$$\Rightarrow (n+1)^2+4 \geq 200$$

$$\Rightarrow (n+1)^2 \geq 196$$

$$\Rightarrow n+1 \geq \cancel{16} \rightarrow 14$$

$$\Rightarrow n \geq \cancel{15} \rightarrow 13.$$

**6.** Estimate the error bound when you use the first 30 terms to approximate series

$$1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \cdots$$

A. 0 B. $\sqrt{31}$ C. $\frac{2}{\sqrt{31}}$

D. $\frac{1}{\sqrt{31}}$ E. $\frac{2}{3\sqrt{31}}$ F. $\frac{2}{\sqrt{30}}$

G. none of the above

Start from 1

so n is 30

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$$

~~(You can treat $a_0 = 0$)~~

$$R_n \leq \int_n^{\infty} \frac{1}{x^{\frac{3}{2}}}\, dx = 2x^{-\frac{1}{2}} \Big|_{\infty}^{n} = \frac{2}{\sqrt{n}}$$

$$\frac{2}{\sqrt{n}} \Big|_{n=30} = \frac{2}{\sqrt{30}}$$