SOLUTION

Tutorial #2: Consumer's problem.

Given: $\begin{cases} h = 16 \\ w = 10 \\ \Pi = 16 \end{cases}$ 1. $\begin{cases} \text{tax rate } \tau = 0.20 \\ T = 0 \quad \text{(no lump-sum tax)} \end{cases}$

• $\frac{2c}{\ell} = MRS_{\ell,c}$

a) After tax wage $= w(1-\tau) = 10(1-0.2) = 8$

$c + w(1-\tau)\ell = w(1-\tau)h + \Pi - T$ (T → 0)

$\Rightarrow c + 8\ell = 8 \times 16 + 16$ — (1)

$\Rightarrow \boxed{c + 8\ell = 144}$ ~ BC

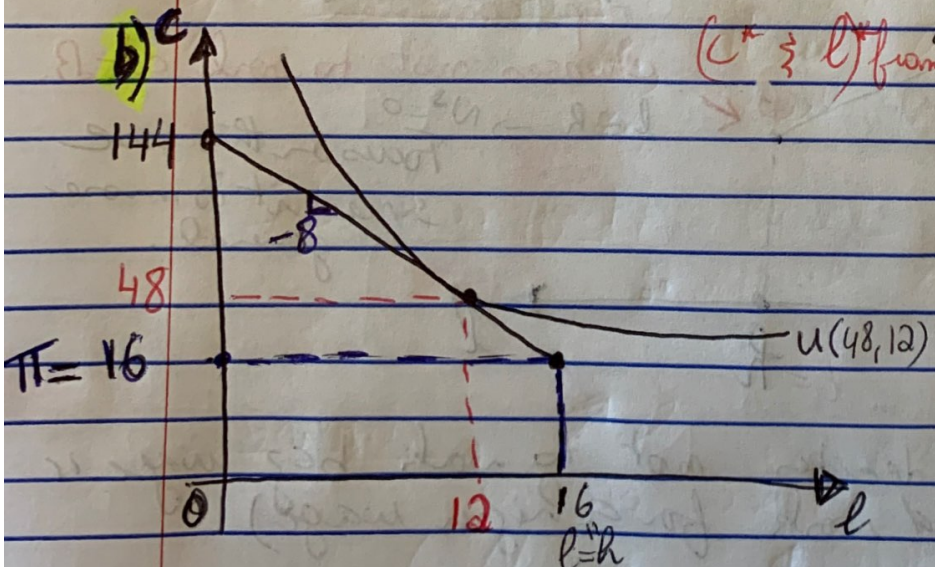
* To max utility, Paul should set c^* & ℓ^* so that:

$MRS_{\ell,c} = w$

(value of leisure in terms of c goods) ↔ (cost of leisure)

& his BC is "met".

b)



(c^*, ℓ^*) from $\begin{cases} c + 8\ell = 144 \\ MRS_{\ell,c} = w \end{cases}$

$\Rightarrow \frac{2c}{\ell} = 8$

$\Rightarrow \boxed{c^* = 4\ell^*}$

$4\ell^* + 8\ell^* = 144$

$12\ell^* = 144$

$\boxed{\ell^* = 12}$

• $c^* = 4 \times 12$

$\boxed{c^* = 48}$

& $N^s = h - \ell^* = 16 - 12$

$\boxed{N^s = 4}$

For Proof:

At $N^s = 6$, $\ell = R - N^s = 16 - 6$

$\Rightarrow \boxed{\ell = 10} \neq 12$

- $c = 144 - 8\ell$
  $= 144 - 80$
  $\boxed{c = 64} \neq 48$

Optimizing condition not met.

- $MRS_{\ell c} = \frac{2c}{\ell} = \frac{2 \times 64}{10} = 12.8 > w = 8$

value of leisure exceeds cost of leisure

$\Rightarrow$ Paul should buy more leisure.

c) Expenditure Approach: $GDP = C + \bar{I} + G$

i) - total income taxes collected from Paul $= \tau \cdot w \cdot N^s$
  $= 0.2 \times 10 \times 4$
  $\Rightarrow \boxed{T = 8}$

- Total Gvt spending $\boxed{G} = 50 \times 8 = 400$
  # × T

ii) Paul's consumption is 48,

- Agg. consumption $\boxed{C} = 50 \times 48 = 2400$

$GDP = C + G = 2400 + 400 = 2800.$

Income Approach: $GDP = wN^s + \Pi + \bar{R}$

i) Paul's labor income (before tax) $= 10 \times 4 = 40.$ ($w \cdot N^s$)
Total labor income $= 50 \times 40 = 2000$

ii) Paul's profit income $= 16$
Total profit income $\Pi = 50 \times 16 = 800$

$GDP = wN + \Pi = 2000 + 800 = 2800$

d) $\tau \uparrow$ 20% to 40%

∴, After tax wage $= (1-\tau^N)\,w = 0.6 \times 10 = 6$ ≠8

- $c + 6l = 6 \times 16 + 16$

  $c + 6l = 112$ ⇝ new BC

- $MRS_{lc} = \frac{2c}{l} = \frac{w}{6} \Rightarrow c = 3l^*$

- $3l + 6l = 112$

  $9l = 112$

  $l^* \cong 12.44$ ↑

  ∴, N^s ↓

  $c^* \cong 3(12.44)$

  $\Rightarrow c^* \cong 37.33$ ↓

  ∴, GDP ↓

You can use income approach to prove it

e) - Income effect: ↑ tax rate ⇒ ↓ income

  ∴, Paul l ↓ (↑ c)

  ∴, Paul wants to ↑ N^s

- Substitution effect: leisure is less expensive now (↓ w)

  ∴, Paul ↑ l (↓ c)

- Overall, ↑ l ∴ substitution effect is stronger in this case.

  (N^s ↓, ↓ GDP)