MATH 204/2 MIDTERM TEST OCTOBER 2012

All questions have equal value.

1. Using the Gauss-Jordan method, find all the solutions of the system of equations

$$\begin{aligned} 3x + 6y - 2z + u \phantom{{}+2v} &= 5 \\ 2x + 4y \phantom{{}-2z} + 2u + 2v &= 0 \\ 2x + 4y + z + 3u + v &= 5. \end{aligned}$$

2. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 1 & -2 & 0 \end{bmatrix}$.

   a) Calculate A^{-1}.

   b) Using the above result solve the system

$$\begin{aligned} x - y \phantom{{}+z} &= 2 \\ 2x \phantom{{}-y} + z &= 3 \\ x - 2y \phantom{{}+z} &= -1. \end{aligned}$$

   (No marks if you don't use the result of part (a).)

3. Calculate the determinant of the matrix $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \end{bmatrix}$.

4. Using Cramer's rule solve the system

$$\begin{aligned} 3x \phantom{{}+2y} + z &= 1 \\ 2y - z &= -1 \\ 2x - y \phantom{{}-z} &= 1. \end{aligned}$$

   (No marks if you don't use Cramer's rule.)

5. Let $\mathbf{u} = (2, -1, 3)$ and $\mathbf{v} = (3, -2, 1)$.

   a) Find numbers x and y such that $x\,\mathbf{u} + y\,\mathbf{v} = (1, -2, -9)$.

   b) Show that there exist no numbers x and y such that $x\,\mathbf{u} + y\,\mathbf{v} = (1, 2, -1)$.

6. Determine the values of a for which the system has no solution, exactly one solution or infinitely many solutions:

$$\begin{aligned} x + y + 7z &= -7 \\ 2x + 3y + 17z &= -16 \\ x + 2y + (a^2 + 1)z &= 3a. \end{aligned}$$

Math 204 Answers Midterm Oct 2012 (UNEDITED)

1. $V=-3$
$u=s$
$Y=t$
$Z=2-s$
$X=3-2t-s$

2a) $A^{-1}=\begin{bmatrix}2&0&-1\\1&0&-1\\-4&1&2\end{bmatrix}$

2b) $x=5$
$y=3$
$z=-7$

3. -8

4. $x=\frac{2}{7}$
$y=-\frac{3}{7}$
$z=\frac{1}{7}$

5a) $x=-4$
$y=3$

5b) No solution (See last Row of ACM.)

6. ACM becomes $\begin{bmatrix}1&0&4&-5\\0&1&3&-2\\0&0&(a^2-9)&3a+9\end{bmatrix}$ OR $\begin{bmatrix}1&0&4&-5\\0&1&3&-2\\0&0&(a-3)(a+3)&3(a+3)\end{bmatrix}\begin{matrix}(1)\\(2)\\(3)\end{matrix}$

Case1 if $a=3$ Row (3) becomes $0 \quad 0 \quad 0 \quad (3-3)(3+3) \quad 3(3+3)$

OR $0 \quad 0 \quad 0 \quad 0 \quad 18$

$\Longrightarrow$ No solution Exists

Case2 if $a=-3$ Row (3) becomes

$0 \quad 0 \quad (-3-3)(-3+3) \quad 3(-3+3)$

or $0 \quad 0 \quad 0 \quad 0$

$\Longrightarrow$ Infinite number of sol.

Case3 if $a\neq 3$ and $a\neq -3$

we can continue with ERO: $R3=\frac{1}{a^2-9}R3$

then $R2=R2+(-3)R3$
$R1=R1+(-4)R3$

to get the RRE and we have a unique solution.