CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	204	All
Examination	**Date**	**Pages**
Midterm	October 2017	2

Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.

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(1) [3 *marks*]
Use the Gauss-Jordan method to find all solutions of the following system:

$$\begin{aligned} u + v - 3w &= 6 \\ 2u - v + 6w &= 7 \\ 3u - 9w &= 9 \end{aligned}$$

(2) [4 *marks*]

(a) Find $\begin{pmatrix} 3 & 4 & -8 \\ 2 & 1 & 8 \\ 5 & 5 & 1 \end{pmatrix}^{-1}$

(b) Solve for X: $\begin{pmatrix} 3 & 4 & -8 \\ 2 & 1 & 8 \\ 5 & 5 & 1 \end{pmatrix}$ X$=$ $\begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{pmatrix}$

(3) [3 *marks*]

Find the determinant $\begin{vmatrix} 6 & -1 & 0 & 4 \\ 3 & 3 & -2 & 0 \\ 0 & 1 & 8 & 6 \\ 2 & 3 & 0 & 4 \end{vmatrix}$

(4) [3 *marks*]
Solve using Cramer's rule:

$$\begin{aligned} x + y + z &= 2 \\ 4x - 8y + 3z &= -2 \\ 2x - 2y + 2z &= 1 \end{aligned}$$

(5) [4 *marks*]

For $A = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 1 & 0 \\ 2 & 7 & 1 \end{pmatrix}$ Compute $a_{11}C_{11} + a_{13}C_{13} - a_{21}C_{21} - a_{31}C_{31}$

(6) [3 *marks*]

Solve the system:

$$\begin{aligned} 2x + y + z &= 1 \\ x - y + 2z &= 5 \\ 3x - 4y - z &= -2 \end{aligned}$$

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1. $u = 4$

$v = 3$

$w = \frac{1}{3}$.

2.
$$A^{-1} = \begin{bmatrix} \frac{39}{5} & \frac{44}{5} & -8 \\ -\frac{38}{5} & -\frac{43}{5} & 8 \\ -1 & -1 & 1 \end{bmatrix}.$$

$$X = \begin{bmatrix} \frac{127}{5} & \frac{4}{5} \\ -\frac{124}{5} & -\frac{3}{5} \\ -3 & 0 \end{bmatrix}.$$

3.

$$\det(A)=\begin{vmatrix} 6 & -1 & 0 & 4 \\ 3 & 3 & -2 & 0 \\ 0 & 1 & 8 & 6 \\ 2 & 3 & 0 & 4 \end{vmatrix} \xrightarrow{4R_2+R_3\to R_3} \begin{vmatrix} 6 & -1 & 0 & 4 \\ 3 & 3 & -2 & 0 \\ 12 & 13 & 0 & 6 \\ 2 & 3 & 0 & 4 \end{vmatrix} \Rightarrow$$

$$\det(A) = (-1)^{(2+3)}(-2)\begin{vmatrix} 6 & -1 & 4 \\ 12 & 13 & 6 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= 2\left[(312-12+144)-(104+108-48)\right]$$

$$= 2[280]$$

$$\det(A) = 560.$$

4. $|A| = -4$

$|A_1| = -1$

$|A_2| = -3$

$|A_3| = -4$

$$x = \frac{|A_1|}{|A|} = \frac{1}{4}$$

$$y = \frac{|A_2|}{|A|} = \frac{-3}{-4} = \frac{3}{4}.$$

$$z = \frac{|A_3|}{|A|} = 1.$$

5. $C_{11} = (-1)^{(1+1)} \begin{vmatrix} 1 & 0 \\ 7 & 1 \end{vmatrix} = 1.$

$C_{13} = (-1)^{(1+3)} \begin{vmatrix} 1 & 1 \\ 2 & 7 \end{vmatrix} = 5.$

$C_{21} = (-1)^{(2+1)} \begin{vmatrix} 3 & 4 \\ 7 & 1 \end{vmatrix} = 25.$

$C_{31} = (-1)^{(3+1)} \begin{vmatrix} 3 & 4 \\ 1 & 0 \end{vmatrix} = -4.$

$\therefore$ $a_{11} C_{11} + a_{13} C_{13} - a_{21} C_{21} - a_{31} C_{31}$

$= (2)(1) + (4)(5) - (1)(25) - (2)(-4)$

$= 2 + 20 - 25 + 8$

$= 5.$

6. $x = -\frac{1}{2}$

$y = -\frac{1}{2}$

$z = \frac{5}{2}.$