Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENTS OF APPLIED MATHEMATICS AND MATHEMATICS

Calculus 1000A Midterm Examination

Friday, October 23, 2015 **Code 111** 7:00 p.m. – 9:00 p.m.

INSTRUCTIONS

1. The first part of the exam (Part A) is MULTIPLE CHOICE. This part is to be answered on the SCANTRON answer sheet. As well, make sure that you circle your selected answer in the examination booklet. The answer you give on the scantron sheet is taken as being your intended choice in the event that the answer in the examination booklet is not the same. The second part (Part B) has questions to be answered in the space provided. Be sure to answer each part of a question in the space provided for that part of the question. INDICATE YOUR ANSWER CLEARLY.
2. SHOW ALL OF YOUR WORK FOR PROBLEMS IN PART B. All results must be justified unless you are instructed otherwise. Unjustified answers will receive little or no credit.
3. Print your name and your instructor's name on the SCANTRON answer sheet. Sign the answer sheet, and mark your student number, section and exam code on the scantron sheet. USE AN HB PENCIL to mark your answers on the scantron answer sheet.
4. DO NOT UNSTAPLE THE BOOKLET.
5. Questions start on Page 1 and continue to Page 12. Questions are printed on both sides of the paper. Should you require extra space for any answer, Pages 13 and 14 are provided for this purpose. BE SURE YOUR BOOKLET IS COMPLETE.
6. CALCULATORS ARE NOT PERMITTED.
7. Leave your scantron sheet on top of your examination booklet when turning them in.
8. Total Marks = 70.
9. Fill in the top of this page. CIRCLE YOUR SECTION NUMBER IN THE LIST BELOW.

001	N. Kiriushcheva (8:30 - 9:30am)	006	J. Turnbull
002	C. Bibby	007	P. Gupta (7 - 9pm)
003	J. Adamus	008	P. Gupta (2:30 - 3:30pm)
004	N. Kiriushcheva (1:30 - 2:30pm)	009	P. Yu
005	A. MacIsaac	011	I. Kobyzev
570	D. Meredith (King's)		

Student Number (**Print**)

Student's Name (**Print**)

FOR GRADING ONLY

PAGE	MARK
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TOTAL	

PART A (36 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET. FOR SAFETY, ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

2 marks A1. The exponential function $f(x) = a^x + b$ whose graph passes through points $(1, 1)$ and $(2, 3)$ is

A: $e^x + 1$	B: $e^x + \ln 3$	C: $2^x - 1$	D: $2^x + 1$	E: $(\frac{1}{2})^x + \frac{1}{2}$

2 marks A2. If $f(x) = e^{\sqrt[3]{x/2}}$, then $f^{-1}(x)$ is

A: $2(\ln x)^3$	B: $3\log_2 x$	C: $2\log_3 x$	D: $3(\ln x)^2$	E: the inverse does not exist

2 marks A3. The domain of the function $f(x) = \cos^{-1}(e^x)$ is

A: $[-1, 1]$	B: $(-\infty, 1]$	C: $[0, \infty)$	D: $(-\infty, \ln(\cos 1)]$	E: $(-\infty, 0]$

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2 marks A4. $\cos(2\arcsin(\frac{1}{3})) =$

A: $\frac{2}{3}$	B: $\frac{3}{2}$	C: $\frac{2}{9}$	D: $\frac{7}{9}$	E: $\frac{2\sqrt{2}}{3}$

2 marks A5. If $x = \log_3(\log_2 8)$, $y = 2\arccos(1)$, and $z = \log_4(\log_3 9)$, then

A: $x < y < z$	B: $y < z < x$	C: $z < x < y$	D: $x < z < y$	E: $y < x < z$

2 marks A6. Which one of the following functions has an inverse?

A: $f(x) = 2015$	B: $g(x) = 2015^{x^2}$	C: $h(x) = \log_{2015}(x^3)$	D: $k(x) = \sqrt[3]{x^2+3}$
E: none of the above			

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2 marks A7. If $\lim_{x\to 4^+} \dfrac{f(x)+3}{x-4} = e^{\pi}$, then $\lim_{x\to 4^+} f(x) =$

A: -4	B: -3	C: 0	D: $\frac{3e^{\pi}}{4}$	E: $e^{\pi}-3$

2 marks A8. For the graph of the function $f(x) = \dfrac{x^2-x}{x^2+x-2}$, which of the following statements is true?

A: The graph has two vertical asymptotes and one horizontal asymptote
B: The line $x = 1$ is a vertical asymptote
C: The line $y = 2$ is a horizontal asymptote
D: The graph has one vertical asymptote and one horizontal asymptote
E: None of the above is true

2 marks A9. $\lim_{x\to -\infty} \arctan((\frac{1}{2})^{2x}) =$

A: $-\pi/2$	B: $-1/2$	C: 0	D: $\pi/2$	E: ∞

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2 marks A10. $\lim_{x\to 0^+} \dfrac{\sin(4x)}{\sqrt{x}\sin(2\sqrt{x})} =$

A: 4	B: 2	C: $\sqrt{2}$	D: 0	E: the limit does not exist

2 marks A11. $\lim_{x\to 0} \dfrac{\sqrt{1-x^2}-\sqrt{1+x^2}}{4x^2} =$

A: $-\infty$	B: $-\frac{1}{2}$	C: $-\frac{1}{4}$	D: 0	E: $\frac{1}{2}$

2 marks A12. $\lim_{x\to -\infty} \dfrac{x^3(4-x^{1/3})}{5x^3-3x^2+1} =$

A: $-\infty$	B: 0	C: $\frac{1}{5}$	D: $\frac{4}{5}$	E: ∞

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2 marks A13. $\dfrac{d}{dx}\, e^{\arctan(\sqrt{2})} =$

A: $-\frac{\pi}{2}$	B: $-\sqrt{2}$	C: 0	D: $\sqrt{2}$	E: $\tan\sqrt{2}$

2 marks A14. If $x^4 + y^4 = 4$, then $\dfrac{dy}{dx} =$

A: $-\frac{x^3}{y^3}$	B: $-\frac{y}{x}$	C: 0	D: $\sqrt[4]{4-x^4}$	E: 1

2 marks A15. $\dfrac{d}{dx}\cos(3^x) =$

A: $\sin(3^x)$	B: $3^x \cdot \cos(3^x)$	C: $-3\sin(3^x)$	D: $3^x \cdot \cos(3^x) \cdot \ln 3$
E: $-3^x \cdot \sin(3^x) \cdot \ln 3$			

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2 marks A16. The slope of the tangent line to the curve $xy = e^{x^2}$ at the point $(1, e)$ is

A: e^{-1}	B: $\frac{1}{2}$	C: 0	D: $\sqrt{e}$	E: e

2 marks A17. If f is continuous and one-to-one on the interval $[-1, 1]$, $f(0) = 2015$, and $f^{-1}(-1) = 1$, then which one of the following statements is necessarily true?

A: f has only negative values on the interval $[\frac{1}{2}, 1]$
B: f is differentiable on $(-1, 1)$
C: f is differentiable on $(0, 1)$
D: There exists at least one x between -1 and 0 such that $f(x) = 0$
E: There exists precisely one x between 0 and 1 such that $f(x) = 0$

2 marks A18. If $f(x) = 2x - 3$, and g is a function such that $g''(0) = 4$ and $(fg)''(0) = -4$, then $g'(0) =$

A: -4	B: -2	C: 0	D: 2	E: 4

PART B (34 marks)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B19. Use the Squeeze Theorem to find $\lim_{x\to 0}\left(x^2\sin(\frac{25}{7x})\right)$. Justify your answer.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B20. Use the Intermediate Value Theorem to show that the equation

$$\sin\left(\frac{\pi x}{2}\right) = e^{\cos(\pi x)}$$

has a solution in the interval $(0, 1)$. Justify your answer.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

B21. Let

$$f(x) = \begin{cases} \frac{\sin(3x)}{x}\,, & x > 0 \\ A\,, & x = 0 \\ \frac{5-\sqrt{25-x}}{Cx}\,, & x < 0\,. \end{cases}$$

4 marks (a) Find $\lim\limits_{x\to 0^-} f(x)$ and $\lim\limits_{x\to 0^+} f(x)$.

3 marks (b) For what values of A and C is the function f continuous at 0? Justify your answer.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B22. Let $f(x) = \arcsin(e^{7/x})$. Find $f'(x)$.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

6 marks B23. Find the value of C for which the tangent line to the graph of $f(x) = 2e^{Cx}$ at the point $(0, 2)$ coincides with the tangent line to the curve $(x+1)^2 + (y-3)^2 = 2$ at the same point. Justify your answer.

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B24. Let $f(x) = \sin\left(\pi \cdot \dfrac{2 - e^x}{2 + e^x}\right)$. Find $f'(x)$.

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