MAT1330 C – Instructor: C. ~~Rada~~ PÜNISHER
Wednesday, October 3, 2019 : Midterm Test 1

Duration: 75 minutes

Family name: – SOL –

First name:

Student number :

Faculté des sciences
Mathématiques et statistique

Faculty of Science
Mathematics and Statistics

613–562–5864
613–562–5776
www.uOttawa.ca
585 King Edward
Ottawa
ON K1N 6N5

Please read the following instructions carefully.

- You have 75 minutes to complete this exam.
- This is a closed book exam. Except for Faculty-approved calculators (models: Texas Instruments TI-30* and TI-34*, Casio FX-260* and Casio FX-300*), no electronics, no notes, cell phones, smartwatches or related devices of any kind are permitted. All such devices, including cell phones, **must be stored in your bag under your desk or at the front or back of the room for the duration of the exam.**
- Read each question carefully — you will save yourself time and grief later on.
- Questions 1 through 6 are multiple choice, worth a total of 6 points. **Record your answers to the multiple choice questions in the boxes provided.**
- Questions 7–10 are long answer, with number of points as indicated. **You must show your work, your work must be legible and well-justified, and you must record your answers in the spaces provided.**
- Where it is possible to check your work, do so.
- Good lück!

Marker's use only:

Question	Marks
1–6 (/6)	
7 (/2)	
8 (/2)	
9 (/5)	
10 (/5)	
Total (/20)	

uOttawa

1. (1 point) What is the domain of the function $f(x) = \dfrac{\ln(100 - x^2)}{\sqrt{1-x}}$?

A. all x **B.** $0 < x \leq 10$ **C.** $\boxed{-10 < x < 1}$

D. $1 < x < 10$ **E.** $-10 < x < 10$ **F.** $x < -10$

Your answer: C

- $1 - x > 0 \Rightarrow 1 > x$
- $100 - x^2 > 0 \Rightarrow (10-x)(10+x) > 0 \Rightarrow$

$-10 < x < 10$

So: $-10 < x < 1$

2. (1 point) Which of the following functions is the inverse of $f(x) = \dfrac{x-2}{x+5}$?

A. $f^{-1}(x) = \dfrac{2x+5}{1-x}$ **B.** $f^{-1}(x) = \dfrac{2x-5}{1-x}$ **C.** $f^{-1}(x) = \dfrac{2x+5}{1-2x}$

D. $f^{-1}(x) = \dfrac{5x+2}{-x+1}$ **E.** $f^{-1}(x) = \dfrac{2x-1}{1-5x}$ **F.** None of the previous answers.

Your answer: D

$y = \dfrac{x-2}{x+5} \Rightarrow yx + 5y = x - 2 \Rightarrow$

$x[y-1] = -5y-2 \Rightarrow x = \dfrac{-5y-2}{y-1} \Rightarrow$

$f^{-1}(x) = \dfrac{5x+2}{1-x}$ D

3. (1 point) Which of the following is equal to $\ln(\sqrt{x^3+7x}) - \frac{1}{2}\ln(x)$ on its domain?

A. $\ln(\sqrt{x^2+7})$ **B.** $\sqrt{\ln(x+7)}$ **C.** $\ln(x+7)$

D. $\ln(7)+\ln(\sqrt{x^2})$ **E.** $\ln(7x)+\ln(x^2)$ **F.** $\ln(x^2+7)$

Your answer: A

$\ln\sqrt{x^3+7x} - \ln\sqrt{x} =$

$\ln\left(\frac{\sqrt{x^3+7x}}{\sqrt{x}}\right) = \ln\sqrt{x^2+\frac{7x}{x}} = \ln\sqrt{x^2+7}$

4. (1 point) Which of the following expressions is equal to $\dfrac{(x^{80}+80)^{1/3}}{x^2}$ for $x \neq 0$?

A. $\left(x^{78}+\dfrac{80}{x^2}\right)$ **B.** $\sqrt[3]{x^{74}+\dfrac{80}{x^6}}$ **C.** $\sqrt[3]{x^{74}+\dfrac{80}{x^2}}$

D. $x^{74}+\dfrac{\sqrt[3]{80}}{x^2}$ **E.** $\left(x^{74}+\dfrac{80}{x^5}\right)$ **F.** $\sqrt[3]{x^{76}+80}$

Your answer: B

$$\frac{\sqrt[3]{x^{80}+80}}{\sqrt[3]{x^6}} = \sqrt[3]{x^{74}+\frac{80}{x^6}}$$

5. (1 point) Given that a is a positive constant, what is the value of the following limit?

$$\lim_{x \to a} \frac{a^2 - x^2}{\sqrt{x} - \sqrt{a}}$$

A. $-4a^2\sqrt{a}$ **B.** ∞ **C.** 0 **D.** $4a$ **E.** $-2\sqrt{a}$ **F.** $-4a\sqrt{a}$

Your answer: F

$$\lim_{x \to a} \frac{(a-x)(a+x)}{\sqrt{x}-\sqrt{a}} = \lim_{x \to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})(a+x)}{-(\sqrt{a}-\sqrt{x})}$$

$$= \lim_{x \to a} -\frac{(\sqrt{a}+\sqrt{x})(a+x)}{1} = -\frac{(2\sqrt{a})(2a)}{1}$$

$$= \boxed{-4a\sqrt{a}}$$

6. (1 point) A study on a particular forest has tracked the forest's area over several decades. Over the course of this study, scientists observed that the forest's area had a natural growth rate of 6% each decade. Furthermore, an average of 20 km^2 of the forest was cut down each decade. If S_t represents the forest's area t decades after the study began (measured in km^2), which of the following DTDS models the dynamics of this forest?

A. $S_{t+1} = 0.94S_t - 20$ C. $S_{t+1} = 1.06S_t - 10$ E. $S_{t+1} = 1.06S_t - 20$

B. $S_{t+1} = 1.06S_t + 20$ D. $S_{t+1} = 0.06S_t + 30$ F. $S_{t+1} = 0.94S_t + 20$

Your answer:

Long-answer questions: You must show your work, your work must be legible and well-justified, and you must record your answers in the spaces provided.

7. (2 points) Evaluate the following limit, if it exists. **You must use algebraic methods and justify your answer mathematically to earn credit for this question.**

Left LiM :

$$\lim_{x\to 4}\frac{x^2-6x+8}{|x-4|}$$

$$\lim_{x\to 4^-}\frac{(x-4)(x-2)}{-(x-4)} = \lim_{x\to 4^-} -\frac{x-2}{1} = -\frac{4-2}{1} = -2$$

Right lim :

$$\lim_{x\to 4^+}\frac{(x-4)(x-2)}{(x-4)} = \lim_{x\to 4}(x-2) = 4-2 = 2$$

since $2 \neq -2$

$\nexists$

Your answer: DNE

8. **(2 points)** Find all solutions of the following inequality. **Show your work and justify your reasoning.**

$$\frac{x-5}{x+3} < \frac{x+2}{x}$$

$$0 < \frac{x+2}{x} - \frac{x-5}{x+3} = \frac{(x+2)(x+3) - x(x-5)}{x(x+3)} \Rightarrow$$

$$0 < \frac{x^2+5x+6-x^2+5x}{x(x+3)} = \frac{10x+6}{x(x+3)} = \frac{10\left(x+\frac{6}{10}\right)}{x(x+3)} = E$$

x	$-\infty$	-3	$-\frac{6}{10}$	0	∞
x	$-\ -\ -$	$-$	$-\ -$	$+\ +\ +\ +$	
$x+3$	$-\ -\ -$	$+\ +\ +$	$+\ +$	$+\ +\ +\ +$	
$x+\frac{6}{10}$	$-\ -\ -$	$-\ -$	$0\ +\ +$	$+\ +\ +$	
E	$-$	$+$	$0\ -$	$+$	

Your answer: $\left(-3, -\frac{6}{10}\right) \cup (0, \infty)$

9. (1+1+2+1=5 points) The DTDS governing the growth of a population's density is given by

$$x_{t+1} = 0.9x_t + 9$$

where t is in years, and x_t denotes the density (in number of individuals per km^2) at time t. In this model, a repopulation strategy has been implemented to ensure that the population's density increases by $c = 9$ individuals per km^2 each year. The growth rate factor 0.9 was determined following several years of study.

(a) Find the fixed point x^* of this DTDS. Show your work. $r = 0.9$; $c = 9$

Fixed point: $\boxed{x^* = \frac{c}{1-r} = \frac{9}{1-0.9} = 90}$

(b) Give the general solution to this DTDS if the initial density is $x_0 = 20$ individuals/km^2.

General solution: $\boxed{x_t} = \boxed{(0.9)^t\left[20-90\right] + 90}$

(c) If the initial density is $x_0 = 20$ individuals/km^2, find the number of whole years it will take for the density to reach at least 40 individuals/km^2. Solve an appropriate **inequality** below.

$(0.9)^t(-70) + 90 > 40 \Rightarrow (0.9)^t(-70) > -50 \Rightarrow (0.9)^t < \frac{5}{7}$

$\rightarrow \ln(0.9)^t < \ln\left(\frac{5}{7}\right) \Rightarrow t \cdot \ln(0.9) < \ln\left(\frac{5}{7}\right) \Rightarrow$

$t > \frac{\ln\left(\frac{5}{7}\right)}{\ln(0.9)} \Rightarrow t > 3.19$

Minimum number of whole years required: $\boxed{4}$

(d) If we would like this population's fixed point to be $x^* = 100$ individuals/km^2 (instead of what we found in part (a)), by what density c (in individuals per km^2) should the repopulation strategy aim to increase this population's density each year?

$\frac{c}{1-r} = 100$

$c = 100\,(1-0.9)$

$c = 100\,(0.1) \Rightarrow c = 10$

$c = \boxed{10}$

10. (0.5+ 1.5 +2+1=5 points) Consider the following non-linear DTDS: $x_{t+1} = \dfrac{5x_t^2}{2x_t + 6}$

(a) Give the updating function for this DTDS: $f(x) = \boxed{\dfrac{5x^2}{2x+6}}$

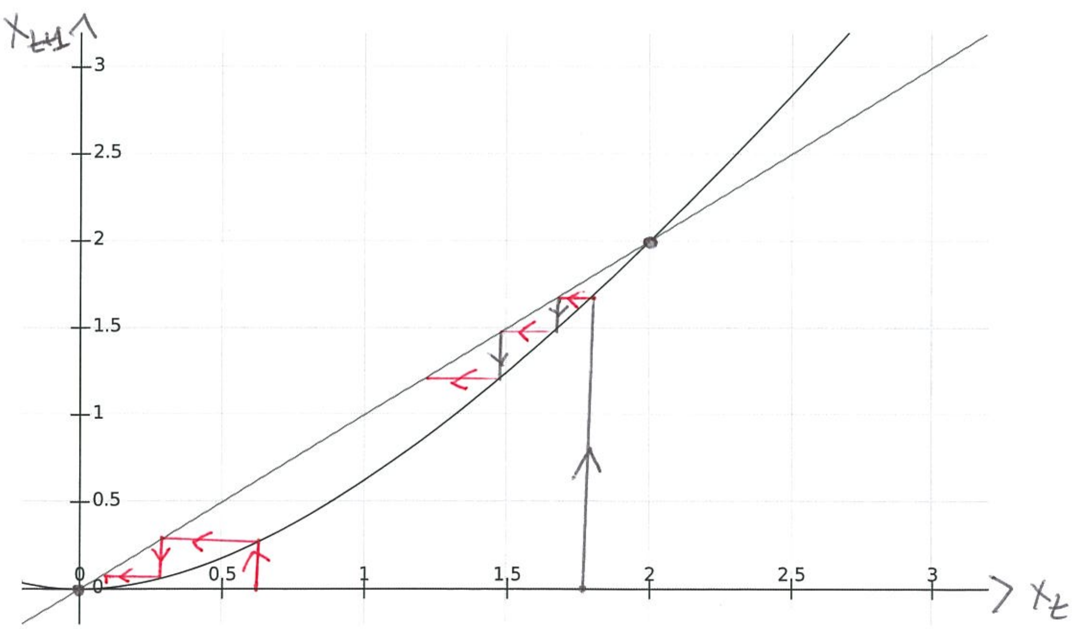
(b) Find, algebraically, all fixed points x^* for this DTDS. Show your work!

$\frac{x}{1} = \frac{5x^2}{2x+6} \Rightarrow 5x^2 = x(2x+6) \Rightarrow x[5x-2x-6]=0$

$\Rightarrow x(3x-6)=0 \rightarrow \begin{cases} x_1^*=0 \\ x_2^* = \frac{6}{3} = 2 \end{cases}$

Fixed points: $\boxed{0; 2}$

(c) The graph of $y = f(x)$ is drawn below. **Label the axes**, identify the **fixed points** of the DTDS on the graph, then **draw a cobweb** for at least three iterations, starting with an initial population density of $x_0 = 1.75$. *Use your student card as a straight edge.*



(d) Which of the fixed points of this DTDS are **stable** ? Briefly justify your answer for each fixed point below.

0 = STABLE : SOLUTIONS starting CLOSE TO 0, move closer to 0

2 = Unstable: SOLUTIONS starting close to 2, MOVE AWAY from 2.

MAT1330 C – Instructor: C. Rada
Wednesday, October 3, 2019 : MIDTERM TEST 1

Duration: 75 minutes

Family name: ________________________________

First name: ________________ HA-HA

Student number : ________________________________

Faculté des sciences
Mathématiques et statistique

Faculty of Science
Mathematics and Statistics

613–562–5864
613–562–5776
www.uOttawa.ca

585 King Edward
Ottawa
ON K1N 6N5

Please read the following instructions carefully.

- You have 75 minutes to complete this exam.
- This is a closed book exam. Except for Faculty-approved calculators (models: Texas Instruments TI-30* and TI-34*, Casio FX-260* and Casio FX-300*), no electronics, no notes, cell phones, smartwatches or related devices of any kind are permitted. All such devices, including cell phones, **must be stored in your bag under your desk or at the front or back of the room for the duration of the exam.**
- Read each question carefully — you will save yourself time and grief later on.
- Questions 1 through 6 are multiple choice, worth a total of 6 points. **Record your answers to the multiple choice questions in the boxes provided.**
- Questions 7–10 are long answer, with number of points as indicated. **You must show your work, your work must be legible and well-justified, and you must record your answers in the spaces provided.**
- Where it is possible to check your work, do so.
- Good luck!

Marker's use only:

Question	Marks
1–6 (/6)	
7 (/2)	
8 (/2)	
9 (/5)	
10 (/5)	
Total (/20)	

uOttawa

1. (1 point) What is the domain of the function $f(x) = \dfrac{\ln(64 - x^2)}{\sqrt{2 - x}}$?

A. all x **B.** $0 < x \leq 2$ **C.** $-8 < x < 2$

D. $-8 < x < 8$ **E.** $2 < x < 8$ **F.** $x < 2$

Your answer: C

- $2 - x > 0 \Rightarrow 2 > x$
- $64 - x^2 > 0 \Rightarrow (8-x)(8+x) > 0 \Leftrightarrow -8 < x < 8$

so: $-8 < x < 2$

2. (1 point) Which of the following functions is the inverse of $f(x) = \dfrac{3 + x}{x - 6}$?

A. $f^{-1}(x) = \dfrac{6x + 3}{-1 + x}$ **B.** $f^{-1}(x) = \dfrac{6x}{1 - x}$ **C.** $f^{-1}(x) = \dfrac{6x + 3}{1 - x}$

D. $f^{-1}(x) = \dfrac{6x - 3}{x - 1}$ **E.** $f^{-1}(x) = \dfrac{6x - 1}{1 - x}$ **F.** None of the previous answers.

Your answer: A

$y = f(x) \rightarrow y = \dfrac{3+x}{x-6} \Rightarrow yx - 6y = 3 + x$

$\Rightarrow x(y-1) = 6y + 3 \Rightarrow x = \dfrac{6y+3}{y-1}$

$f^{-1}(x) = \dfrac{6x+3}{x-1}$

3. (1 point) Which of the following is equal to $\ln(\sqrt{x^9+9x}) - \frac{1}{2}\ln(x)$ on its domain?

A. $\sqrt{\ln(x^8+9)}$ **B.** $\ln(\sqrt{x^8+9})$ **C.** $\ln(x^8+9)$

D. $\ln(9)+\ln(x^8)$ **E.** $\ln(9)+\ln(\sqrt{x^8})$ **F.** $\ln(x+9)$

Your answer: B

$\ln\sqrt{x^9+9x} - \ln\sqrt{x}$

$\ln\left(\frac{\sqrt{x^9+9x}}{\sqrt{x}}\right)$

$\ln\left(\sqrt{x^8+9}\right)$

4. (1 point) Which of the following expressions is equal to $\frac{(x^{18}+7)^{1/3}}{x^2}$ for $x \neq 0$?

A. $\sqrt[3]{x^{12}+7}$ **B.** $\left(x^{12}+\frac{7}{x^5}\right)$ **C.** $x^{12}+\frac{\sqrt[3]{7}}{x^2}$

D. $\sqrt[3]{x^{12}+\frac{7}{x^6}}$ **E.** $\sqrt[3]{x^{16}+\frac{7}{x^6}}$ **F.** $\left(x^{12}+\frac{7}{x^2}\right)$

Your answer: D

$$\frac{\sqrt[3]{x^{18}+7}}{\sqrt[3]{x^6}} = \sqrt[3]{x^{12}+\frac{7}{x^6}}$$

5. (1 point) Given that a is a positive constant, what is the value of the following limit?

$$\lim_{x \to a} \frac{\sqrt{a} - \sqrt{x}}{x^2 - a^2}$$

A. $\frac{-1}{4a\sqrt{a}}$ **B.** $-4a^2\sqrt{a}$ **C.** $-4a$ **D.** $\frac{-1}{\sqrt{a}}$ **E.** 0 **F.** ∞

Your answer: A

$$\lim_{x \to a} \frac{\sqrt{a} - \sqrt{x}}{(x-a)(x+a)} =$$

$$\lim_{x \to a} \frac{(\sqrt{a} - \sqrt{x})}{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})(x+a)} = \lim_{x \to a} \frac{-1}{(\sqrt{x} + \sqrt{a})(x+a)}$$

$$= \frac{-1}{(2\sqrt{a}) \cdot 2a} = \frac{-1}{4a\sqrt{a}}$$

6. (1 point) A study on a particular forest has tracked the forest's area over several decades. Over the course of this study, scientists observed that the forest's area had a natural growth rate of 5% each decade. Furthermore, an average of 30 km^2 of the forest was cut down each decade. If S_t represents the forest's area t decades after the study began (measured in km^2), which of the following DTDS models the dynamics of this forest?

A. $S_{t+1} = 0.95S_t - 40$ C. $S_{t+1} = 1.05S_t - 40$ E. $S_{t+1} = 1.05S_t - 30$

B. $S_{t+1} = 1.05S_t + 30$ D. $S_{t+1} = 0.05S_t + 30$ F. $S_{t+1} = 0.95S_t + 40$

Your answer: E

Long-answer questions: You must show your work, your work must be legible and well-justified, and you must record your answers in the spaces provided.

7. (2 points) Evaluate the following limit, if it exists. **You must use algebraic methods and justify your answer mathematically to earn credit for this question.**

$$\lim_{x\to 5} \frac{x^2+2x-35}{|x-5|}$$

Left lim :

$$\lim_{x\to 5^-} \frac{(x-5)(x+7)}{-(x-5)} = \lim_{x\to 5^-} -(x+7) = -\lim_{x\to 5^-} x+7 = -(12)$$

Right Lim :

$$\lim_{x\to 5^+} \frac{(x-5)(x+7)}{+(x-5)} = \lim_{x\to 5^+} x+7 = 5+7 = 12$$

Since $12 \neq -12$

$$\Downarrow$$

Your answer: D.N.E

8. **(2 points)** Find all solutions of the following inequality. **Show your work and justify your reasoning.**

$$\frac{x-7}{x+4} < \frac{x+2}{x}$$

$$0 < \frac{x+2}{x} - \frac{x-7}{x+4} = \frac{x^2+6x+8 - x^2+7x}{x(x+4)} = \frac{13x+8}{x(x+4)}$$

$\cancel{x}$	$-\infty$	-4	$-\frac{8}{13}$	0	∞
x	$-$ $-$	$-$ $-$ $-$ $-$		$+$ $+$ $+$ $+$ $+$	
$x+4$	$-$ $-$ $-$ $-$	$+$ $+$ $+$ $+$ $+$		$+$ $+$ $+$ $+$	
$13x+8$	$-$ $-$ $-$	$-$ $-$ 0 $+$ $+$ $+$		$+$ $+$ $+$	
$\mathcal{E}$	$-$ $-$	$+$ 0 $-$		$+$	

Your answer: $\left(-4, -\frac{8}{13}\right) \cup (0, \infty)$

9. (1+1+2+1=5 points) The DTDS governing the growth of a population's density is given by

$$x_{t+1} = 0.8x_t + 8$$

where t is in years, and x_t denotes the density (in number of individuals per km^2) at time t. In this model, a repopulation strategy has been implemented to ensure that the population's density increases by $c = 8$ individuals per km^2 each year. The growth rate factor 0.8 was determined following several years of study.

(a) Find the fixed point x^* of this DTDS. Show your work.

$r = 0.8$

$c = 8$

Fixed point: $\boxed{\frac{c}{1-r} = \frac{8}{1-0.8} = 40}$

(b) Give the general solution to this DTDS if the initial density is $x_0 = 16$ individuals/km^2.

General solution: $\boxed{x_t} = \boxed{(0.8)^t(16-40) + 40}$

(c) If the initial density is $x_0 = 16$ individuals/km^2, find the number of whole years it will take for the density to reach at least 30 individuals/km^2. Solve an appropriate **inequality** below.

$(0.8)^t(-24) + 40 > 30 \Rightarrow (0.8)^t(-24) > -10 \Rightarrow$

$(0.8)^t < \frac{10}{24} \Rightarrow \ln(0.8)^t < \ln\left(\frac{5}{12}\right) \Rightarrow t\ln(0.8) < \ln\left(\frac{5}{12}\right)$

$\Rightarrow t > \frac{\ln(5/12)}{\ln(0.8)} \Rightarrow t > 3.92$

Minimum number of whole years required: $\boxed{4}$

(d) If we would like this population's fixed point to be $x^* = 50$ individuals/km^2 (instead of what we found in part (a)), by what density c (in individuals per km^2) should the repopulation strategy aim to increase this population's density each year?

$\frac{c}{1-r} = 50 \Rightarrow$

$c = 50(1-0.8) \Rightarrow$

$c = \boxed{10}$

10. (0.5+ 1.5 +2+1=5 points) Consider the following non-linear DTDS: $x_{t+1} = \dfrac{5x_t^2}{4+4x_t}$

(a) Give the updating function for this DTDS: $f(x) = \boxed{\dfrac{5x^2}{4+4x}}$

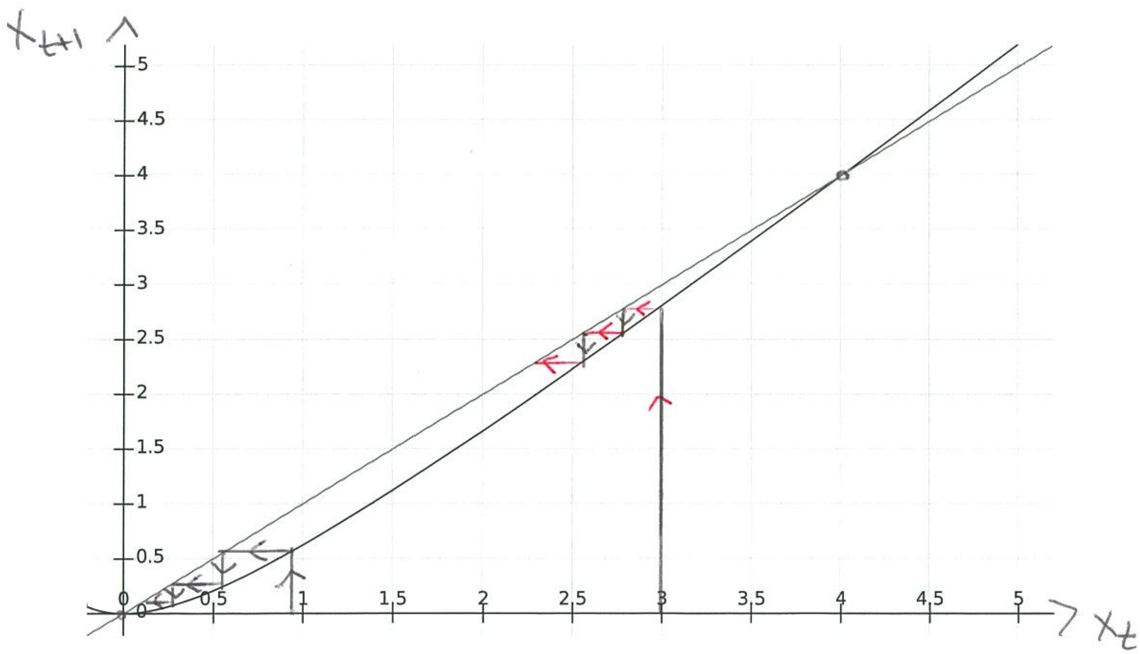
(b) Find, algebraically, all fixed points x^* for this DTDS. Show your work!

$\frac{x}{1} = \frac{5x^2}{4+4x} \Rightarrow 5x^2 = x(4+4x) \Rightarrow x[5x-4-4x]=0$

$x(x-4)=0 \Rightarrow \begin{cases} x_1^* = 0 \\ x_2^* = 4 \end{cases}$

Fixed points: $\boxed{0, 4}$

(c) The graph of $y = f(x)$ is drawn below. **Label the axes**, identify the **fixed points** of the DTDS on the graph, then **draw a cobweb** for at least three iterations, starting with an initial population density of $x_0 = 3$. *Use your student card as a straight edge.*



(d) Which of the fixed points of this DTDS are **stable** ? Briefly justify your answer for each fixed point below.

0=Stable : Solutions starting close to 0, move closer to 0

4=Unstable: Solutions starting close to 2, move AWAY from 4.