MATH 1300 A & B-MIDTERM # 1 Fall-2015

Professors: Richard Blute and Termeh Kousha

Last Name: ____________________ **First Name:** ____________________

ID# Solutions, V1

INSTRUCTIONS: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 50 points.
Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

You must read this and sign below. Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam or an **F** in the course.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

SIGN HERE:____________________________

For long answer questions, YOU MUST SHOW YOUR WORK.

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

Multiple Choice Answers:

#1	#2	#3	#4

Multiple Choice Questions (1-4)

Question 1 *Solve the following logarithmic equation.*

$$\ln(5x) - \ln(x+1) = -1$$

A) $\frac{5e+1}{e}$ **B)** $\frac{5}{e-1}$ **C)** $\frac{e+1}{5e-1}$ **D)** $\frac{1}{5e-1}$ **E)** $\frac{5}{5e+1}$

$\ln(5x) - \ln(x+1) = \ln\left(\frac{5x}{x+1}\right)$

So $\frac{5x}{x+1} = e^{-1}$ (using $e^{\ln(A)} = A$)

or $5x = xe^{-1} + e^{-1}$ $\Rightarrow$ $5ex - x = 1$ (multiply by e)

$5x - xe^{-1} = e^{-1}$ $\quad x = \frac{1}{5e-1}$

Question 2 *Find a and b so that $f(x)$ is continuous everywhere.*

$$f(x) = \begin{cases} ax+b & x \le 0 \\ x^2 - 2 & ;0 < x \le 1 \\ bx + a & ;x > 1 \end{cases}$$

A) $a = 1$ and $b = 3$

B) $a = 1$ and $b = -1$

C) $a = b = -2$

D) $a = 1$ and $b = -2$

E) $a = -3$ and $b = -2$

F) None of the answers above.

$$\left.\begin{aligned} \lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-} ax+b = b \\ \lim_{x\to 0^+} f(x) &= \lim_{x\to 0^-} x^2-2 = -2. \end{aligned}\right\} \Rightarrow b = -2$$

$$\left.\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} x^2-2 = -1 \\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} -2x+a = -2+a \end{aligned}\right\} \Rightarrow a = 1$$

Question 3 *Find the slope of the tangent line to the graph* $y = -6\sqrt{8x+1}$ *when* $x = 1$.

A) 2 **B)** -6 **C)** -4 **D)** -8 **E)** -1

Question 4 *Find the following limit.*

$$\lim_{x\to 3} \frac{x^2 - 2x - 3}{x^2 - 5x + 6}.$$

A) 1 **B)** -1 **C)** 4 **D)** 2 **E)** The limit does NOT exist.

Q3 $f(x) = -6(8x+1)^{1/2} \Rightarrow f'(x) = (-6)\left(\frac{1}{2}\right)(8x+1)^{-1/2} \cdot 8$

$\Rightarrow f'(1) = -3\left(\frac{1}{3}\right)(8) = -8$

Q4

$$\lim_{x\to 3} \frac{x^2-2x-3}{x^2-5x+6} = \lim_{x\to 3} \frac{(x-3)(x+1)}{(x-3)(x-2)}$$

$$= \lim_{x\to 3} \frac{x+1}{x-2} = \frac{3+1}{3-2} = 4$$

Long Answer Questions (5-7)

Question 5 (10 points)

- *Using* **only** *the definition of derivative as a limit, calculate $f'(x)$ where*

$$f(x) = \frac{3}{2-x}.$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{3}{2-(x+\Delta x)} - \frac{3}{2-x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{3(2-x) - 3(2-(x+\Delta x))}{[2-(x+\Delta x)][2-x]}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{3\Delta x}{(2-(x+\Delta x))(2-x)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{3}{[2-(x+\Delta x)][2-x]}$$

$$= \frac{3}{(2-x)^2}$$

- *Find the equation of the tangent line to the curve $f(x) = x\ln(x-4)$ at $x = 5$.*

$$f'(x) = \ln(x-4) + \frac{x}{x-4}$$

When $x=5$, $f(x)=0$

So $(5,0)$ is on the line.

$$f'(5) = \ln(1) + 5 = 5$$

So line equation is

$y = mx+b = 5x+b$. Plug in $(5,0)$

$b = -25$

$$\boxed{y = 5x-25}$$

Question 6 (10 points) For the following, you do not need to simplify your answers

- *Suppose* 4,000 *dollars is invested at a rate of* 2 *percent, compounded* 6 *times per year. How much money is in the account after* 4 *years?*
- *Suppose* 1,000 *dollars is invested in an account that compounds continuously. The interest is unknown, but you do know that the money doubled in* 5 *years. What must the interest rate have been?*

- $P(t) = P_0\left[1+\frac{r}{n}\right]^{nt}$

$$= 4000\left[1+\frac{.02}{6}\right]^{24}$$

- $P(t) = P_0 e^{rt}$

$$= 1000\, e^{rt}$$

We know

$$2000 = 1000 e^{5r}$$

So $2 = e^{5r}$

$$\ln(2) = 5r$$

$$r = \frac{\ln(2)}{5}$$

Question 7 (10 points) *Suppose that x and y are related by the equation*

$$e^{1-x} + (x-1)y^2 = 9 - y^3$$

- *Use implicit differentiation to find $\frac{dy}{dx}$ at the point (1,2).*

$$\frac{d\left[e^{1-x} + (x-1)y^2 = 9-y^3\right]}{dx}$$

$$e^{1-x}(-1) + y^2 + (x-1)\left(2y\frac{dy}{dx}\right) = -3y^2\frac{dy}{dx}$$

Plug in (1,2)

$$-1 + 4 + 0 = -12\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3}{-12} = \frac{-1}{4}$$

MATH 1300 A & B-MIDTERM # 1 Fall-2015

Professors: Richard Blute and Termeh Kousha

Last Name: ____________________ **First Name:** ____________________

ID# Solutions, V2

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 50 points.
Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

You must read this and sign below. Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam or an **F** in the course.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

SIGN HERE: ______________________________

For long answer questions, YOU MUST SHOW YOUR WORK.

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

Multiple Choice Answers:

#1	#2	#3	#4

Multiple Choice Questions (1-4)

Question 1 *Solve the following logarithmic equation.*

$$\ln(2x) - \ln(x+2) = -1$$

A) $\frac{2e+1}{e}$ **B)** $\frac{2}{2e-1}$ **C)** $\frac{e+1}{2e-1}$ **D)** $\frac{1}{2e+1}$ **E)** $\frac{4}{2e+1}$

$\ln\left(\frac{2x}{x+2}\right) = -1$

$\frac{2x}{x+2} = \frac{1}{e}$

$2xe = x+2$

$x(2e-1) = 2$

$x = \frac{2}{2e-1}$

Question 2 *Find a and b so that $f(x)$ is continuous everywhere.*

$$f(x) = \begin{cases} ax+b & x \le 0 \\ x^2+4 & ;0 < x \le 1 \\ bx+a & ;x > 1 \end{cases}$$

A) $a = 1$ and $b = 4$

B) $a = 1$ and $b = -1$

C) $a = b = 4$

D) $a = -2$ and $b = 3$

E) $a = -1$ and $b = 4$

F) None of the answers above.

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} ax+b = b$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2+4 = 4$

So $b = 4$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2+4 = 5$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 4x+a = 4+a$. So $a = 1$

Question 3 *Find the slope of the tangent line to the graph* $y = -6\sqrt{4x+1}$ *when* $x = 2$.

A) 2 **B)** -6 **C)** -4 **D)** -8 **E)** -1

Question 4 *Find the following limit.*

$$\lim_{x\to 3} \frac{x^2 - x - 6}{x^2 - 5x + 6}.$$

A) 1 **B)** -1 **C)** 4 **D)** 5 **E)** The limit does NOT exist.

$f(x) = -6(4x+1)^{1/2} \Rightarrow f'(x) = -6\left(\frac{1}{2}\right)(4x+1)^{-1/2}(4)$

So $f'(2) = -6\left(\frac{1}{2}\right)(9)^{-1/2}(4) = -4$

$\lim_{x\to 3} \frac{x^2-x-6}{x^2-5x+6} = \lim_{x\to 3} \frac{(x-3)(x+2)}{(x-3)(x-2)} = \lim_{x\to 3} \frac{x+2}{x-2} = 5$

Long Answer Questions (5-7)

Question 5 (10 points)

- *Using* **only** *the definition of derivative as a limit, calculate $f'(x)$ where*

$$f(x) = \frac{2}{1-x}.$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{2}{1-(x+\Delta x)} - \frac{2}{1-x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{2(1-x) - 2(1-(x+\Delta x))}{[1-(x+\Delta x)][1-x]}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{2\Delta x}{[1-(x+\Delta x)][1-x]}}{\Delta x} = \lim_{\Delta x \to 0} \frac{2}{[1-(x+\Delta x)][1-x]}$$

$$= \frac{2}{(1-x)^2}$$

- *Find the equation of the tangent line to the curve $f(x) = x\ln(x-3)$ at $x = 4$.*

$$f'(x) = \ln(x-3) + \frac{x}{x-3}$$

$f(4) = 0$

So $(4,0)$ is on the line

$f'(4) = 4.$

So $y = mx+b = 4x+b$ Plug in $(4,0)$

$0 = 4\cdot 4 + b \Rightarrow b = -16$

$y = 4x - 16$

Question 6 (10 points) For the following, you do not need to simplify your answers

- *Suppose* 2,000 *dollars is invested at a rate of* 3 *percent, compounded* 3 *times per year. How much money is in the account after* 4 *years?*
- *Suppose* 2,000 *dollars is invested in an account that compounds continuously. The interest is unknown, but you do know that the money doubled in* 4 *years. What must the interest rate have been?*

- $P(t) = P_0\left[1+\frac{r}{n}\right]^{nt}$

  $= 2000\left[1+\frac{.03}{3}\right]^{12}$

- $P(t) = P_0 e^{rt}$

  $= 2,000\, e^{rt}$

We know

$4,000 = 2,000\, e^{r(4)}$

So $2 = e^{4r}$

$4r = \ln(2)$

$r = \frac{\ln(2)}{4}$

Question 7 (10 points) *Suppose that x and y are related by the equation*

$$e^{x^2-1} + (x-1)y = 28 - y^3.$$

- *Use implicit differentiation to find $\frac{dy}{dx}$ at the point (1,3).*

$$\frac{d}{dx}\left[e^{x^2-1} + (x-1)y = 28 - y^3\right]$$

$$e^{x^2-1}(2x) + (x-1)\frac{dy}{dx} + y = -3y^2\frac{dy}{dx}$$

Plug in (1,3)

$$2 + 3 = -27\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-5}{27}$$