CVG 4110

Hydraulics of Open Channels

Gradually-varied flow – Part A

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Gradually-varied flow

Introduction

Gradually-varied flow: A non-uniform flow is called gradually-varied flow if the changes in the flow depth are gradual.

An expression for gradually-varied flow

$$\frac{dE}{dx} = S_0 - S_f$$

$$\frac{dE}{dx} = \frac{dy}{dx}\left(1 - F_r^2\right)$$

$$\frac{dE}{dx} = \frac{S_0 - S_f}{\left(1 - F_r^2\right)}$$

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Classification of Channels for Gradually-varied Flow

Mild channels	$y_n > y_c$
Steep channels	$y_n < y_c$
Critical channels	$y_n = y_c$
Horizontal channels	$S_0 = 0$
Adverse channels	$S_0 < 0$

where y_n = normal depth and y_c = critical depth

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Classification of Gradually-varied Flow Profiles

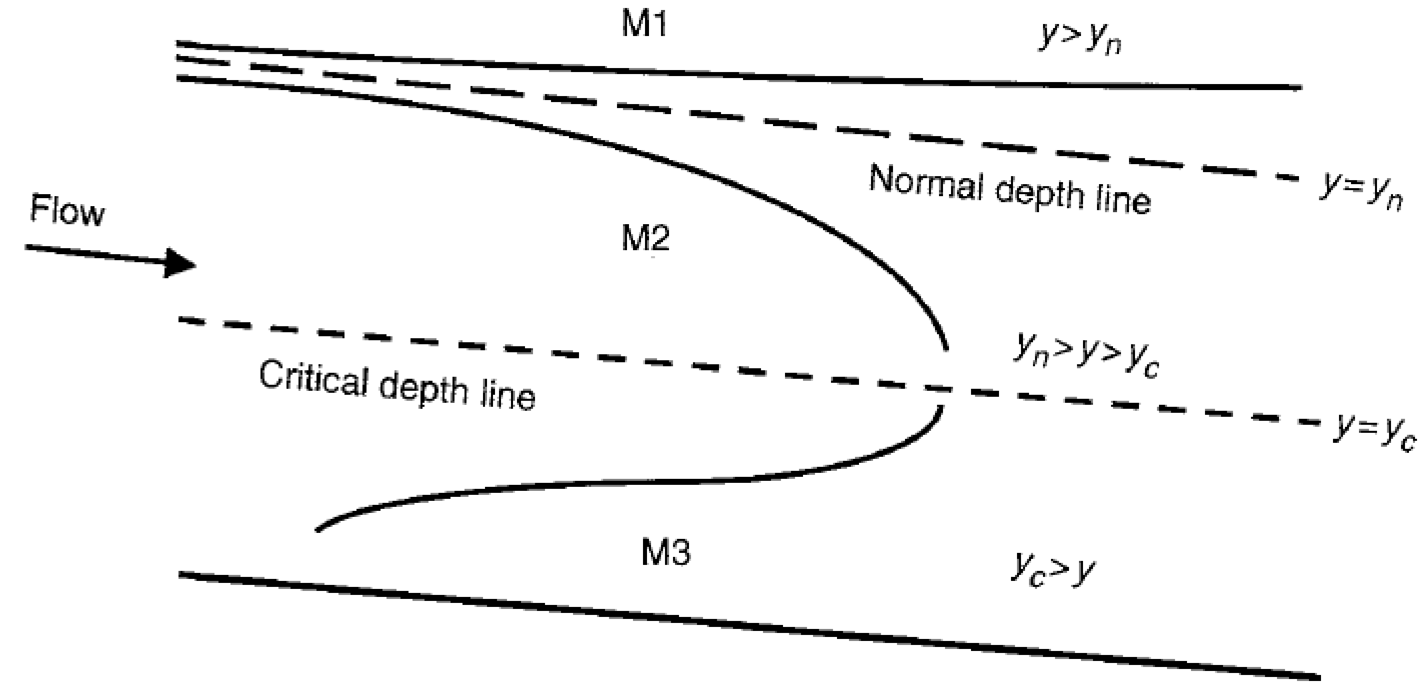

FIGURE 4.1 profiles in mild channels

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Classification of Gradually-varied Flow Profiles

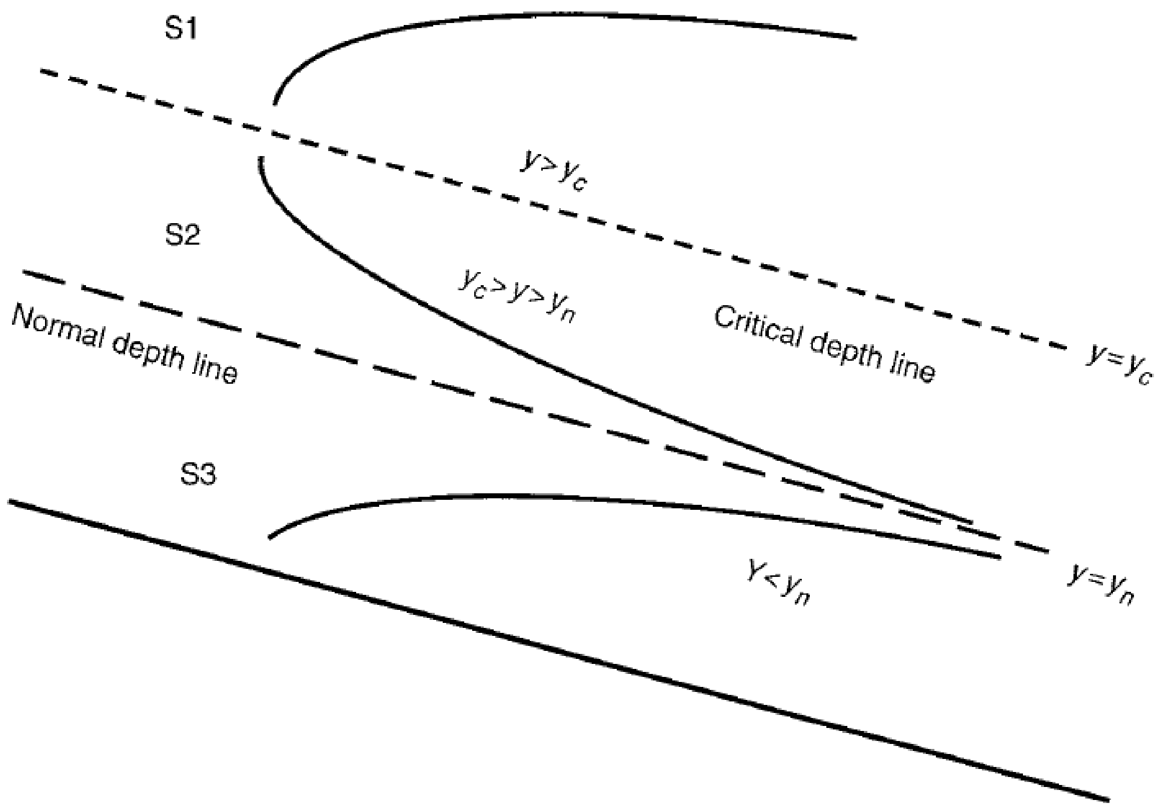

FIGURE 4.2 profiles in steep channels

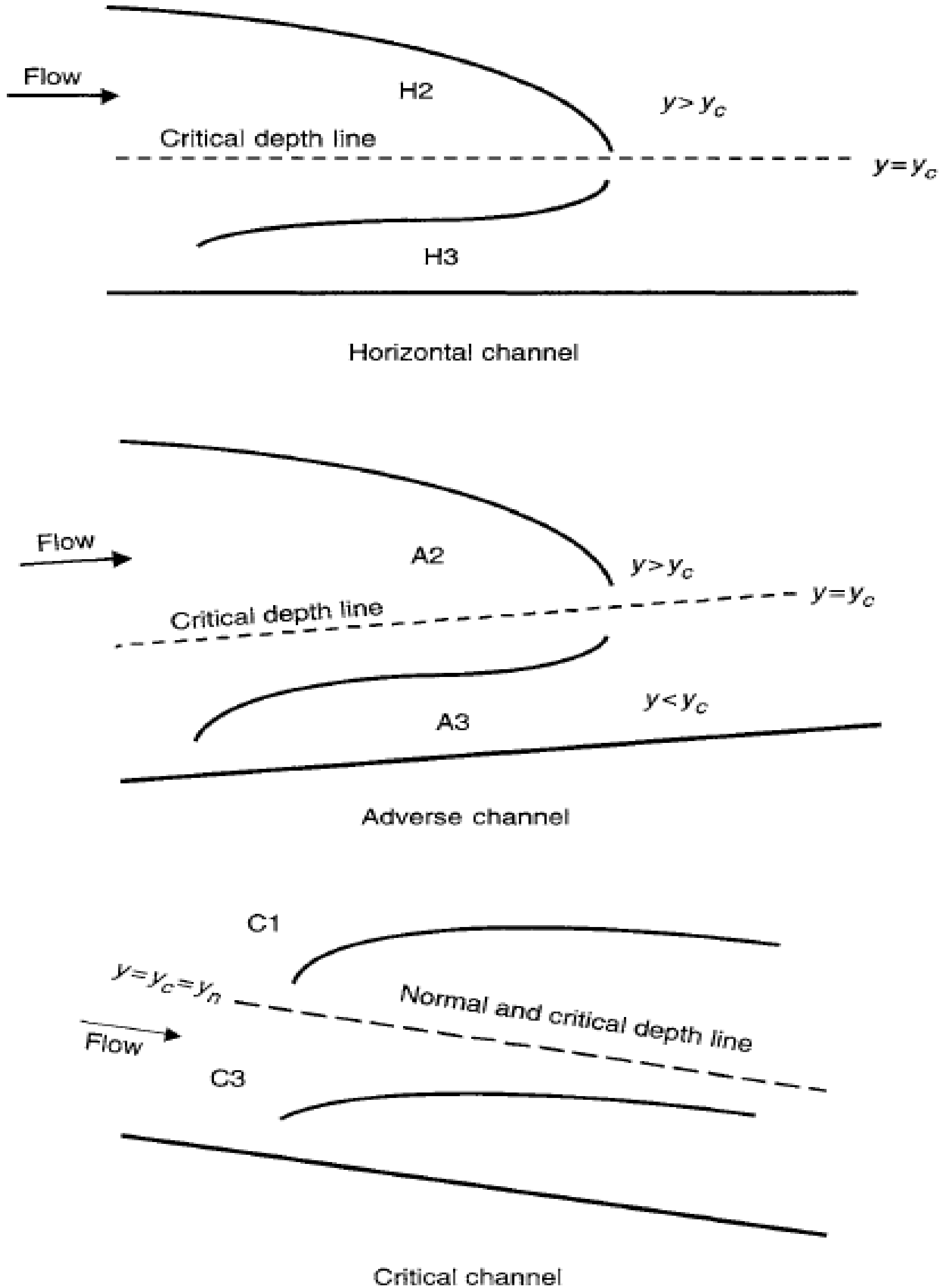

FIGURE 4.3 Flow profiles in horizontal, adverse, and critical channels

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Significance of Froude Number in Gradually-varied Flow Calculations

Supercritical flow is subject to upstream control, and we need an upstream boundary condition to solve the gradually-varied flow equations.

Subcritical flow is subject to downstream control, and we need a downstream boundary condition to solve the gradually-varied flow equations.

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Qualitative Determination of Expected Gradually-varied Flow Profiles

To specify this boundary condition, however, we first need to determine, qualitatively, the types of profiles that will occur. Some general rules:

1. Subcritical flow is subject to downstream control
2. Supercritical flow is subject to upstream control
3. In the absence of (or far away from) flow controls, flow tends to become normal in prismatic channels

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Qualitative Determination of Expected Gradually-varied Flow Profiles

4. The only possible shapes for the water surface profile occurring in the different zones labeled as Ml, S2, etc., are those shown in Figures 4.1, 4.2, and 4.3

5. Normal flow in a mild channel is subcritical and that in a steep channel is supercritical

6. Flow is subcritical upstream of a sluice gate and supercritical downstream

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Qualitative Determination of Expected Gradually-varied Flow Profiles

7. When subcritical flow is present in a channel terminating at a free fall, the depth at the free fall will be equal to the critical depth (this is an assumption, and a good one, since the critical depth actually occurs a short distance, about $4y_c$, upstream of the free fall)

8. The change from supercritical to subcritical flow is possible only through a hydraulic jump in a prismatic channel.

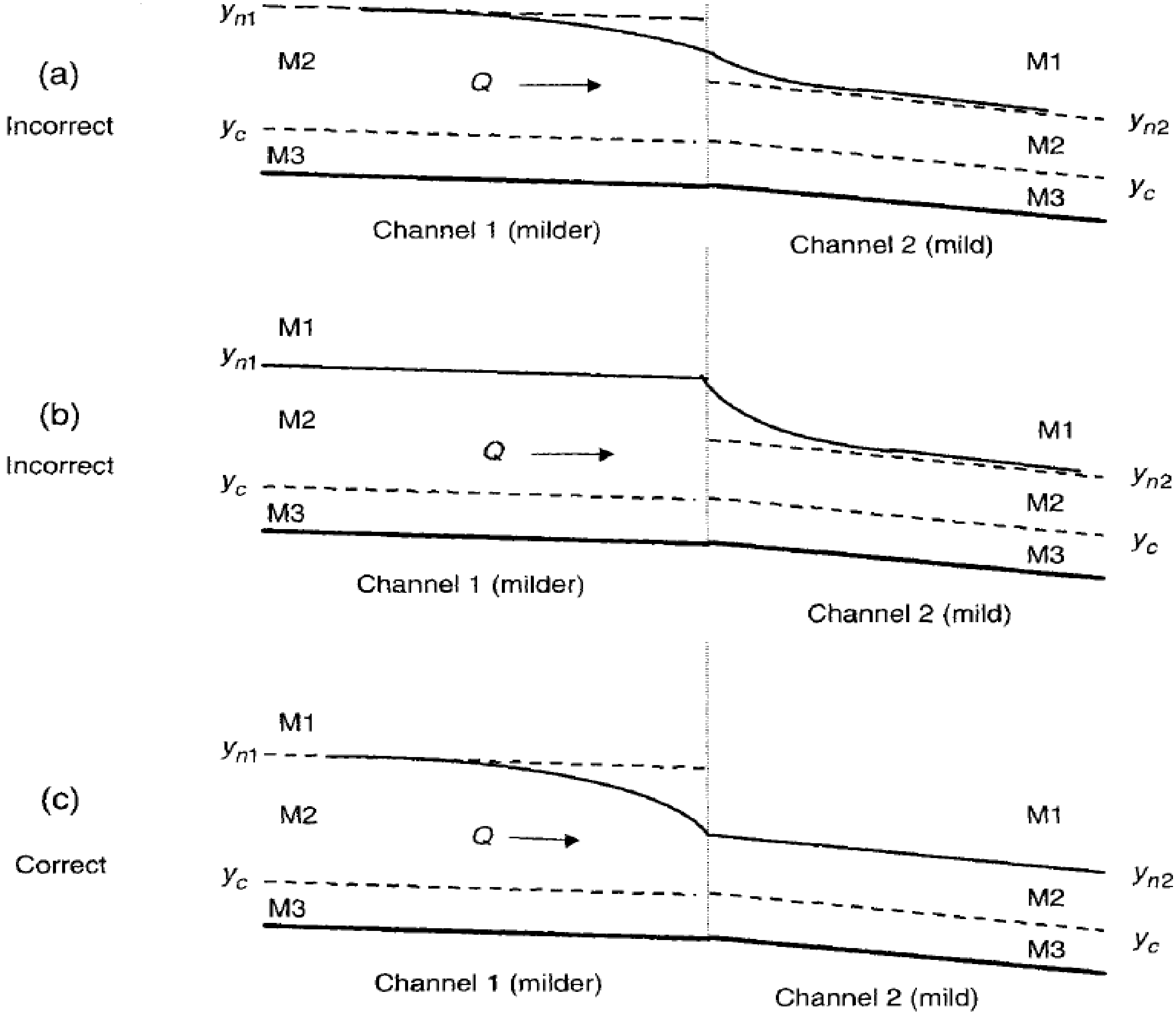

FIGURE 4.7
Composite profile in milder and mild channels

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Qualitative Determination of Expected Gradually-varied Flow Profiles

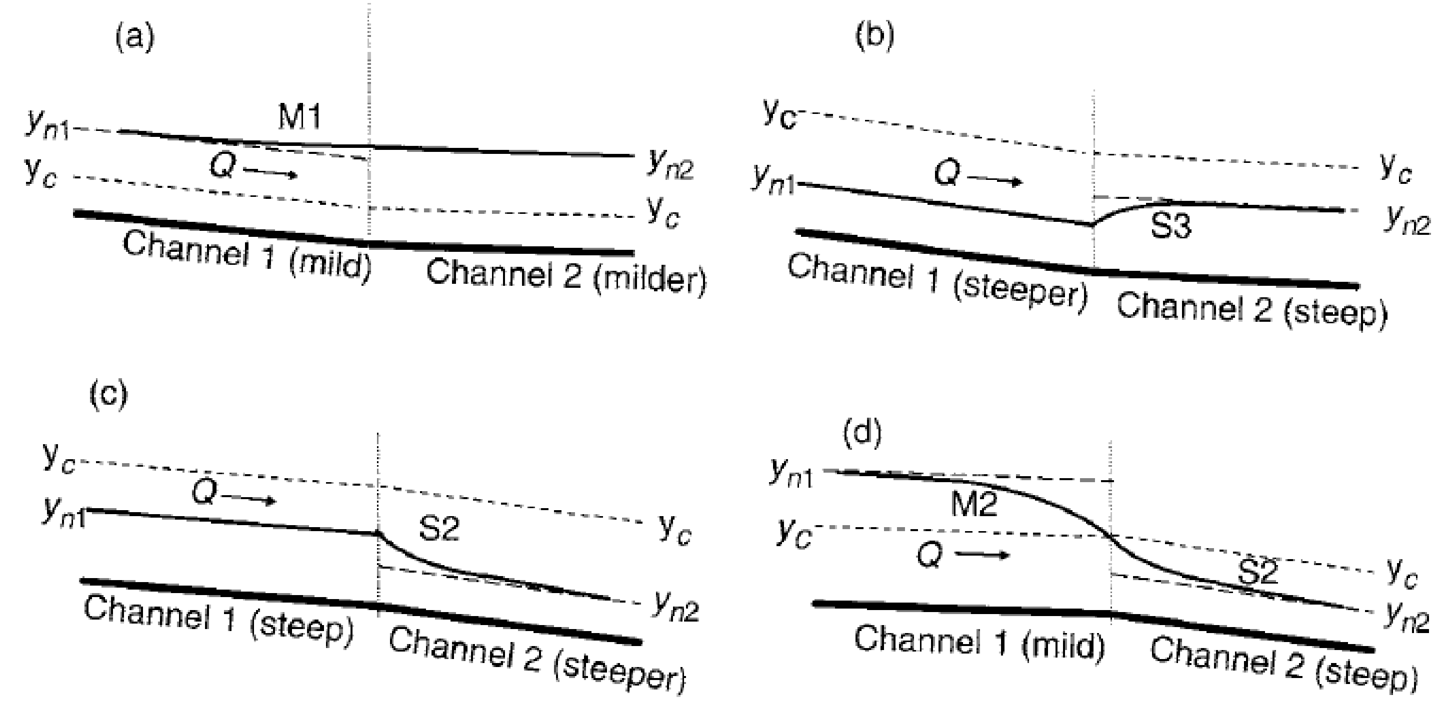

FIGURE 4.8 Various composite flow profiles

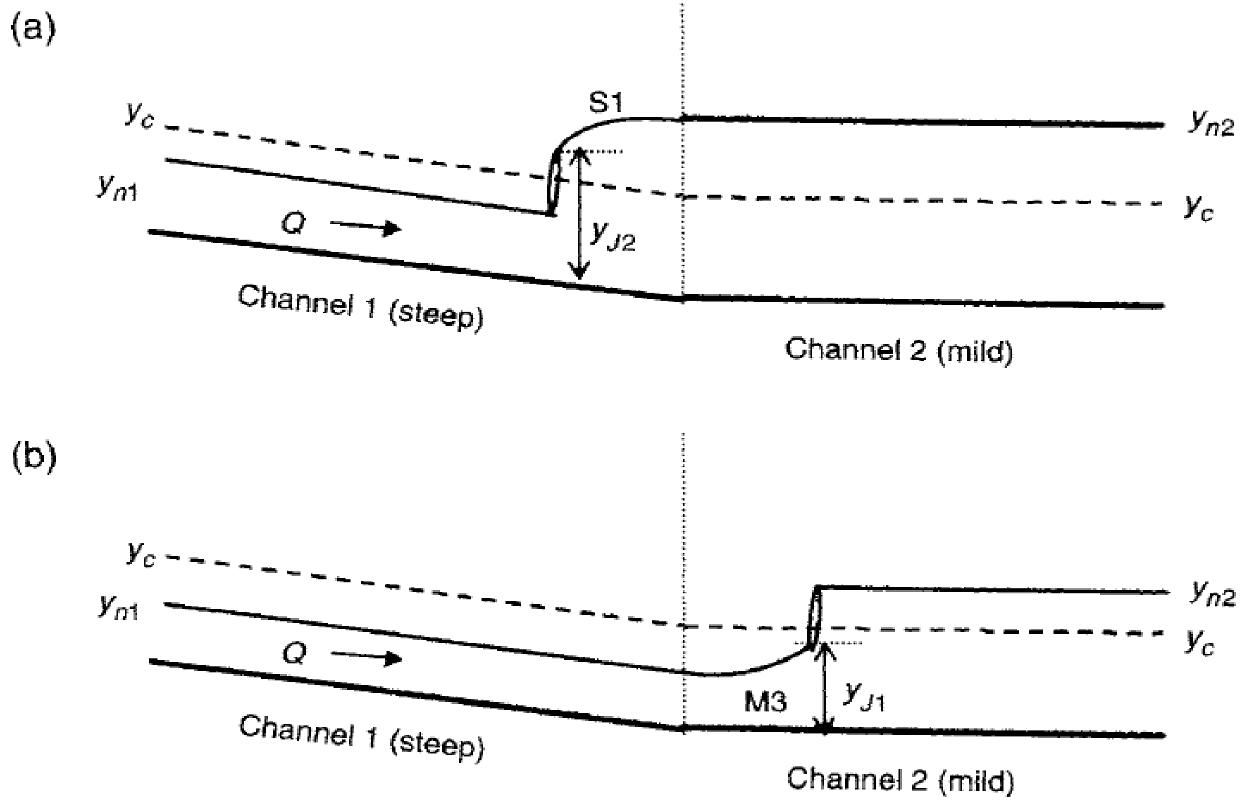

FIGURE 4.9 Flow profiles in a steep channel followed by a mild channel

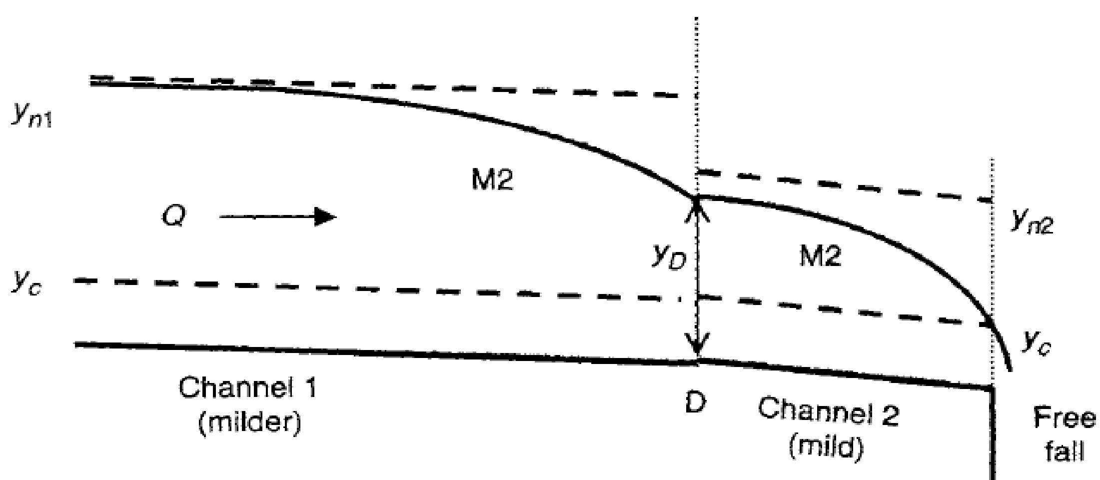

FIGURE 4.10 Example composite profiles involving a short channel

Example 4.1

A very long rectangular channel (channel 1) has a width of b = 3 m, Manning roughness factor of n= 0.018, and a bottom slope of S_0 = 0.015. It carries a discharge of Q = 65 m^3/s. This channel joins another channel (channel 2) downstream, as in Figure 4.9, that has identical properties except for a slope of S0 = 0.009. Determine the type of water surface profile occurring in these two channels.

Gradually-varied flow

Example 4.1

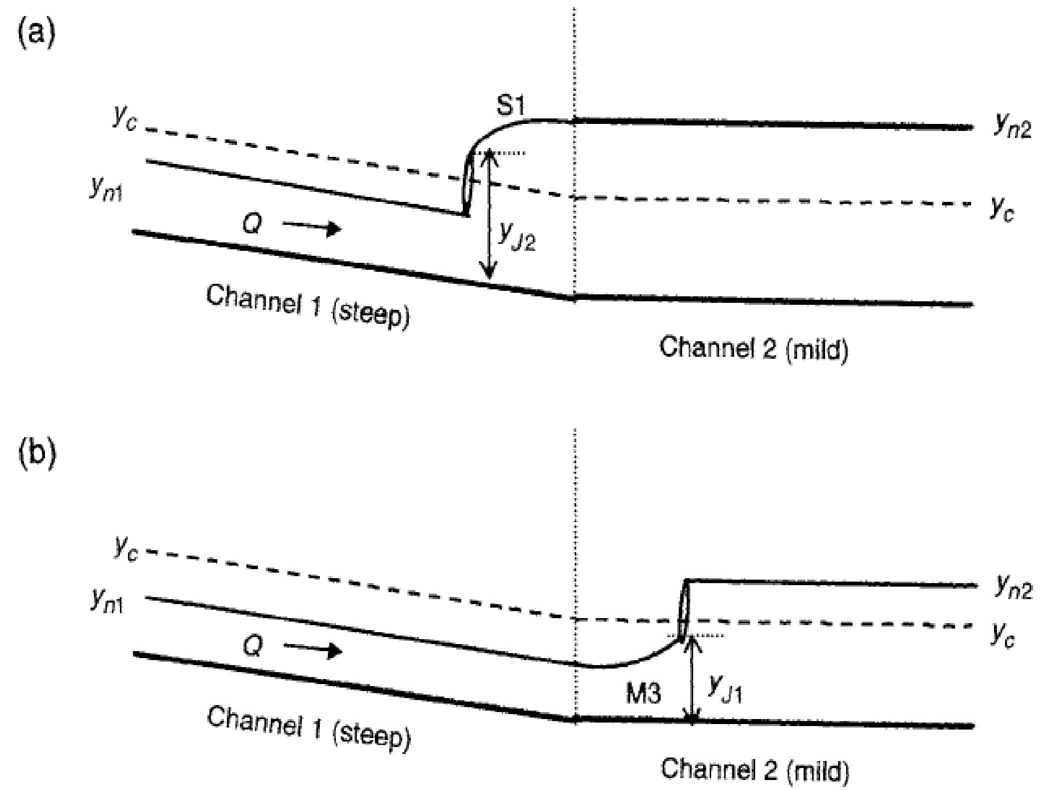

FIGURE 4.9 Flow profiles in a steep channel followed by a mild channel

Example 4.2

A very long rectangular channel (channel 1) has a width of b = 3 m, Manning roughness factor of n= 0.018, and a bottom slope of S_0 = 0.015. It carries a discharge of Q = 65 m^3/s. This channel joins another channel (channel 2) downstream, as in Figure 4.9, that has identical properties except for a slope of S0 = 0.005. Determine the type of water surface profile occurring in these two channels.

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Example 4.2

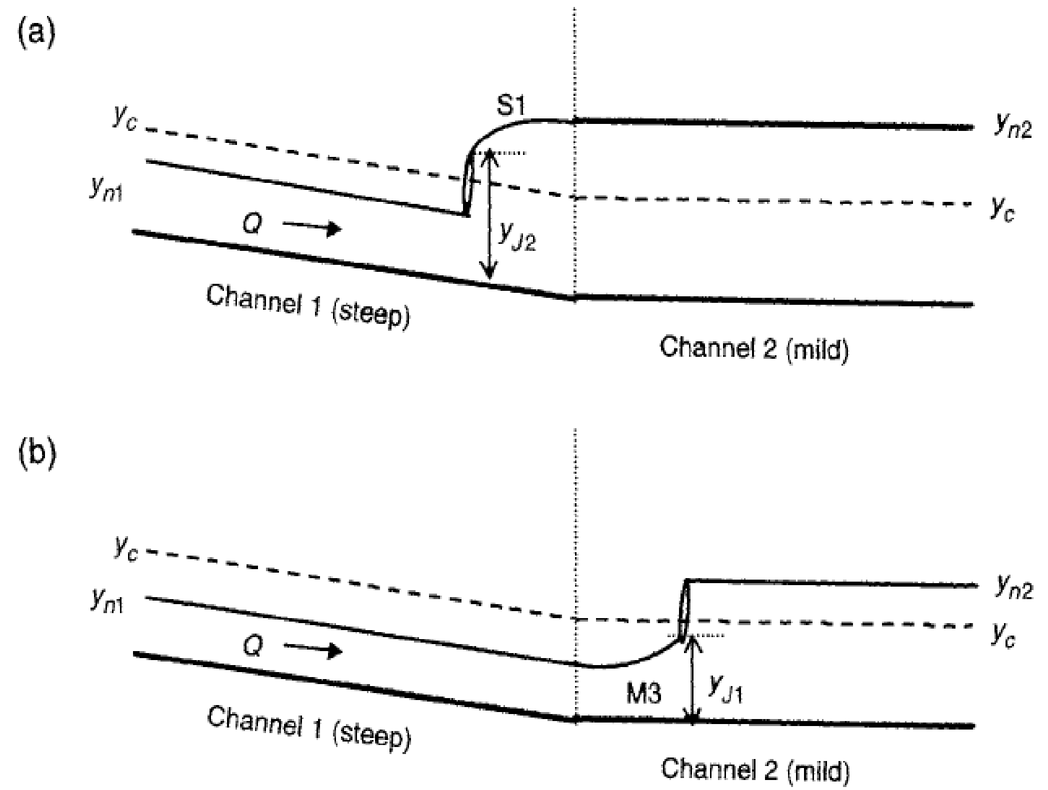

FIGURE 4.9 Flow profiles in a steep channel followed by a mild channel

Gradually-varied flow

Gradually-varied Flow Computations

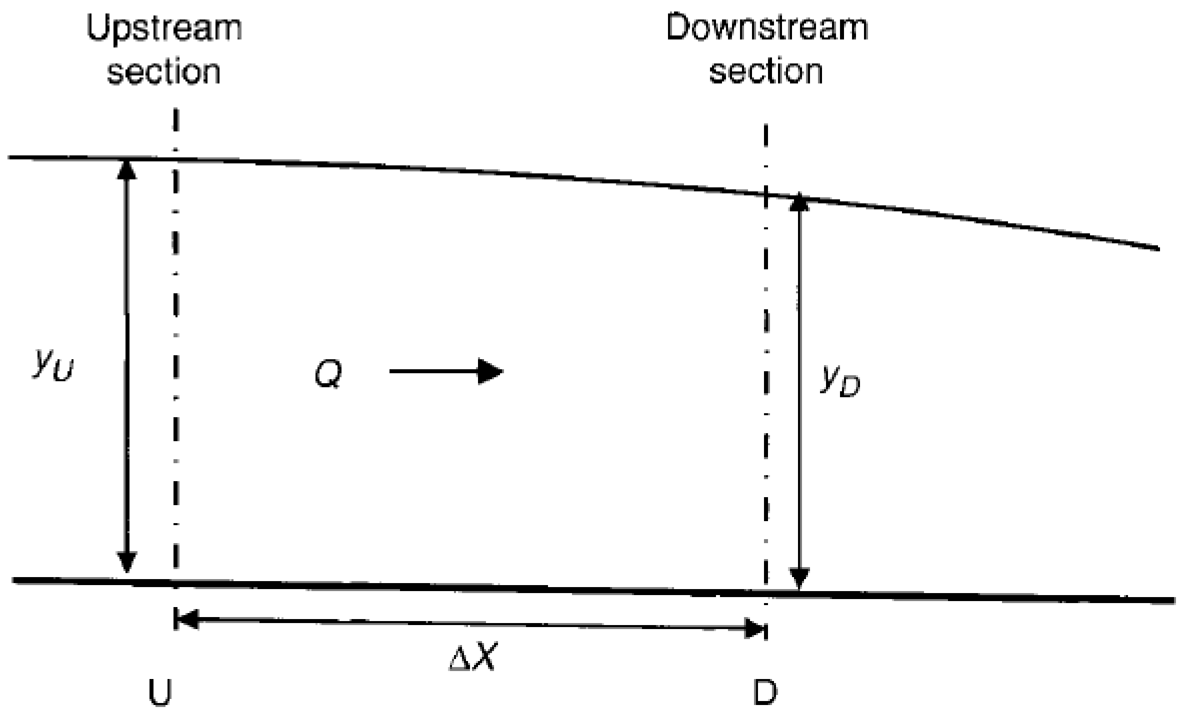

FIGURE 4.11
Definition sketch for gradually-varied flow formulation

Gradually-varied flow

Gradually-varied Flow Computations

A form of the gradually-varied flow equation is

$$\frac{dE}{dx} = S_0 - S_f$$

We can write it in finite difference form as

$$\frac{E_D - E_U}{\Delta X} = S_0 - S_{f_m}$$

with

$$S_{f_m} = \frac{S_{fU} + S_{f_D}}{2}$$

Gradually-varied Flow Computations

By rearranging the Manning formula, the friction slopes at Sections U and D are obtained as

$$S_{fU} = \frac{n^2 V_U^2}{k_n^2 R_U^{4/3}}$$

and

$$S_{fD} = \frac{n^2 V_D^2}{k_n^2 R_D^{4/3}}$$

The two most common methods used to perform the gradually-varied flow calculations are the direct step method and the standard step method.

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Gradually-varied Flow Computations – Direct Step Method

The main equation for direct step method is

$$\Delta X = \frac{E_D - E_U}{S_0 - S_{fm}} = \frac{\left(y_D + V_D^2/2g\right) - \left(y_U + V_U^2/2g\right)}{S_0 - S_{fm}}$$

In a typical subcritical flow problem, the condition at the downstream section D is known. We pick an appropriate value for y_U, and calculate ΔX. Where supercritical flow is involved, conditions at Section U are known; we pick a value for y_D to calculate the reach length.

Example 4.3

A very long trapezoidal canal has b = 5 m, m = 1.5, S_0 = 0.0005, and n = 0.015, and it carries Q = 65 m^3/s. The canal terminates at a free fall. Calculate the water surface profile.

Example 4.4

Flow enters a long, rectangular flume at its upstream end from under a sluice gate. The flume has b = 2 n, n=0.012, and S_0 = 0.015. The flow depth at the entrance is 1 m and the discharge is 10 m^3/s. Determine the water surface profile.

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Gradually-varied Flow Computations – Standard Step Method

The main equation for standard step method is

$$y_U + \frac{V_U^2}{2g} - \frac{1}{2}(\Delta X)S_{fU} = y_D + \frac{V_D^2}{2g} + \frac{1}{2}(\Delta X)S_{fD} - (\Delta X)S_0$$

For subcritical flow

$$\Delta y_k = \frac{(LHS)_k - (RHS)}{\left(1 - F_{rU}^2 + 3(\Delta X)S_{fU}/2R_U\right)_k}$$

$$(y_U)_{k+1} = (y_U)_k - \Delta y_k$$

Gradually-varied Flow Computations – Standard Step Method

For supercritical flow

$$\Delta y_k = \frac{(RHS)_k - (LHS)}{\left(1 - F_{rD}^2 + 3(\Delta X) S_{fD} / 2R_D\right)_k}$$

$$(y_D)_{k+1} = (y_D)_k - \Delta y_k$$

Example 4.5

A very long trapezoidal canal has b = 10 m, m = 1.5, S_0 = 0.0012, and n = 0.015, and it carries Q=250 m^3/s. The canal terminates at a free fall. Calculate the water surface profile in this channel using the standard step method and a constant space increment of dX= 5 m.