Student No.:_____

Name: _____

FACULTY OF ENGINEERING AND COMPUTER SCIENCE
CONCORDIA UNIVERSITY

MIDTERM EXAMINATION	
COURSE**:** ENGR 311	TRANSFORM CALCULUS
INSTRUCTOR: M. ESHAGHI	MAX. MARKS: 100
DATE: 17 OCTOBER 2017	DURATION: 75 MINUTES

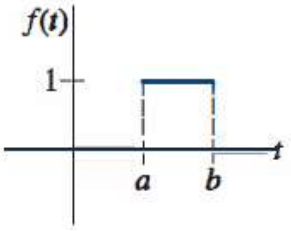
- *Write directly on this questions booklet. Show your work in sufficient details and clearly to qualify for partial marks*
- *DO NOT UNSTAPLE THE QUESTIONS BOOKLET*
- *YOU CAN FIND FORMULA SHEET IN THE LAST PAGE*
- *TOTAL NUMBER OF PAGES: 8*

Question #	1:	☐
Question #	2:	☐
Question #	3:	☐
Question #	4:	☐

Question #1 (20 Marks)

Find Laplace transform of following function using:

(a) Laplace transform definition
(b) Unit step function



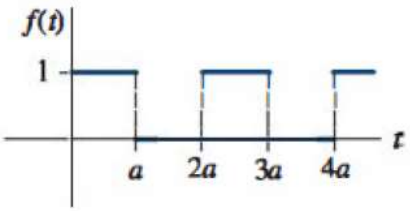
Question #2 (40 Marks)

Find Laplace transform of following four functions.

$f(t) = 8t^{2.5}$

$f(t) = t\int_0^t \tau e^{-\tau} d\tau$

$$f(t) = \begin{cases} 2 & , \quad 0 \le t < 4 \\ t^2 & , \quad t \ge 4 \end{cases}$$



Square wave

Question #3 (30 Marks)

Solve following equation and system of equations using Laplace transform.

$y'' + 2y' + y = \delta(t-1), \qquad y(0) = 0, \qquad y'(0) = 0$

$$\begin{cases} y_1' - y_2 = 0 \\ y_1 + y_2' = 0 \end{cases} \qquad y_1(0) = 0 \; and \; y_2(0) = 1$$

Question #4 (20 Marks)

Solve following equation and find $f(t)$.

$$1 - f(t) = \int_0^t (e^{\tau} - e^{-\tau}) f(t-\tau) d\tau$$

=

Function	Laplace Transform
1	$1/s$
t^n	$n!/s^{n+1}$
e^{at}	$1/(s-a)$
sin kt	$k/(s^2+k^2)$
cos kt	$s/(s^2+k^2)$
sinh kt	$k/(s^2-k^2)$
cosh kt	$s/(s^2-k^2)$
t^α $\alpha > -1$	$\Gamma(\alpha+1)/s^{\alpha+1}$ $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$
$\Gamma(\frac{1}{2})$	$\sqrt{\pi}$.
$f^{(n)}(t)$	$s^nF(s)- s^{n-1} f(0)- s^{n-2} f'(0)-... -f^{(n-1)}(0)$
$e^{at} f(t)$	$F(s-a)$
$f(t-a)\,\mathcal{U}(t-a)$	$e^{-as}F(s)$
$g(t)\,\mathcal{U}(t-a)$	$e^{-as}\mathcal{L}\{g(t+a)\}$
$t^n f(t)$	$(-1)^n d^nF(s)/ds^n$
$f*g$	$F(s).G(s)$
$\int_0^t f(\tau)\,d\tau$	$F(s)/s$
f (t) where f(t+T)=f(t)	$\frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt$
$\delta(t-t_0)$	e^{-st_0}

Trigonometric Identities:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin A \cdot \sin B = \frac{1}{2}\left[-\cos(A+B) + \cos(A-B)\right]$$

$$\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \cdot \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$