CONCORDIA UNIVERSITY DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP 232 Mathematics for Computer Science Winter 2017

Assignment 2

Due date: February 27, 2017, 11:59 AM.

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1. For each of the arguments below, formalize them in propositional logic. If the argument is valid identify which inference rule was used, and formulate the tautology underlying the rule. If the argument is invalid, state whether the inverse or converse error was made.

   a) All cheaters sit in the back row.
   George sits in the back row.
   George is a cheater.

   b) For all students x, if x studies discrete math, then x is good at logic.
   Dawn studies discrete math.
   Dawn is good at logic.

   c) If the compilation of a computer program produces error messages, then the program is not correct or the compiler is faulty.
   The compilation of this program does not produce error messages.
   this program is correct and the compiler is not faulty.

   d) All students who do not do their homework and do not study the course material will not get a good course grade.
   John gets a good course grade.
   John did his homework or studied the course material.

2. Use rules of inference to show that

   (a) $\forall x \left(R(x) \to \left(S(x) \lor Q(x)\right)\right)$
   $\exists x \left(\neg S(x)\right)$

   $\exists x \left(R(x) \to Q(x)\right)$

   (b) $\forall x\left(P(x) \lor Q(x)\right)$
   $\forall x\left(\left(\neg P(x) \land Q(x)\right) \to R(x)\right)$

   $\forall x\left(\neg R(x) \to P(x)\right)$

3. Prove that the product of two odd numbers is odd, using

i) a direct proof;
ii) an indirect proof;
iii) a proof by contradiction.

4. For any integer n, prove that

a) 3 divides $n^3 - n$.
b) 3 divides one of the integers n, $n+1$, or $2n+1$.
c) 3 divides one of n, $n+2$ or $n+4$.
d) 3 divides one of n, $2n-1$ or $2n+1$.

5. a) Prove that $\lfloor n/2 \rfloor \lceil n/2 \rceil = \lfloor n^2/4 \rfloor$ for all integers n.
b) Show that for any integer n, $5|(n^5 - n)$.
c) Show that if n is odd integer, then $n^3 - n$ is a multiple of 24.

6. For each of the statements below state whether it is True or False. If True then give a proof. If False then explain why, *e.g.*, by giving a counterexample.
a) For all positive $x, y \in$, if x is irrational and y is irrational then $x + y$ is irrational.
b) $\forall x, y \in$, if x is irrational and y is rational then $x - y$ is irrational.
c) $\sqrt{3}$ is irrational.
d) $\log_5(2)$ is irrational.

7. a) Show that if x and y are positive rational numbers with $x < y$ then there is a rational number r with $x < r < y$.
b) Prove that if x is an integer then $\lceil x \rceil = \lfloor x \rfloor$
c) Prove that if x is a real number but not an integer then $\lceil x \rceil - \lfloor x \rfloor = 1$

8. a) Prove that for any integer n, $n(n^2 - 1)(n + 2)$ is divisible by 12.
b) Prove that $n \in Z^+$ is divisible by 3 if and only if the sum of its digits is divisible by 3.
c) $\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor$