Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Calculus I MAT 1320 3X

Midterm 1

June 1, 2017

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You must **sign below** to confirm that you have read, understand, and will follow these **instructions**:

- This is an 80-minute **closed-book** exam; no notes are allowed. **Calculators are permitted, but must be non-programmable, non-graphing calculators with no differentiation or integration functions.** This test has been designed so that a calculator will not be needed.
- The exam consists of 14 questions on 9 pages. Page 9 provides additional work space. *Please do not detach it.*
- Questions 1-5 are **multiple-choice**. There is a series of boxes on Page 2 where you should enter your choices. It may be a good idea to also circle your choices in case you forget to put them in the boxes.
- Questions 6-10 are **short-answer**. You will be graded only on your final answer, and you do not need to show any extra steps.
- Questions 11-14 are **long-answer**. You must show all relevant steps in order to receive full marks.
- For rough work or additional work space, you may use Page 9 or the backs of the pages. **Do not use scrap paper of your own**.
- **Cellular phones** and other unauthorized electronic devices **are not permitted** during this exam. Phones and other devices must be turned off completely and stored out of reach. Do not keep them in your possession, such as in your pockets. If you are caught with such a device, the following may occur: academic fraud allegations will be filed which may result in your obtaining a 0 (zero) for the exam.

LAST NAME:_____

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Question	1-5	6-10	11	12	13	14	Total
Max	5	5	3	2	3	2	20
Marks							

[5pts]

Multiple-choice: please write your selection in the boxes provided here. You do not need to show any work. A correct answer is worth 1 point; an incorrect answer is worth 0 points.

Question	1	2	3	4	5
Answer					

1. Let $f(x) = 3x\sqrt{\log(x-1)}$. What is the range of f^{-1}?

   A. $\{y \in \mathbb{R} : y > 1\}$

   B. $\mathbb{R}$

   C. $\{y \in \mathbb{R} : y \geq 1\}$

   D. $\{y \in \mathbb{R} : y \geq 2\}$

   E. $\{y \in \mathbb{R} : y \geq 0\}$

2. Let $f(x) = \ln x$ and $g(x) = \sqrt[3]{x^2}$. What is the domain of $f \circ g$?

   A. $\{x \in \mathbb{R} : x \geq 0\}$

   B. $\mathbb{R}$

   C. $\{x \in \mathbb{R} : x \neq 0\}$

   D. $\{x \in \mathbb{R} : x > 0\}$

   E. $\{x \in \mathbb{R} : x \geq 1\}$

3. Let $f(x) = \frac{2+x}{1-x}$. What is $f^{-1}(x)$? (Keep in mind that you may need to simplify your answer.)

   A. $f^{-1}(x) = \frac{x-2}{x-1}$

   B. $f^{-1}(x) = \frac{x+2}{x+1}$

   C. $f^{-1}(x) = \frac{x-2}{x+1}$

   D. $f^{-1}(x) = \frac{2-x}{x+1}$

   E. $f^{-1}(x) = \frac{x-2}{1-x}$

4. Let $f(x) = \begin{cases} \sqrt{x} & : \ x > 1 \\ e^x & : \ x \leq 1. \end{cases}$

For what values of x is $f(x)$ **not** differentiable?

A. $\{x \in \mathbb{R} : x < 0\}$

B. f is not differentiable anywhere.

C. $x = 1$

D. f is differentiable everywhere.

E. $\{x \in \mathbb{R} : x < 0 \text{ or } x = 1\}$

5. What is the value of $\lim_{x \to \infty} \frac{3x^2 - 5x + 2}{2x^3 + 4x}$?

A. 0

B. ∞

C. $\frac{3}{2}$

D. $-\infty$

E. The limit does not exist and is neither ∞ nor $-\infty$.

[5pts] **Differentiate the following.**

Answer in the space provided. You do not need to show any intermediate steps or simplify your answers.

6. $f(x) = \frac{1-x^2}{1+e^x}$

7. $f(x) = 2^{\sin x} \cdot 3^x$

8. $f(x) = \tan^2(x^2 - 4)$

9. $f(x) = \frac{1}{\sqrt{1+x}}$

10. $f(x) = (x+1)(x+2)\cos(x+3)$

Long-answer questions: in questions 11-14, you must show all relevant steps. A correct answer without any justification will not receive full marks.

[3pts]

11. Find the value of

$$\lim_{x \to 1} \frac{x^2 - 4x + 5}{x - 1}.$$

If the limit does not exist, determine whether it is ∞, $-\infty$, or neither.

[2pts]

12. Find all values $c \in \mathbb{R}$ such that the function

$$f(x) = \begin{cases} (x-1)^2 & : \quad x \geq 1 \\ cx^2 + cx - 2 & : \quad x < 1 \end{cases}$$

is **continuous**.

[3pts] 13. Find the derivative of $f(x) = \sqrt{x^2 - 2x - 1}$ by using the definition of the derivative as a limit. *You* **must** *use the definition of the derivative. A correct answer obtained by any other method will not receive any points.*

[2pts] 14. Find the equation of the line tangent to $y = \sin(e^{x-1} - 1)$ at $x = 1$.

Additional work space. Please do not detach this page.