April 2018

#1 Find the derivative for each of following fn:

(a) $g(x) = \left(e\sqrt{x} - \frac{7}{x^2}\right)\left(e^4 - x^4\right)$

$u = e\sqrt{x} - 7x^{-2}$ $\quad v = e^4 - x^4$

$u' = \frac{e}{2\sqrt{x}} + 14x^{-3}$ $\quad v' = 0 - 4x^3$

$g'(x) = \left(\frac{e}{2\sqrt{x}} + 14x^{-3}\right)\left(e^4 - x^4\right) + \left(e\sqrt{x} - \frac{7}{x^2}\right)\left(-4x^3\right)$

(b) $h(x) = x^3 \ln(3x) - e^{(-x^3 + x)}$

$u = x^3$ $\quad v = \ln 3x = \ln 3 + \ln x$

$u' = 3x^2$ $\quad v' = \frac{1}{3x} \cdot 3 = \frac{1}{x}$

$h'(x) = 3x^2 \ln(3x) + x^3\left(\frac{1}{3x} \cdot 3\right) - e^{-x^3 + x}\left(-3x^2 + 1\right)$

#2 Graph $x^2 - 100 = y^2$, find y' by implicit differentiation, and find the slopes of the graph when $x = -10$

↳ means, $y = 0$

$2x = 2yy'$

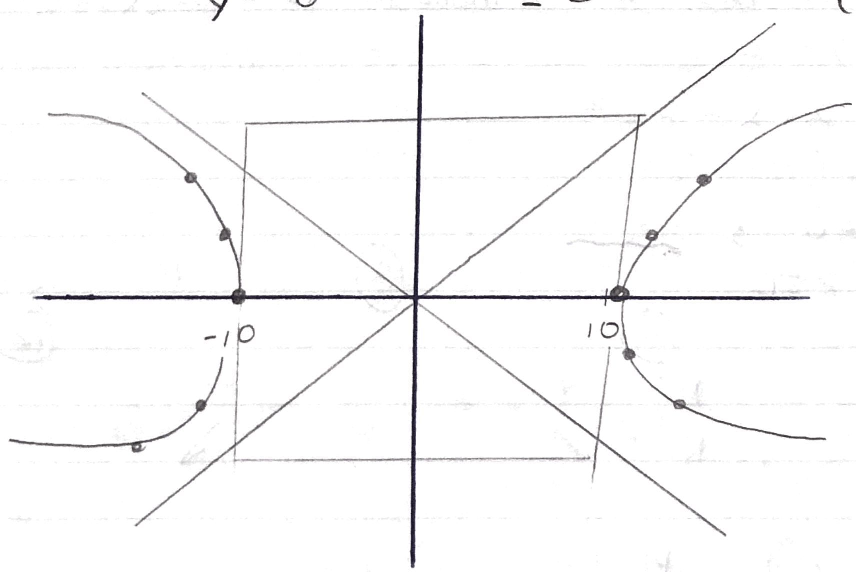
$\left[y' = \frac{x}{y}\right]$

Slopes @ $x = -10$: $y' = \frac{-10}{\pm 0} = \infty$ infinate slope (vertical slope)

$y = 0$

$y^2 = x^2 - 100$

x	y
10	0
-10	0
11	$\sqrt{21}$
11	$-\sqrt{21}$
-11	$\sqrt{21}$
-11	$-\sqrt{21}$



#3 Use the price-demand equation $x = (40-p)\,1000$ to find values of p for which the demand is elastic and for which the demand is inelastic.

$$E = \frac{-px'}{x} = \frac{-p(-1000)}{40\,000 - 1000p} = \frac{p}{40-p} = 1$$

max price

Inelastic: $(0, 20)$

Elastic: $(20, 40)$

min demand

$$p = 40 - p$$

$$\frac{2p}{2} = \frac{40}{2}$$

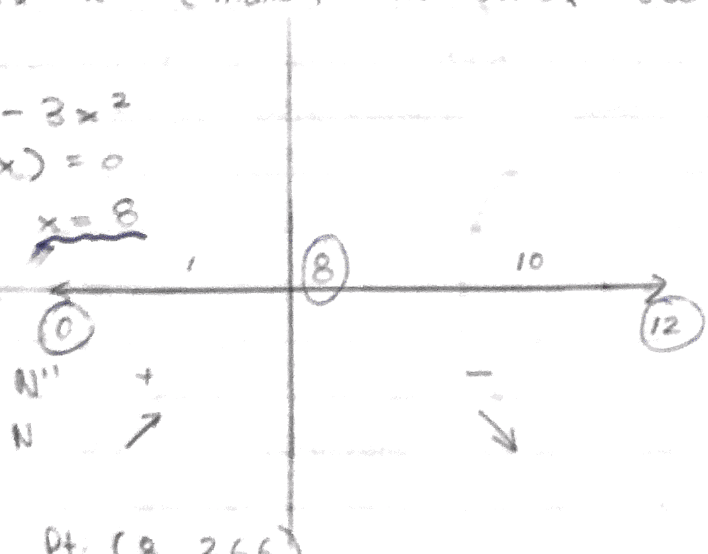
$$p = 20$$

#4 A discount store is presently selling 200 televisions set monthly. If the store invests x thousand dollars in an advertising campaign, the ad company estimates that sales will increase to $N(x) = 4x^3 - 0.25x^4 + 500$, for $0 \le x \le 12$. When is the rate of change of sales increasing and when is it decreasing? What is the point of diminishing returns?

- $N'(x) = 12x^2 - x^3$ (main fn $4x^3 - 0.25x^4 + 500$)
- $N'(10) = 0$
- $N'' = 24x - 3x^2$

  $3x(8 - x) = 0$

  $x = 0$; $x = 8$
- $N'(8) = 256$



Inc: $[0, 8)$ Pt: $(8, 266)$

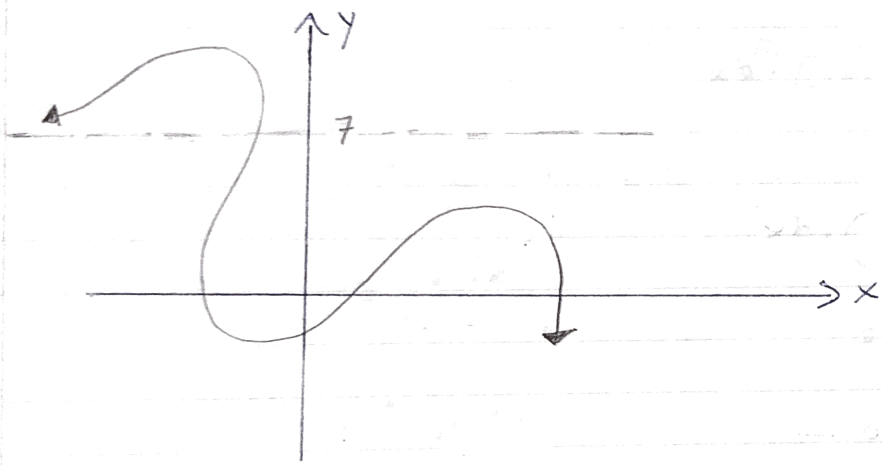
Dec: $(8, 12]$

(a)

#5 Find $\lim\limits_{x\to\infty} \frac{7-x^2}{x^4+13x} = \lim\limits_{x\to\infty} \frac{\frac{7}{x^4} - \frac{x^2}{x^4}}{\frac{x^4}{x^4} + \frac{13x}{x^4}} = \lim\limits_{x\to\infty} \frac{\frac{7}{x^4} - \frac{1}{x^2}}{1 + \frac{13}{x^3}}$

$= \frac{0-0}{1+0} = 0$

(b) Give an example of a fn f defined for all real numbers which has the property that $\lim\limits_{x\to-\infty} f = 7$ and $\lim\limits_{x\to-\infty} f = -\infty$



#6 A point is moving on the graph of $x^2y = 12$. When the point is at $(-2, 3)$, its x-coordinate is increasing by 7 units/sec. How fast is the y-coordinate changing at that moment?

METHOD ①

→ Step ① and ② given

→ Step ③

$x^2 y = 12$

method ①

$u = x^2 \quad v = y$

$u' = 2xx' \quad v' = y'$

$2xx'y + x^2y' = 0$

→ Step ④: $2(-2)(+7)(3) + (-2)^2 y' = 0$

$y' = 21$ units / sec

Method ②

→ Step ① and ② Given

→ Step ③

$x^2y = 12$

$x^2 = \frac{12}{y}$

$x^2 = 12y^{-1}$

$2xx' = -12y^{-2}y'$

→ Step ④: $2(-2)(7) = -12(3)^{-2}y'$

$\frac{-28}{-4/3} = -4/3\, y'$

$= 21$ units/sec

#7 Find the differential dh if $h = 2x^2 - 3x$, $x = 2$ and the change in x is 0.1.

$dh = h'\Delta x$

$2x^2 - 3x$

$4x - 3$

$(4(2) - 3)(0.1) = 0.5$

#8 You are told a country has Lorenz curve $y = \frac{x^2}{10}$. You want to find its Gini index.

>• What conclusion can you draw?

$$G.I = 2\int_0^1 [x - f(x)] \cdot dx$$

$$= 2\int_0^1 \left(x - \frac{x^2}{10}\right) \cdot dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{30}\right]_0^1$$

$$= 2\left(\frac{1}{2} - \frac{1}{30}\right) = \frac{14}{15} = 0.9\overline{33}$$

>• Conclusion: Since the G.I index is 0.933 and is close to 1, the country has a large wealth inequality gaps where the wealthy are very wealthy and poor, very poor.

* Summary: • the closer G.I is to zero, more equal wealth.
• the closer G.I is to one, more inequal wealth.

$G.I \leq 0$ = equality
$G.I \geq 1$ = inequality

#9 For $f(x) = 12x - x^3$, find the abs max and min, if either exists, on the interval $[-3, 3]$.

$f' = 0$: $12 - 3x^2 = 0$

$3(4 - x^2) = 0$

$4 = x^2$

$\sqrt{4} = \sqrt{x^2}$

$\pm 2 = x$

$f(-3) = 12(-3) - (-3)^3 = -9$

$f(-2) = 12(-2) - (-2)^3 = -16$

$f(2) = 12(2) - (2)^3 = 16$

$f(3) = 12(3) - (3)^3 = 9$

Abs max: $f(2) = 16$, Abs min: $f(-2) = -16$

#10 Find the following

a) $\int \left(\frac{2}{\sqrt{x}} - \frac{1}{x^4}\right) \cdot dx$

$= \int \left(2x^{-1/2} - x^{-4}\right) \cdot dx$

$= \frac{2x^{1/2}}{1/2} - \frac{x^{-3}}{-3} + C$

b) $\int_3^6 \frac{-7}{x+3} \cdot dx$

$u = x + 3$; $x = 6$, $u = 9$; $x = 3$, $u = 6$

$dx = du$

$= \int_6^9 \frac{-7}{u} \cdot du$

$= \Big[-7 \ln|u|\Big]_6^9 = -7\ln 9 + 7\ln 6$

$= 7(-\ln 9 + \ln 6)$

$= 7 \ln \frac{6}{9} = 7 \ln \frac{2}{3}$

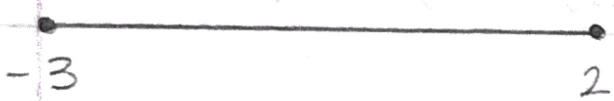
#11 Find the area bounded by $f(x) = 6 - x^2$ and $g(x) = x$

$6 - x^2 = x$

$0 = x^2 + x - 6$ Factor

$0 = (x+3)(x-2)$

$x = -3, \quad x = 2$



Plug in 0 in $f(x)$ goes on top

$$A = \int_{-3}^{2} (6 - x^2 - x) \cdot dx$$

$$= \left[6x - \frac{x^3}{3} - \frac{x^2}{2} \right]_{-3}^{2}$$

$$= \left(12 - \frac{8}{3} - 2\right) - \left(-18 + 9 - \frac{9}{2}\right)$$

$$= \frac{125}{6}$$