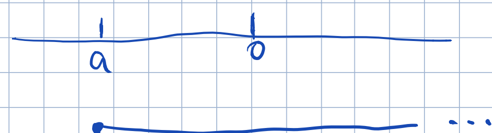
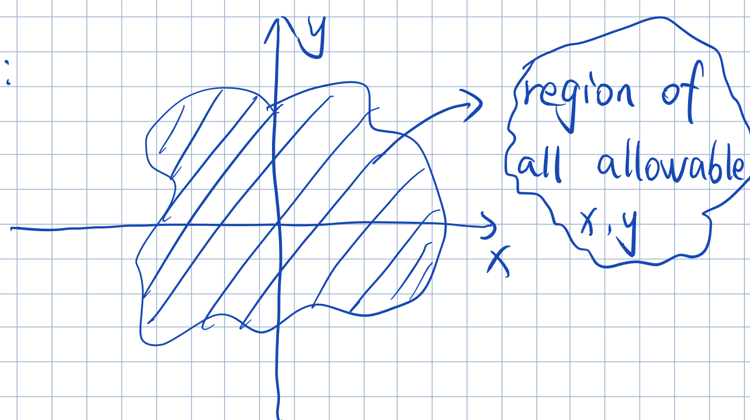
In the context $z=f(x,y)$, the ordered pair (x,y) is mapped to the number z. So z is the image of the pair (x,y) by f.

So all the <u>allowable</u> (x,y) combinations yield a single value z, and these x,y belong to $\mathrm{dom}(f)$. The domain is not a set of numbers, but a set of ordered pair.

In 2-D : $y=f(x)$:



In 3-D: $y=f(x,y)$:



Particular graphical objects that "live" in 3-D space :

Iso sections, other sets of points.

Def : An <u>iso section</u> is an equation (or a rule) associated to a graphical object, that is obtained by setting one of the 3 variables in $z = f(x,y)$ to a constant.

ex. $z = f(x,y) = x^2 + y^2$. now if we set $z = 16$.

$$\Downarrow$$

$$\underbrace{x^2 + y^2 = 16.}_{\text{iso section}}$$

↳ $x^2 + y^2 = 16$ is a circle, centered at $(0,0)$, radius $= 4$.

Set z = constant : iso - z section => an equation involving x and y.

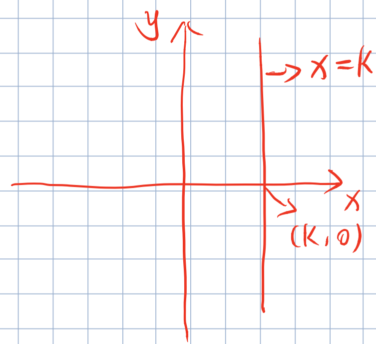
Set y = constant : iso - y section => an equation between x and z.

Set x = constant : iso- x section => an equation between x and z.

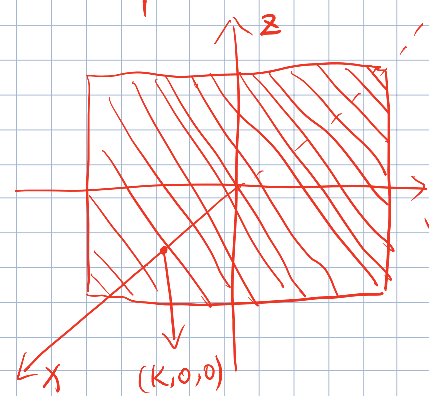
ex. $x = K$, where K is a constant : so 2 of the variables are fixed.

Recall that, in 2-D (Cartesian plane), $x=K$ corresponds to a vertical line.

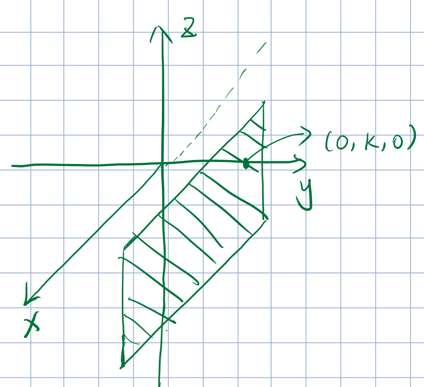
2-D:

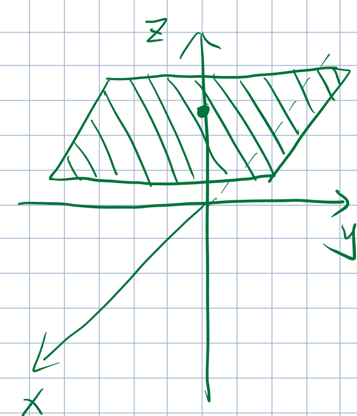


3-D: $x=K$ corresponds to a vertical wall:



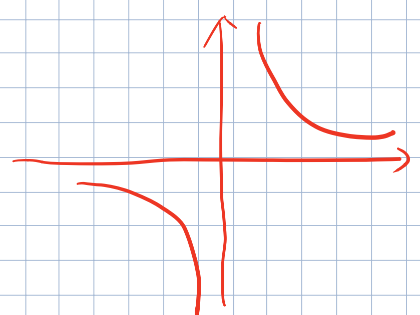
ex. $y=K$ (vertical plane) $z=K$ (horizontal plane)



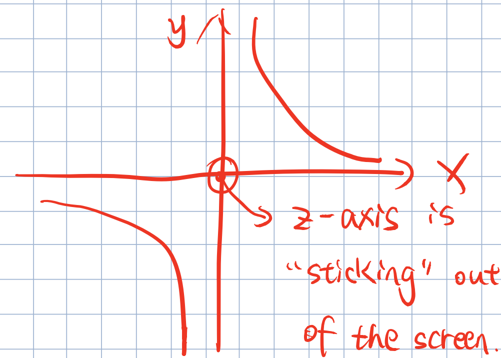


$y=\frac{1}{x}$:

Recall that 2-D:



3-D: vertical walls:



The domain of 2-variable functions :

Goal: we want to determine and correctly represent the set of x, y ordered pairs that are in the domain of the function. Ie: which x, y combined x_s yield a number?

The same algebraic constants hold as in the one variable case:

- argument under an even root cannot be <0
- division by zero is undefined.
- argument of a logaritheric function must be >0.

Graphically, the domain of 2-variable function is corresponds to a subset of the Cartesian plane.

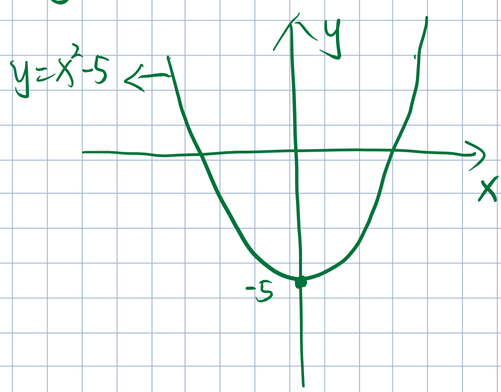
ex. Let $z = f(x,y) = \sqrt{y - x^2 + 5}$

because of $\sqrt{\ }$: $y - x^2 + 5 \geq 0$

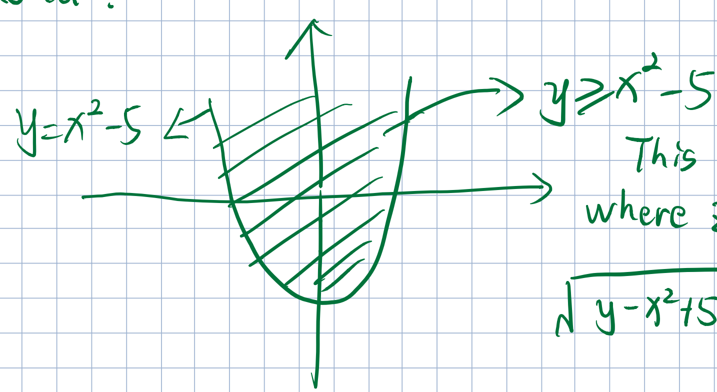
$$y \geq x^2 - 5$$

$\therefore \text{dom}(f) = \{(x,y) \in \mathbb{R}^2 \mid y \geq x^2 - 5\}$

Graphically, $y = x^2 - 5$ is a parabola:



... but $y \geq x^2 - 5$ is a region "inside and including" the parabola:



This is the region where $z = f(x,y) = \sqrt{y - x^2 + 5}$ "lives".

... all allowable x, y pairs in this region yield a number z.

ex. Let $z = f(x,y) = \dfrac{1}{y-2x-10}$

because of the devision bar: $y-2x-10 \neq 0$

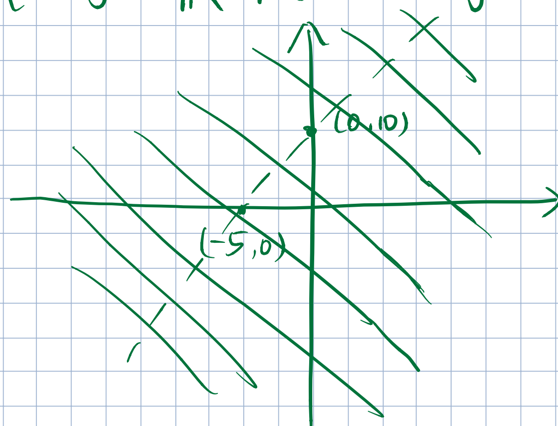
$y \neq 2x+10$

- $y = 2x+10$ (straight line): $\rightarrow$ slope of 2

$\rightarrow$ y-intercept is 10.

- $y \neq 2x+10 \rightarrow \mathbb{R}^2$, with the points on the line excluded

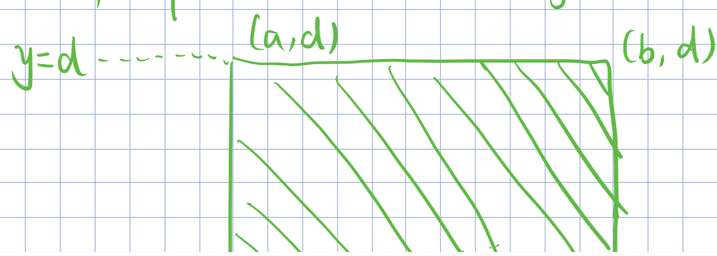
$\operatorname{dom}(f) = \{(x,y) \in \mathbb{R}^2 \mid y \neq 2x+10\}$

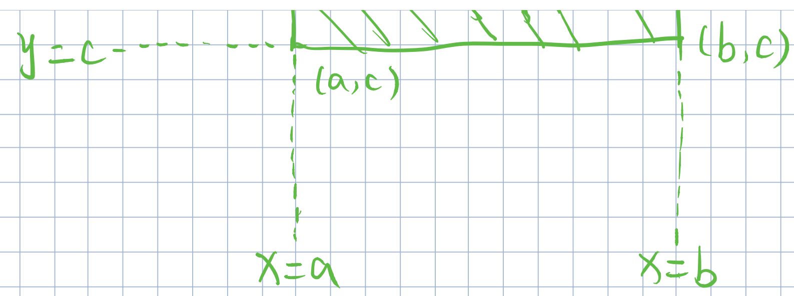


Special Case: ... where it is possible to use interval notation to represent dom(f).

$\rightarrow$ The case of a rectangle domain region

ex. Suppose the domain region is:





The domain can be described using the Cartesian multiplication of the sub-intervals $(a,b]$ and $[c,d]$.

$$\therefore \operatorname{dom}(f) = (a,b] \times [c,d]$$

ex. $z = \sqrt{x} + \dfrac{1}{y-x} - \sqrt[4]{y-x^2+10}$

3 restrictions : $\rightarrow \sqrt{}$: $x \geq 0$

$\rightarrow \dfrac{1}{y-x}$: $y \neq x$

$\rightarrow \sqrt[4]{}$: $y \geq x^2 - 10$