Document #1:

Discount and Compound Interest Factor Applications

University of Ottawa
Economics Department

Prepared by
Claude Théoret

September 2016

This document provides simple applications of single-sum, uniform and arithmetic gradient interest factors.

Financial Table						
	MARR = 10.0%					
N	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410
5	1.6105	0.6209	0.2638	3.7908	0.1638	6.1051
6	1.7716	0.5645	0.2296	4.3553	0.1296	7.7156
7	1.9487	0.5132	0.2054	4.8684	0.1054	9.4872
8	2.1436	0.4665	0.1874	5.3349	0.0874	11.4359
9	2.3579	0.4241	0.1736	5.7590	0.0736	13.5795
10	2.5937	0.3855	0.1627	6.1446	0.0627	15.9374
11	2.8531	0.3505	0.1540	6.4951	0.0540	18.5312
12	3.1384	0.3186	0.1468	6.8137	0.0468	21.3843
13	3.4523	0.2897	0.1408	7.1034	0.0408	24.5227
14	3.7975	0.2633	0.1357	7.3667	0.0357	27.9750
15	4.1772	0.2394	0.1315	7.6061	0.0315	31.7725
16	4.5950	0.2176	0.1278	7.8237	0.0278	35.9497
17	5.0545	0.1978	0.1247	8.0216	0.0247	40.5447
18	5.5599	0.1799	0.1219	8.2014	0.0219	45.5992
19	6.1159	0.1635	0.1195	8.3649	0.0195	51.1591
20	6.7275	0.1486	0.1175	8.5136	0.0175	57.2750
21	7.4002	0.1351	0.1156	8.6487	0.0156	64.0025
22	8.1403	0.1228	0.1140	8.7715	0.0140	71.4027
23	8.9543	0.1117	0.1126	8.8832	0.0126	79.5430
24	9.8497	0.1015	0.1113	8.9847	0.0113	88.4973
25	10.8347	0.0923	0.1102	9.0770	0.0102	98.3471

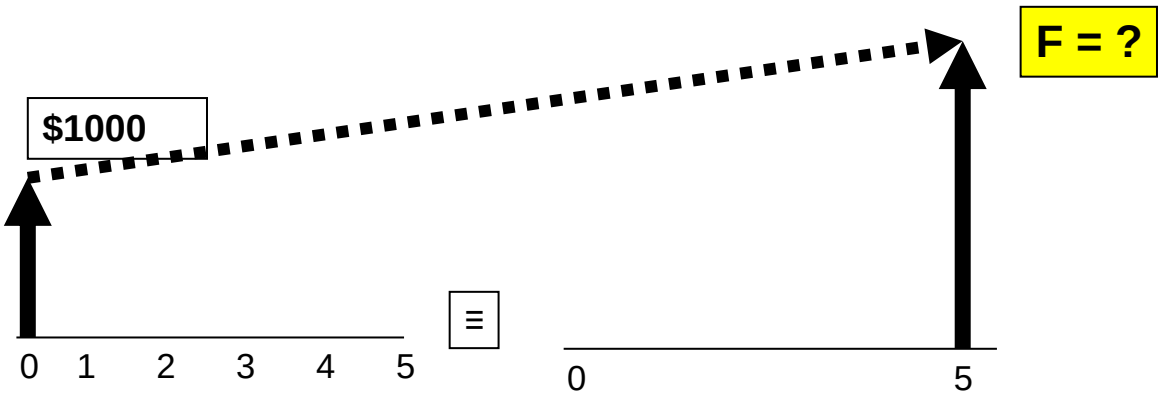
Single Sum (single cash flow): relating discount (P) and compound (F) amounts

(F/P, i%, n): Compound Amount Factor

Find the future value (F) of an immediate value (P) knowing the annual rate of interest, the frequency of compounding and the timeframe between P and F.

Example: Find the value in five (5) years of $1000 today with a 10% rate of interest compounding annually.

F = P(F/P, 10%, 5) = 1000(1.6105) = $1610.50



Financial Table

MARR(%) = 10.0%

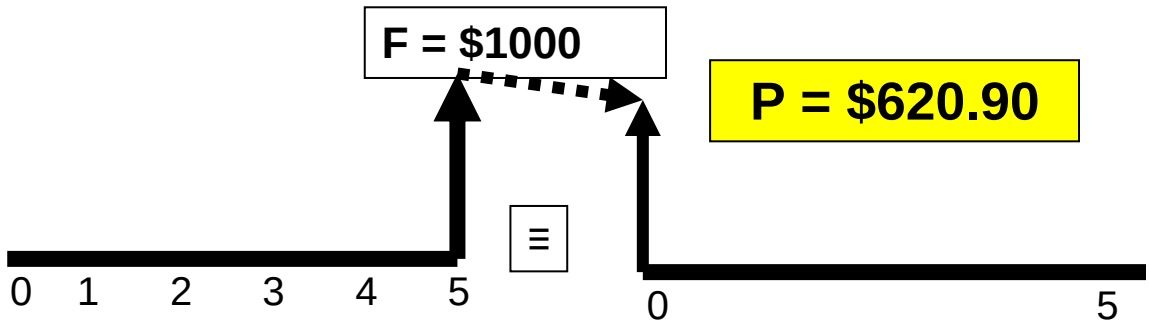
n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410
5	***1.6105***	0.6209	0.2638	3.7908	0.1638	6.1051
6	1.7716	0.5645	0.2296	4.3553	0.1296	7.7156
7	1.9487	0.5132	0.2054	4.8684	0.1054	9.4872
8	2.1436	0.4665	0.1874	5.3349	0.0874	11.4359
9	2.3579	0.4241	0.1736	5.7590	0.0736	13.5795
10	2.5937	0.3855	0.1627	6.1446	0.0627	15.9374
11	2.8531	0.3505	0.1540	6.4951	0.0540	18.5312
12	3.1384	0.3186	0.1468	6.8137	0.0468	21.3843

(P/F, i%, n): Discount Amount Factor

Find the value today (P) of a future value (F) knowing the annual rate of interest, the frequency of compounding and the timeframe between P and F.

Example: Find the value today of $1000 in 5 years today with a 10% rate of interest compounding annually.

P = F(P/F, 10%, 5) = 1000(0.6209) = $620.90



Financial Table

MARR = 10.0%

n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410
5	1.6105	***0.6209***	0.2638	3.7908	0.1638	6.1051
6	1.7716	0.5645	0.2296	4.3553	0.1296	7.7156
7	1.9487	0.5132	0.2054	4.8684	0.1054	9.4872
8	2.1436	0.4665	0.1874	5.3349	0.0874	11.4359
9	2.3579	0.4241	0.1736	5.7590	0.0736	13.5795
10	2.5937	0.3855	0.1627	6.1446	0.0627	15.9374
11	2.8531	0.3505	0.1540	6.4951	0.0540	18.5312
12	3.1384	0.3186	0.1468	6.8137	0.0468	21.3843

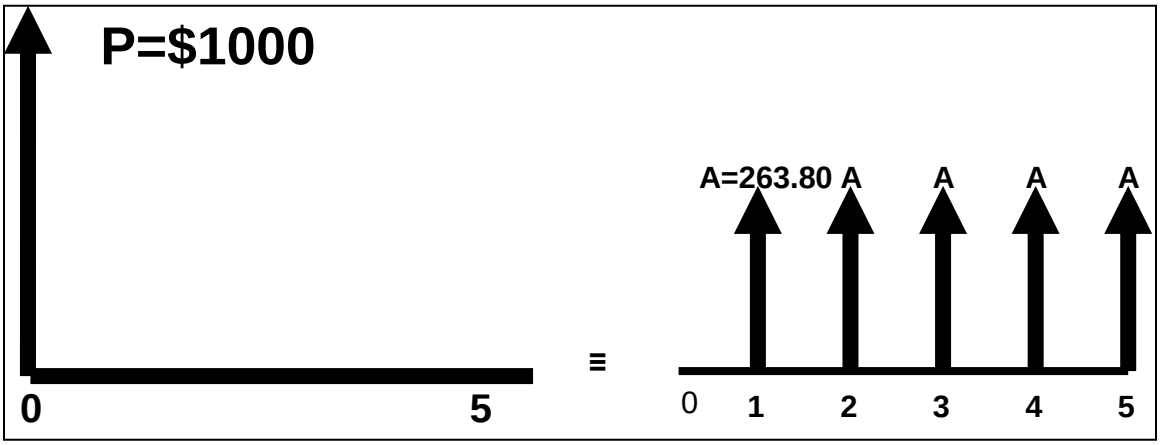
Uniform Annuities: (A/P, i%, n) = Capital Recovery Factor

Find the annual withdrawal that a bank balance (P) today can generate over "N" years given an interest rate compounded annually.

Assumption: The first withdrawal occurs in exactly one year from today.

Example: What annual withdrawal can be provided each of 5 years from a current bank balance of $1000 given a 10% rate of interest compounded annually?

A = P(A/P, 10 %, 5) = 1000(0.2638) = $263.80



Financial Table

MARR = 10.0%

n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410
5	1.6105	0.6209	***0.2638***	3.7908	0.1638	6.1051
6	1.7716	0.5645	0.2296	4.3553	0.1296	7.7156
7	1.9487	0.5132	0.2054	4.8684	0.1054	9.4872
8	2.1436	0.4665	0.1874	5.3349	0.0874	11.4359
9	2.3579	0.4241	0.1736	5.7590	0.0736	13.5795
10	2.5937	0.3855	0.1627	6.1446	0.0627	15.9374
11	2.8531	0.3505	0.1540	6.4951	0.0540	18.5312
12	3.1384	0.3186	0.1468	6.8137	0.0468	21.3843

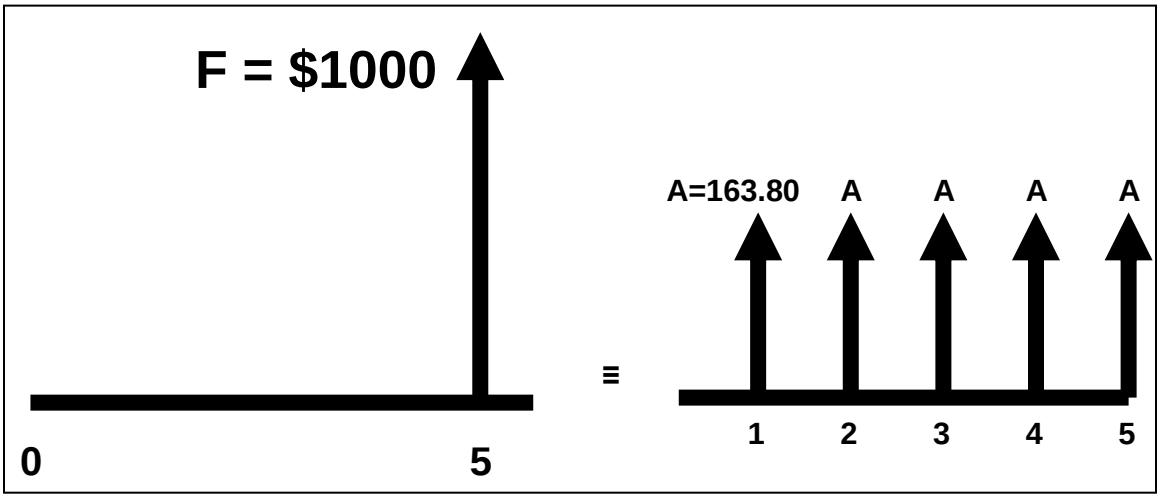
Uniform Annuities: (A/F, i%, n) = Sinking Fund Factor

Find the annual deposit required to reach a specific bank balance given a timeframe and a rate of interest compounded annually on deposits.

Assumption: The first deposit occurs in exactly one year from today.

Example: Find the annual deposit required to reach a $1000 bank balance in 5 years from today given a 10% rate of interest compounded annually?

A = F(A/F, 10%, 5) = 1000(0.1638) = $163.80



Financial Table

MARR = 10.0%

n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410
5	1.6105	0.6209	0.2638	3.7908	***0.1638***	6.1051
6	1.7716	0.5645	0.2296	4.3553	0.1296	7.7156
7	1.9487	0.5132	0.2054	4.8684	0.1054	9.4872
8	2.1436	0.4665	0.1874	5.3349	0.0874	11.4359
9	2.3579	0.4241	0.1736	5.7590	0.0736	13.5795
10	2.5937	0.3855	0.1627	6.1446	0.0627	15.9374
11	2.8531	0.3505	0.1540	6.4951	0.0540	18.5312
12	3.1384	0.3186	0.1468	6.8137	0.0468	21.3843

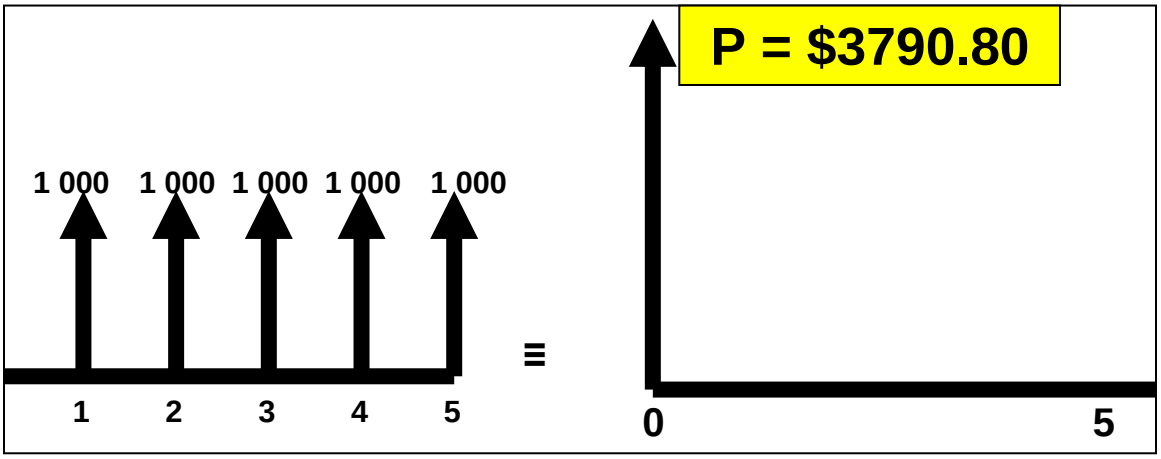
Uniform Annuities:
(P/A, i%, n) = Uniform Series Compound Amount Factor

Find the present worth (P) of an annual cash flow (A) over "N" years given an interest rate compounded annually.

Assumption: The first cash flow occurs exactly one year from today.

Example: Find the value today (Present Worth) of a uniform $1000 annual cash flow (A) at the end of each of 5 years given a 10% rate of interest compounded annually?

P = A(P/A, 10%, 5) = 1000(3.7908) = $3790.80



Financial Table

MARR = 10.0%

n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410
5	1.6105	0.6209	0.2638	***3.7908***	0.1638	6.1051
6	1.7716	0.5645	0.2296	4.3553	0.1296	7.7156
7	1.9487	0.5132	0.2054	4.8684	0.1054	9.4872
8	2.1436	0.4665	0.1874	5.3349	0.0874	11.4359
9	2.3579	0.4241	0.1736	5.7590	0.0736	13.5795
10	2.5937	0.3855	0.1627	6.1446	0.0627	15.9374
11	2.8531	0.3505	0.1540	6.4951	0.0540	18.5312
12	3.1384	0.3186	0.1468	6.8137	0.0468	21.3843

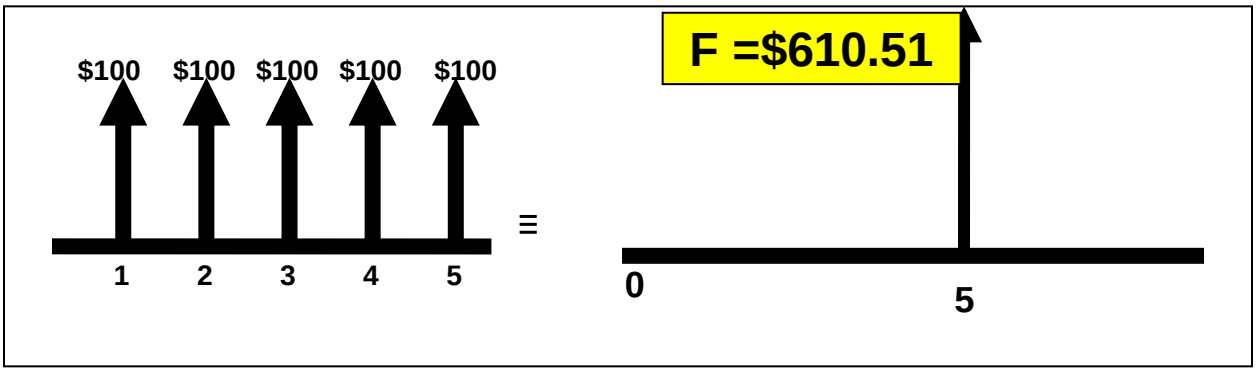
Uniform Annuities:
(F/A, i%, n) = Uniform Series Compound Amount Factor

Find the future worth (F) of an annual cash flow (A) over "N" years given an interest rate compounded annually.

Assumption: The first cash flow occurs exactly one year from today.

Example: Find the future worth in 5 years of a uniform $1000 annual cash flow (A) at the end of each year given a 10% rate of interest compounded annually?

F = A(F/A, 10%, 5) = 100(6.1051) = $610.51



Financial Table

MARR = 10.0%

n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410
5	1.6105	0.6209	0.2638	3.7908	0.1638	***6.1051***
6	1.7716	0.5645	0.2296	4.3553	0.1296	7.7156
7	1.9487	0.5132	0.2054	4.8684	0.1054	9.4872
8	2.1436	0.4665	0.1874	5.3349	0.0874	11.4359
9	2.3579	0.4241	0.1736	5.7590	0.0736	13.5795
10	2.5937	0.3855	0.1627	6.1446	0.0627	15.9374
11	2.8531	0.3505	0.1540	6.4951	0.0540	18.5312
12	3.1384	0.3186	0.1468	6.8137	0.0468	21.3843

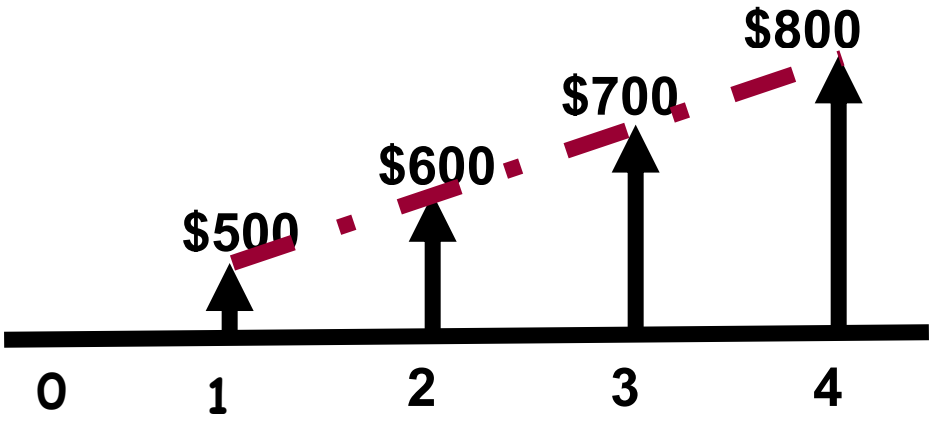
Arithmetic Gradient Series: (A/G, i%, n) = Converts an Arithmetic Gradient Series to a Uniform Series

Find the annual annuity equivalent to a series that grows (or declines) by $G annually during "N" years given an interest rate compounded annually.

Assumption: The gradient begins in the second year.

Example: What uniform annuity is equivalent to a series of cash flows beginning with $250 at EOY1 thereafter growing annually by $250 for 3 years (hence a 4-year cash flow) given a 10% rate of interest compounded annually?

A = A+G(A/G, 10%, 4) = 500+100(A/G,10%,4)= $638.12



Financial Table

MARR = 10%

n	(F/P,i%,n)	(P/F,i%,n)	(A/P,i%,n)	(P/A,i%,n)	(A/F,i%,n)	(F/A,i%,n)	(A/G,i%,n)
1	1.1000	0.9091	1.1000	0.9091	1.0000	1.0000	0.0000
2	1.2100	0.8264	0.5762	1.7355	0.4762	2.1000	0.4762
3	1.3310	0.7513	0.4021	2.4869	0.3021	3.3100	0.9366
4	1.4641	0.6830	0.3155	3.1699	0.2155	4.6410	**1.3812**
5	1.6105	0.6209	0.2638	3.7908	0.1638	6.1051	1.8101
6	1.7716	0.5645	0.2296	4.3553	0.1296	7.7156	2.2236
7	1.9487	0.5132	0.2054	4.8684	0.1054	9.4872	2.6216
8	2.1436	0.4665	0.1874	5.3349	0.0874	11.4359	3.0045
9	2.3579	0.4241	0.1736	5.7590	0.0736	13.5795	3.3724
10	2.5937	0.3855	0.1627	6.1446	0.0627	15.9374	3.7255