

MAT1330 A : Instructor Monica Nevins

Wednesday, October 4, 2017 : Test #1

Duration: 75 minutes

Family name: _____

First name: _____

Student number : _____

For picking up your graded test:

Circle the DGD you will attend to pick up your test (whether you are registered or not). Tests take at least one week to grade; watch for announcements in class.

| # : | DGD1 | DGD2 | DGD3 | DGD4 |
|-------------|---------|----------|-----------|----------|
| Day : | Tuesday | Tuesday | Wednesday | Thursday |
| Start time: | 10 am | 1 pm | 1 pm | 11:30 am |
| Room : | SMD 425 | TBT 0019 | MNT 207 | FTX 137 |
| TA : | Ruaridh | Luca | Rabib | Ruaridh |

Please read the following instructions carefully.

- You have 75 minutes to complete this exam.
- This is a closed book exam. Except for Faculty-approved calculators (models: Texas Instruments TI-30* and TI-34*, Casio FX-260* and Casio FX-300*), no notes, cell phones, smartwatches or related devices of any kind are permitted. All such devices, including cell phones, **must be stored in your bag under your desk for the duration of the exam.**
- Read each question carefully — you will save yourself time and grief later on.
- Questions 1 through 4 are multiple choice, worth 1 point each. **Record your answers to the multiple choice questions in the boxes provided.**
- Questions 5 through 9 are short answer, with number of points as indicated. **You must show your work, your work must be legible, and you must record your answers in the boxes provided.**
- Where it is possible to check your work, do so.
- Good luck!

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Marker's use only:

| Question # | Marks |
|-------------|-------|
| 1-4 (/4) | |
| 5, 6 (/4) | |
| 7 (/7) | |
| 8 (/5) | |
| 9 (/4) | |
| Total (/24) | |

1. (1 point) Suppose that a patient receives a daily dose of 50mg/L of a certain drug such that 42% of it is eliminated from the body each day. If on a certain Monday, the concentration of the drug measured in their body (shortly after the daily dose) is 55 mg/L, which of the following Discrete-Time Dynamical Systems describe the dynamics of the concentration x_t of the drug in the body (in mg/L, t days after that Monday)?

A. $x_{t+1} = .42x_t + 55$, with $x_0 = 50$

D. $x_{t+1} = .50x_t + 42$, with $x_0 = 55$

B. $x_{t+1} = .55x_t + 42$, with $x_0 = 50$

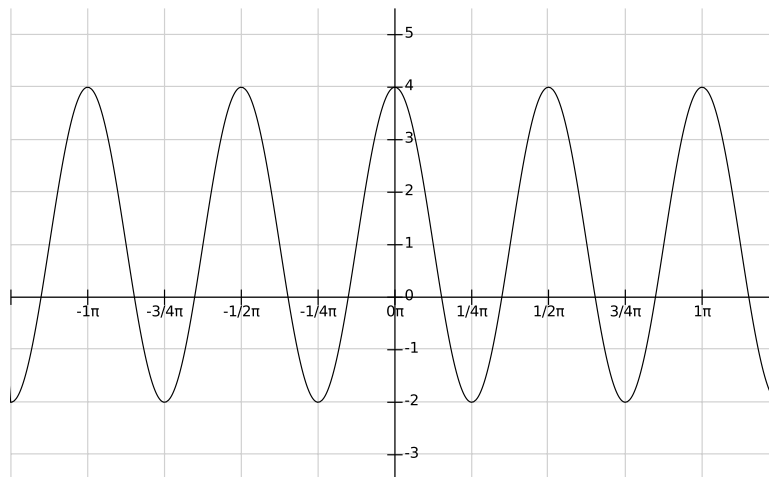
E. $x_{t+1} = .58x_t + 55$, with $x_0 = 50$

C. $x_{t+1} = .58x_t + 50$, with $x_0 = 55$

F. $x_{t+1} = 42x_t + 50$, with $x_0 = 55$

Your answer:

2. (1 point) The following is the graph of a function $y = f(x)$.



Which of the following is a formula for $f(x)$?

A. $f(x) = 3 + 2 \cos\left(2\left(x - \frac{\pi}{2}\right)\right)$

D. $f(x) = 2 + 3 \cos\left(2\left(x - \frac{\pi}{4}\right)\right)$

B. $f(x) = 3 + 3 \cos(4x)$

E. $f(x) = 1 + \cos(x)$

C. $f(x) = 1 + 3 \cos(4x)$

F. $f(x) = 1 + 4 \cos(2x)$

Your answer:

3. (1 point) Find the set of all solutions to the inequality

$$\frac{6}{x} - 8 > 4.$$

A. $(-\infty, \frac{1}{2})$

B. $(0, \infty)$

C. $(\frac{1}{2}, \infty)$

D. $(-\infty, 0)$

E. $(-\infty, 0) \cup (\frac{1}{2}, \infty)$

F. $(0, \frac{1}{2})$

Your answer:

4. (1 point) Find all x for which the following equality holds:

$$|5 - 3x^2| = 4.$$

A. $x = 0$

B. $x = 1/\sqrt{3}$

C. $x = \pm 1/\sqrt{3}$

D. $x = 2 \pm \sqrt{5/3}$

E. $x = \pm \sqrt{5/3}$

F. $x = \pm 1/\sqrt{3}$ or $x = \pm \sqrt{3}$

Your answer:

5. (2 points) Find all solutions x of the following equation. Show your work.

$$\ln(x + 1) + \ln(x - 4) = \ln(2x + 10)$$

Your work:

Your answer:

6. (2 points) For which value of the parameter a is the following function continuous at $x = 3$? Justify your answer by explaining what is required for continuity and solving for it.

$$f(x) = \begin{cases} a \cos(\pi x) & \text{if } x < 3 \\ \frac{x}{2} - 4 & \text{if } x \geq 3. \end{cases}$$

Your work:

$a =$

7. (7 points) The population of fish in Fisher's Pond grow at a steady rate annually, but fishing is so popular that, without restocking, the population would die out. Therefore Fisher's Pond is restocked with fish each spring. Using historical data, we declare that DTDS modeling the population of fish, with x_t representing the average number of fish per m^2 of surface area in year t , is given by

$$x_{t+1} = 0.7x_t + 4.8.$$

(a) (1 point) Give the updating function f for this DTDS. $f(x) =$

(b) (1 point) Find the fixed point x^* of this DTDS.

$x^* =$

(c) (1 point) Suppose that in year zero there were 36 fish/ m^2 . Give the general solution formula to this DTDS.

General solution formula:

(d) (1 point) Find the number of fish per m^2 after two years.

Your answer:

(e) (3 points) Determine the (whole) number of years necessary until the number of fish per m^2 is within 0.5 of the fixed point, that is, until $|x_t - x^*| < 0.5$. **Show your work. Your answer must be clear and well-justified to earn full marks.**

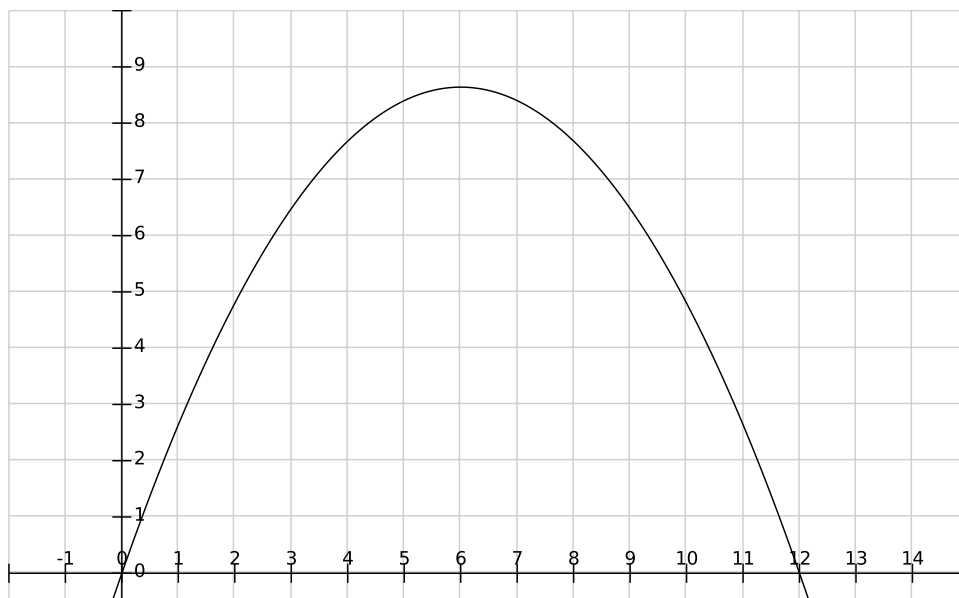
8. (5 points) The DTDS $x_{t+1} = 0.24x_t(12 - x_t)$ models a certain population.

(a) (2 points) Solve for all fixed points of the DTDS.

Your work:

Your answer:

(b) (2 points) The graph of the updating function of this DTDS is given below. Suppose the initial value is $x_0 = 2$. Draw a cobweb diagram on the graph below for this DTDS with at least 4 steps. Label the axes, the functions, the fixed points and the points $x_0 \cdots x_4$.



(c) (1 point) Write a sentence to explain what happens in the long term if $x_0 = 2$. Your sentence should include the word “stable” or “unstable”, as well as the exact fixed point from (a) which is relevant.

9. (4 points) Decide if the following limits exist. For each one, if it exists, evaluate the limit exactly using algebraic methods, showing all steps. If it does not exist, justify your answer clearly using mathematical reasoning.

(a) (2 points) $\lim_{x \rightarrow \infty} \frac{3x^3}{\sqrt{5x^6 - 4x}}$.

Your answer:

(b) (2 points) $\lim_{x \rightarrow 4} \frac{3 - \sqrt{25 - x^2}}{x - 4}$

Your answer: